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MODELLING OF ACOUSTIC HEATING INDUCED BY DIFFERENT TYPES OF SOUND

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Dynamic equation governing acoustic heating is derived by splitting of the conservation laws into acoustic and non-acoustic parts. Numerical simulations result in the general conclusions about efficiency of acoustic heating produced by pulses of different polarity and shape. Efficiency of heating induced by stochastic and regular periodic sound of the identical intensity is numerically investigated.

Keywords: nonlinear acoustics, acoustic heating, the Burgers equation.

1. Introduction

Linear projecting of the total hydrodynamic perturbation into different specific types of motion may be provided algorithmically in many problems. The initial point in the studies of fluid dynamics is the determination of eigenvectors (modes) of a linearized form of the Navier–Stokes equations. Every mode specifies the time-independent links of velocity and two thermodynamic variables (excess pressure and density, for example) for every type of possible motion in a fluid: acoustic, vortical, and entropy ones [1]. Dynamics of any mode in the linear flow is independent of other modes.

By use of properties of the modes, the weakly nonlinear flow may be successfully investigated. The conservation system splits into nonlinear dynamic equations governing every specific mode. The number of equations is equal exactly to the number of modes. The planar flow specifies three dynamic equations, two for the rightwards and leftwards progressing acoustic waves, and one for the entropy mode. In the case of weakly nonlinear flow, the dynamic equations are coupled, including nonlinear terms of all modes. The problem of acoustic heating presupposes the dominative sound, and comparatively small entropy produced by it. Over the time domain, where it holds true, the growth of entropy is governed by the heat transfer equation with an acoustic quadratic source. The sound itself should satisfy the Burgers dynamic equation [2], so that the problem looks fairly complex. Origin of acoustic heating are nonlinear losses in acoustic energy, since it is essentially a thermoviscous nonlinear phenomenon.

The conventional theory of acoustic heating deals exclusively with periodic sound. It uses periodicity as the condition of the energy balance equation, splitting into acoustic and non-acoustic parts while averaging over the period of sound [2, 3]. The temporal averaging subdivides "slow" (entropy) and "quick", progressive (acoustic) modes in a consistent manner, but rigorously applies only to periodic sound. It does not allow to study the delicate temporal structure of the heating dynamics. Moreover, in the three-dimensional flow vortex modes do appear, also "slow", so that the validity of subdivision needs additional justification.

The advantage of the local in time subdivision is obvious. It makes it possible to distinguish parts of the overall increase in heat, corresponding to the acoustic and entropy modes. In spite of difficulty of the problem, numerical simulations basing on the new dynamic equation allow to draw general conclusions about efficiency of heating induced by different types of sound. The resulting equation may be averaged over any time domain in order to verify the experimental data. Some analytical solutions of the Burgers equation governing the sound in the thermoviscous nonlinear flow, are investigated as the role of acoustic source of heating in the Subsec. 2.1 below. The importance of taking into account of thermal conductivity is pointed out in this subsection.

2. Dynamic equations of acoustic instantaneous heating

The starting point is a set of hydrodynamic equations of the plane flow in the differential form:

$$
\begin{aligned}
\rho\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}\right)+\frac{\partial p}{\partial x} &= \frac{4\mu}{3}\frac{\partial^2 v}{\partial x^2},\\
\rho T\left(\frac{\partial S}{\partial t}+v\frac{\partial S}{\partial x}\right) &= \chi\frac{\partial^2 T}{\partial x^2}+\frac{4\mu}{3}\left(\frac{\partial v}{\partial x}\right)^2,\\
\frac{\partial \rho}{\partial t}+\frac{(\partial \rho v)}{\partial x} &= 0,
\end{aligned}
\tag{1}
$$

where ρ, p, v, S, T denote density, pressure, velocity of fluid, entropy per unit volume and temperature, respectively, x, t are spacial co-ordinate and time, and χ, μ denote thermal conductivity and viscosity, both supposed to be constants.

The system (1) should be completed by the equations of state. In the present investigation, only ideal gases will be considered. Internal energy and temperature of an ideal gas are functions of ρ, p as follows (C_v and C_p denote heat capacity under constant volume and pressure per unit mass, $\gamma = C_p/C_v$ denotes the ratio of specific heats):

$$
e = C_v T = \frac{p}{(\gamma-1)\rho}. \tag{2}
$$

Using the dimensionless quantities

$$p' = \frac{p - p_0}{c_0^2 \cdot \rho_0}, \quad \rho' = \frac{\rho - \rho_0}{\rho_0}, \quad v' = \frac{v}{c_0}, \tag{3}$$

$x' = x/\lambda$, $t' = tc_0/\lambda$, with $\lambda, \rho_0, p_0, c_0 = \sqrt{\gamma p_0/\rho_0}$ being respectively the characteristic scale of a flow, unperturbed density and pressure, and infinitely small signal sound velocity, one obtains the following system (4):

$$\begin{gathered}
\frac{\partial v}{\partial t} + \frac{\partial p}{\partial x} - \delta_1 \frac{\partial^2 v}{\partial x^2} = -v\frac{\partial v}{\partial x} + \rho\frac{\partial p}{\partial x} - \delta_1 \rho \frac{\partial^2 v}{\partial x^2}, \\
\begin{aligned}
\frac{\partial p}{\partial t} + \frac{\partial v}{\partial x} - \frac{\delta_2}{\gamma - 1}\frac{\partial^2(\gamma p - \rho)}{\partial x^2} = -v\frac{\partial p}{\partial x} - \gamma p \frac{\partial v}{\partial x} + \delta_1(\gamma - 1)\left(\frac{\partial v}{\partial x}\right)^2 \\
- \frac{\delta_2}{\gamma - 1}\frac{\partial^2(\gamma p\rho - \rho^2)}{\partial x^2},
\end{aligned} \\
\frac{\partial \rho}{\partial t} + \frac{\partial v}{\partial x} = -v\frac{\partial \rho}{\partial x} - \rho\frac{\partial v}{\partial x}.
\end{gathered} \tag{4}$$

The system of Eqs. (4) is equivalent to the initial one (1) with accuracy up to the quadratic nonlinear terms (including those standing at the dissipative coefficients), that are of the major importance in weakly nonlinear acoustics. Expansion in the Taylor series of the term $(1+\rho)^{-1}$ was performed, convergent for small Mach numbers or, equivalently, for weakly nonlinear flows. In the system (4) and everywhere below in the text, primes at perturbations are dropped. Among the already mentioned quantities, $\delta_1 = \dfrac{4\mu}{3\rho_0 c_0 \lambda}$, $\delta_2 = \dfrac{\chi}{\rho_0 c_0 \lambda}\left(\dfrac{1}{C_v} - \dfrac{1}{C_p}\right)$ are dimensionless dissipative (viscous and thermal, relatively) coefficients. Three eigenvectors of the matrix operator of linear evolution take the form [4, 5]:

$$\begin{gathered}
\psi_{a,1} = \begin{pmatrix} 1 - \dfrac{\beta}{2}\partial/\partial x \\ 1 - \delta_2 \partial/\partial x \\ 1 \end{pmatrix} \rho_{a,1}, \\
\psi_{a,2} = \begin{pmatrix} -1 - \dfrac{\beta}{2}\partial/\partial x \\ 1 + \delta_2 \partial/\partial x \\ 1 \end{pmatrix} \rho_{a,2}, \\
\psi_e = \begin{pmatrix} -\dfrac{\delta_2}{\gamma - 1}\partial/\partial x \\ 0 \\ 1 \end{pmatrix} \rho_e,
\end{gathered} \tag{5}$$

where $\psi = \begin{pmatrix} v & p & \rho \end{pmatrix}^{\mathrm{T}}$ is a vector of perturbations, $\beta = \delta_1 + \delta_2$ denotes the total attenuation coefficient. These three eigenvectors manifest the existence of the three basic types of the planar flow: two first ones are acoustic, rightwards and leftwards progressing ones, and the third one is the entropy mode. The specific excess densities $\rho_{a,1}$, $\rho_{a,2}$, ρ_e determine the overall dimensionless perturbations ρ, p, v uniquely:

$$\begin{aligned} v(x,t) &= \rho_{a,1}(x,t) - \rho_{a,2}(x,t) - \frac{\beta}{2}\frac{\partial \rho_{a,1}(x,t)}{\partial x} - \frac{\beta}{2}\frac{\partial \rho_{a,2}(x,t)}{\partial x} \\ &\qquad - \frac{\delta_2}{\gamma-1}\frac{\partial \rho_e(x,t)}{x}, \\ p(x,t) &= \rho_{a,1}(x,t) + \rho_{a,2}(x,t) - \delta_2\frac{\partial \rho_{a,1}(x,t)}{\partial x} + \delta_2\frac{\partial \rho_{a,2}(x,t)}{\partial x}, \\ \rho(x,t) &= \rho_{a,1}(x,t) + \rho_{a,2}(x,t) + \rho_e(x,t). \end{aligned} \tag{6}$$

The dimensionless linear velocity of every mode in the non-viscous flow follows from the linear dispersion relation: $c_{a,1} = -c_{a,2} = 1$, $c_e = 0$.

Let the overall perturbation consist of a rightwards progressing acoustic mode determined by ρ_a, and the entropy mode which is a secondary mode with amplitude much less than that of sound. Links for the rightwards progressing mode should be corrected by involving of quadratic nonlinear terms specific for the Riemann wave [2, 5, 6]. These terms make the sound quasi-isentropic within the corresponding accuracy. The corrected links take the form as follows:

$$\begin{aligned} v(x,t) &= \rho_a(x,t) + \frac{\gamma-3}{4}\rho_a^2(x,t) - \frac{\beta}{2}\frac{\partial \rho_a(x,t)}{\partial x} - \frac{\delta_2}{\gamma-1}\frac{\partial \rho_e(x,t)}{\partial x}, \\ p(x,t) &= \rho_a(x,t) + \frac{\gamma-1}{2}\rho_a^2(x,t) - \delta_2\frac{\partial \rho_a(x,t)}{\partial x}, \\ \rho(x,t) &= \rho_a(x,t) + \rho_e(x,t). \end{aligned} \tag{7}$$

Using Eqs. (7), two dynamic equations follow from Eqs. (4). The first one governs the entropy mode:

$$\begin{aligned} -\frac{\partial \rho_e}{\partial t} + \frac{\delta_2}{\gamma-1}\frac{\partial^2 \rho_e}{\partial x^2} &= \delta_2 \rho_a \frac{\partial^2 \rho_a}{\partial x^2} + ((\gamma-1)\delta_1 + \gamma\delta_2)\left(\frac{\partial \rho_a}{\partial x}\right)^2 \\ &\qquad + \delta_2 \frac{(\gamma-5)}{4}\frac{\partial^2 \rho_a^2}{\partial x^2}. \end{aligned} \tag{8}$$

Acoustic excess density $\rho_a(x,t)$ on the right-hand side, satisfies the Burgers equation which also is a result of splitting:

$$\frac{\partial \rho_a}{\partial t} + \frac{\partial \rho_a}{\partial x} + \frac{\gamma+1}{2}\rho_a\frac{\partial \rho_a}{\partial x} - \frac{\beta}{2}\frac{\partial^2 \rho_a}{\partial x^2} = 0. \tag{9}$$

Only quadratic nonlinear terms are considered in the instantaneous equations (8), (9). They are valid over the time domain, when amplitude of sound $\rho_{a,0}$ is much larger than that of the entropy mode: $\rho_{e,0}$. Otherwise, both equations should be corrected in view of the growing role of the entropy mode. Nonlinear interactions induce the leftwards increasing sound, so it should be taken into account at later stages of the evolution.

The irreversible growth of integral entropy follows from the second equation of the set of Eqs. (1). Dimensional total heat release in unit mass per unit time Q is equal to:

$$\begin{aligned} Q_{\text{total, dim}} &= T\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{c^3}{\lambda}\left(Q_e + Q_a\right) \\ &= \frac{c^3}{\lambda}\frac{\delta_1}{\text{Pr}}\left(\frac{\partial^2\rho_a}{\partial x^2} - \frac{1}{\gamma-1}\frac{\partial^2\rho_e}{\partial x^2} + (\gamma-3)\rho_a\frac{\partial^2\rho_a}{\partial x^2} + (\gamma+\text{Pr}-2)\left(\frac{\partial\rho_a}{\partial x}\right)^2\right), \end{aligned} \tag{10}$$

where Q_e, Q_a are dimensionless quantities associated with entropy and acoustic excess densities:

$$\begin{aligned} Q_e &= -\frac{1}{(\gamma-1)}\frac{\partial\rho_e}{\partial t} \\ &= \frac{\delta_1}{\text{Pr}}\left(\frac{\gamma-3}{2}\rho_a\frac{\partial^2\rho_a}{\partial x^2} - \frac{1}{\gamma-1}\frac{\partial^2\rho_e}{\partial x^2} + \frac{3\gamma+2\text{Pr}-5}{2}\left(\frac{\partial\rho_a}{\partial x}\right)^2\right) \\ &\approx \frac{\delta_1}{\text{Pr}}\left(\frac{\gamma-3}{2}\rho_a\frac{\partial^2\rho_a}{\partial x^2} + \frac{3\gamma+2\text{Pr}-5}{2}\left(\frac{\partial\rho_a}{\partial x}\right)^2\right), \end{aligned} \tag{11}$$

$$Q_a = \frac{\delta_2}{(\gamma-1)}\left(\frac{\partial^2\rho_a}{\partial x^2} - \frac{\gamma-1}{2}\left(\frac{\partial\rho_a}{\partial x}\right)^2 + \frac{\gamma-3}{2}\rho_a\frac{\partial^2\rho_a}{\partial x^2}\right). \tag{12}$$

While going to the last line of Eq. (11), the small term $\frac{1}{\gamma-1}\frac{\partial^2\rho_e}{\partial x^2}$ is ignored. Pr is the Prandtl number:

$$\text{Pr} = \frac{(\gamma-1)\delta_1}{\delta_2}. \tag{13}$$

The right-hand side of Eq. (8) confirms the nonlinear thermoviscous origin of acoustic heating. It is remarkable that the irreversible increase in total entropy in its acoustic part Q_a is proportional exclusively to the thermal conductivity. The meaning of "acoustic heating" denotes the isobaric increase of entropy, so that it associates with Q_e. It is useful to write the dimensional heat release in unit volume per unit time $Q_{e,\text{dim}}$ in terms of dimensional velocity of the rightward acoustic wave V_a:

$$Q_{e,\text{dim}} = \frac{\zeta+4/3\eta}{\text{Pr}}\left(\frac{\gamma-3}{2}V_a\frac{\partial^2 V_a}{\partial x^2} + \frac{3\gamma+2\text{Pr}-5}{2}\left(\frac{\partial V_a}{\partial x}\right)^2\right). \tag{14}$$

2.1. Efficiency of acoustic heating

The formula (11) permits to study efficiency of heating caused by any type of sound which, on the other hand, must satisfy the nonlinear Burgers equation (9) itself. What kind of pulses would stimulate more (or less) effective heating? How to reduce it? The answers may be helpful in biological, medical and technical applications of ultrasound. In Subsec. 2.1.1, 2.1.2 below, we investigate heating produced by some known analytical solutions of the Burgers equation (the pulse self-similar waveform, and the periodic stochastic and regular sound).

2.1.1. The self-similar acoustic pulses

Both pulses of positive and negative polarity are explicit solutions of (9) depending on the parameter C [2]:

$$\rho_a(x,t) = -\frac{4\beta}{\sqrt{\xi+\xi_0}} \cdot \frac{\exp\left(-\dfrac{(\tau+\tau_0)^2}{2(\xi+\xi_0)}\right)}{(\gamma+1)\sqrt{2\pi}\left(C - \mathrm{Erf}\left(\dfrac{(\tau+\tau_0)^2}{2(\xi+\xi_0)}\right)\right)}, \tag{15}$$

where $\xi = \beta x$, $\tau = t - x$, and ξ_0, τ_0 are constants which, without any loss of generality, may be assumed to be zero. Absolute value and sign of the constant C determine the shape and sign of a pulse, respectively. The self-similar waveform (15) is singular at $t = 0$.

We examine the ratio $\Phi(C,t)$ of the integral heating in the unit cross-section per unit time, q_e, and the energy of acoustic wave in the unit cross-section at time $t = 1$, $\int\limits_0^\infty \rho_a^2(x,t=1)\mathrm{d}x$:

$$q_e(t) = \int\limits_0^\infty Q_e \mathrm{d}x = -\frac{1}{\gamma-1}\int\limits_0^\infty \frac{\partial \rho_e}{\partial t}\mathrm{d}x, \quad \Phi(C,t) = \frac{q_e(t)}{\int\limits_0^\infty \rho_a^2(x,t=1)\mathrm{d}x}. \tag{16}$$

The dimensionless energy of acoustic pulse in the unit cross-section in the leading order is equal to $\dfrac{1}{2}\int\limits_0^\infty (p_a^2(x,t=1) + v_a^2(x,t=1))\mathrm{d}x = \int\limits_0^\infty \rho_a^2(x,t=1)\mathrm{d}x$.

Figure 1a demonstrates a set of six pulses as functions of x at $t = 1$, three positive and three negative ones corresponding to different values of C. A positive quantity of C yields a negative pulse, and vice versa. Ratios $r(C,t) = \Phi(C=-\eta,t)/\Phi(C=\eta,t)$ as functions of t for three different values of η are presented in the Fig. 1b. It shows that efficiency of heating is considerably larger being produced by a negative pulse, and that the difference increases for smaller $|C|$, i.e., for more asymmetric shapes of a pulse. The relative efficiency r is greater than 1 for all $t > 1$ and tends to 1 for large t. Examinations of relative efficiency of pure positive or pure negative pulses give the similar results: more effective is a pulse with smaller value of $|C|$. Figure 1 refers to quantities $\gamma = 1.4$, $\beta = 0.1$. Examinations undertaken by the author show that general conclusions hold true for any other values.

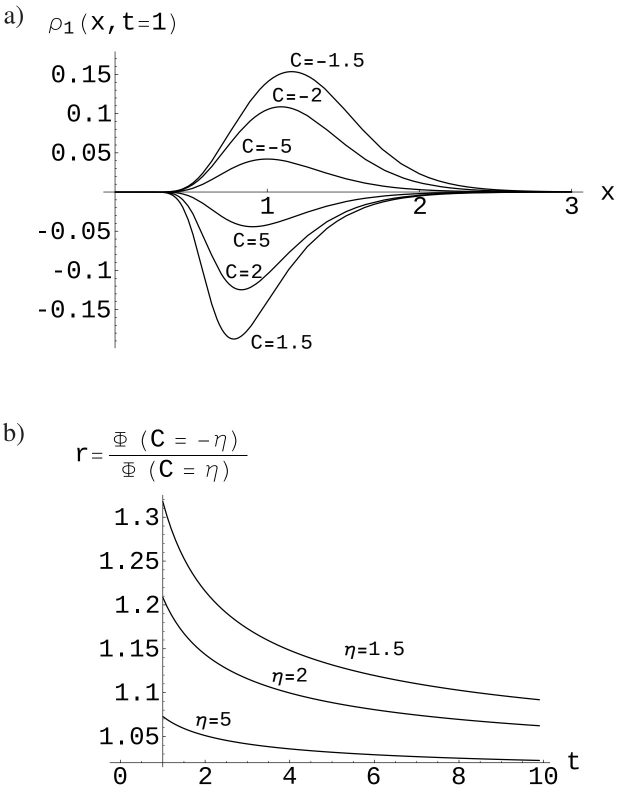

Fig. 1. a) Excess density of acoustic pulse $\rho_a(x, t = 1)$ corresponding to the following quantities of C: $1.5, 2, 5, -5, -2, -1.5$, from the lower curve to the upper one. b) A ratio $r(\eta, t) = \Phi(C = -\eta, t)/\Phi(C = \eta, t)$ for values of η as follows: 5 (lowest curve), 2 (middle curve) and 1.5 (upper curve). $\Phi(C, t) = q_e / \int\limits_0^\infty \rho_a^2(x, t = 1)\mathrm{d}x$ denotes the efficiency of heating.

It might be erroneously concluded that a difference in attenuation of pulses themselves stipulates rigorously a difference in the efficiency of relative acoustic heating. Numerical simulations of pulse dynamics shows that relative attenuation of any acoustic pulse

$$\Phi_a(t) = \frac{\int\limits_0^\infty \rho_a^2(x, t)\mathrm{d}x}{\int\limits_0^\infty \rho_a^2(x, t = 1)\mathrm{d}x} \tag{17}$$

depends neither on value of C, nor on value of β. Figure 2 represents Φ_a as a function of time.

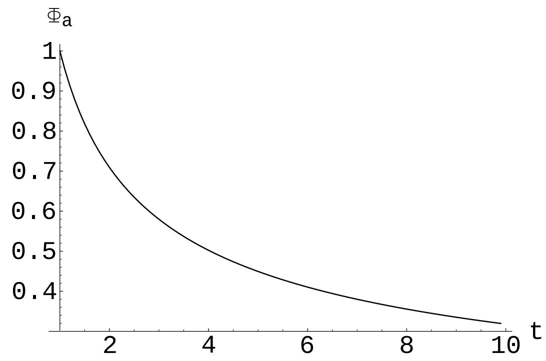

Fig. 2. Relative attenuation of acoustic pulse $\Phi_a = \int\limits_0^\infty \rho_a^2(x,t)\mathrm{d}x / \int\limits_0^\infty \rho_a^2(x,t=1)\mathrm{d}x$ as a function of time.

In spite of the uniformity of relative attenuation of any pulse independently of β and C, the efficiency of heating caused by different pulses is different. The reason is that the acoustic heating is an integral of the rather complex quadratic function of excess density in the right-hand side of (11), which includes second-order spatial derivatives.

2.1.2. Periodic stochastic and regular waveforms

The solution of the Burgers equation (9) known to be valid over the domain of waveform stabilization, where nonlinear and dissipative distortions hold equilibrium, is the FAY solution [7]. The dimensionless perturbation of velocity in the Fay waveform is:

$$v_a(\theta, z) = \frac{2A}{(\gamma+1)\mathrm{Re}} \sum_{n=1}^{\infty} \frac{\sin(n\theta + n\varphi)}{\sinh\left(\dfrac{n(1+Az)}{\mathrm{Re}(\gamma+1)}\right)}, \tag{18}$$

where $A(\theta) = v_{a,0}/\sigma$ is the dimensionless amplitude with $\sigma = \sqrt{\langle v_a^2 \rangle}$ being the mean square value of amplitude $v_{a,0}$ over the ensemble, $\theta = t - x$ is the retarded time, $\mathrm{Re} = v_{a,0}/\beta$ is the Reynolds number, $z = \dfrac{\gamma+1}{2}\sigma x$; all the listed quantities are dimensionless.

We consider two types of sound: stochastic and regular. Regular waveform has constant phase φ, amplitude $A = 1$ and $\sigma = v_{a,0} = \mathrm{M}$ (M is the Mach number of flow). Stochastic Gaussian stationary sound possesses amplitude and angle probability functions as follows:

$$W(A) = A\exp(-A^2/2), \quad W(\varphi) = 1/2\pi. \tag{19}$$

In numerical calculations of (11) with a sound velocity given by (18), we use the equality of quantities ρ_a and v_a in the leading order, and the identity $\langle \sin^2((n\theta + n\varphi)\rangle = 0.5$ for any natural n. Square brackets mean averaging over a period. The final series in the right-hand side of the Eq. (11) is quickly convergent. We stop calculations at the tenth partial sum.

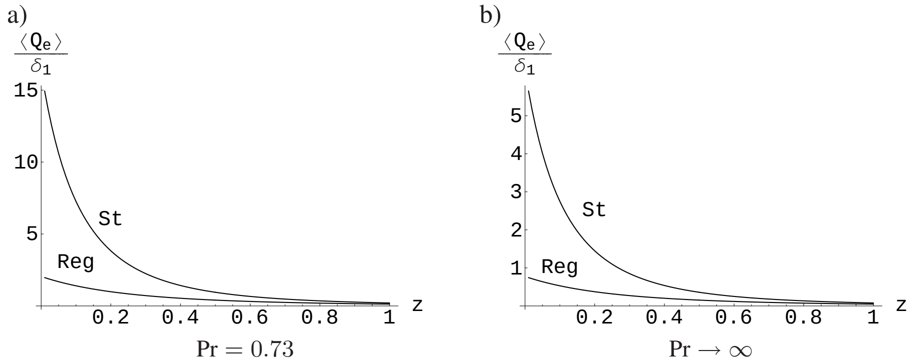

Fig. 3. Heating produced by stochastic (upper curve) and regular (lower curve) periodic sound Q_e, divided by viscosity δ_1, as a function of $z = \dfrac{\gamma+1}{2}\sigma x$; x denotes dimensionless distance from a transducer. Calculations are in accordance to the formula (11) with the acoustic velocity (18).

Figure 3a, b represents results of numerical simulations for the air with Prandtl number Pr $= 0.73$. Nonlinearity specific for the air is used: $\gamma = 1.4$. To reveal importance of taking into account the thermal conductivity, figure b corresponds to the zero thermal conductivity ($\delta_2 = 0$). Formally, in this case Pr $\to \infty$. The heating produced by the stochastic sound is considerably more effective than the one produced by the regular sound of the same energy. That agrees with the general conclusions of nonlinear acoustics proving that the nonlinear interactions are more effective for the stochastic sound than for the regular one with a similar initial energy [2]. Simulations demonstrate the importance of thermal part of the total damping, especially for the stochastic sound: accounting for thermal conductivity leads to considerably larger heating. The Fig. 3 reveals that a difference is not so noticeable for the regular sound. Calculations refer to the value of Reynolds number Re $= 100$. Analysis shows that the results are only slightly dependent on Re.

3. Conclusions

By means of projecting, the initial system of conservation equations for the planar flow splits into dynamic equations governing propagation of the dominant sound and heating induced by it.

Studies of the acoustic heating is a fairly difficult problem. The acoustic excess density in the right-hand side of the basic Eq. (11) must satisfy the nonlinear Burgers equation (9). The well-known Cole–Hopf transformations leads (9) to the linear parabolic equation which may be analytically solved. The problem may be simplified in the two limiting cases: when nonlinearity is essentially smaller than absorption and, vice versa, if it is relatively large. In the first case, Eq. (9) transforms to the equation of thermal conductivity by use of the retarded time $\tau = t - x$:

$$\frac{\partial \rho_a}{\partial \tau} = \frac{\beta}{2}\frac{\partial^2 \rho_a}{\partial x^2} \tag{20}$$

which may be solved analytically knowing the initial and boundary conditions. For initial density perturbation $\rho_a(x, \tau = 0)$ at the whole axis $X = (-\infty, \infty)$, the solution is:

$$\rho(x, \tau) = \int_{-\infty}^{\infty} \rho(\xi, 0) G(x - \xi, \tau) \mathrm{d}\xi, \tag{21}$$

where $G(x, t) = \dfrac{1}{\sqrt{2\pi\beta t}} \exp\left(-\dfrac{x^2}{2\beta t}\right)$ is the Green function.

In the second limiting case of large nonlinearity, the Burgers equation transfers to the Earnshaw equation, the solution of which depends on the initial condition in the following manner:

$$\rho_a(x, t) = F\left(x - \left(1 + \frac{\gamma + 1}{2}\rho_a(x, t)\right) t\right), \tag{22}$$

where $F(x) = \rho_a(x, t = 0)$ is the initial profile. From the point of view of giving illustrations of relative heating, this case is obviously considerably easier that the first one. Note that Eq. (22) is no longer true after shock formation. Both waveforms, for weak or strong relative nonlinearity, may be used in evaluations of heating in the suitable problem.

Fortunately, there exists a simple self-similar solution of the Burgers equation (15) which may be immediately used as acoustic source. In the present investigation, the heating caused by regular sound which is the self-similar solution of (9), is examined. Negative pulses are found to produce more effective heating.

For the purpose to study the heating following the stochastic sound, the solution (18) is most convenient and simple. It is valid in the domain of equilibrium of nonlinear and viscous phenomena. Periodic stochastic sound leads to considerably larger heating than the regular one of the comparative energy. This conclusion is analogous to the well-known property of stochastic sound to produce high harmonics more effectively than the regular one of the same intensity [2]. The importance of taking into account of thermal conductivity in the production of heating should be stressed, especially for gases and metallic liquids.

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