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AXIALLY SYMMETRIC PERIODICAL WAVE PROCESSES IN A ROTATING ELASTIC HOLLOW CIRCULAR CYLINDER SURROUNDED BY AN ACOUSTIC MEDIUM

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In this paper the spectral characteristics of a strain-stress state of an infinitely long elastic hollow circular cylinder with arbitrary thickness rotating about its axis of symmetry with time-dependent angular velocity is investigated. It is assumed that a cylinder is empty inside and surrounded by ideal (non-viscous) compressible fluid or gas. The exact solution of this problem is obtained using the Fourier transform with respect to time. Calculations are carried out for a case of the Armco iron tube immersed in water. The detailed analysis of temporal and spatial spectral characteristics of the elastic displacements and stresses, the material density and the power flow density averaged in the period of vibrations in elastic cylinder, are presented.

Keywords: rotating elastic cylinder, acoustic medium, strain-stress wave process, spectral characteristics, power flow density.

1. Introduction

Many details of such type as a hollow elastic circular cylinder rotating around its axis of symmetry at a variable angular velocity are often encountered in the technology. These bodies as a rule are surrounded by fluid or gas. Since the angular velocity is varying in time, the sound waves (noise) are radiated. The spectral characteristics of these waves have resonance properties [1]. The stresses and displacements in the rotating elastic cylinder have analogous resonance characteristics [2]. The re-reflection of elastic waves on the inside and outside surfaces of the tube and sound radiation into the surrounding medium can be a cause for complicated distributions of displacements and stresses in this solid. Therefore, it is important to take into account the resonance ranges of frequencies, where the strain-stress state of cylinder is considerably different

from its static state, because amplitudes of vibrations can reach the critical values. In real construction systems these elements and the surrounding acoustical medium occupy a finite volume. The investigation of the wave process formation is so complicated that solutions become unclear and do not permit to present the main mechanisms of wave generation for the conditions mentioned above. Therefore, in this paper we consider the simplified mathematical model of that problem.

A non-uniform rotation of elastic cylinder or cylindrical tube in vacuum was studied first in the papers [3, 4], but without detailed numerical analysis. Many works were devoted to rotation of the elastic cylinders and disks in vacuum with constant angular velocity. First solution of this problem was given long time ago by LOVE [5], nevertheless new questions concerning rotary motions of elastic bodies appeared in the field of vision of many authors now (see, e.g. [6–11]).

In this paper we study the dynamical plane strain-stress state of hollow circular cylinder of infinite length rotating in an infinite space, occupied by ideal (non-viscous) compressible fluid or gas. It is, in fact, the second part of paper [12], where the spectral characteristics of the sound waves radiated by a hollow circular elastic cylinder were studied. The particular case of solid cylinder analysis of spectral characteristics of strain-stress state was carried out in [13], and the pulse sound radiation and stresses in the body were studied in [14, 15].

2. Spectral characteristics of the displacement and stresses

An elastic circular hollow cylinder of infinite length, surrounded by a compressible ideal (non-viscous) fluid or gas, rotates with time-varying angular velocity around its axis; as a result, non-uniform centrifugal force is excited as a function of time too [16]. Since the rotating body is in contact with acoustic medium, the centrifugal force plays the role of a source of elastic wave propagation in the cylinder and sound radiation in the exterior space. So, the body and the surrounding liquid have mutual influence on the acoustic wave formation. In this article, main attention is focused on the processes generated in the cylinder material.

The dynamical equilibrium of an elastic hollow cylinder rotating with variable angular velocity about a fixed axis is described by means of the differential equation [3, 4]:

$$(\lambda+2\mu)\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right)+\rho_s^0 r\Omega^2(t)=\rho_s^0\frac{\partial^2 u}{\partial t^2}\quad (b<r<a), \tag{1}$$

where $u\equiv u(r,t)$ is the radial displacement, $\Omega(t)$ is the angular velocity of the body axial rotation, t is the time, λ, μ are the Lamé elastic parameters and ρ_s^0 is the density of elastic material in equilibrium state, r is the radial polar co-ordinate with an origin on the axis of symmetry, a, b ($b<a$) are the radii of the cylinder surfaces.

The radial $\sigma_r(r,t)$, hoop $\sigma_\theta(r,t)$ and axial $\sigma_z(r,t)$ stresses in the circular cylinder are determined from relations [3, 4, 16]:

$$\sigma_r=\lambda e+2\mu e_r,\qquad \sigma_\theta=\lambda e+2\mu e_\theta,\qquad \sigma_z=\lambda e, \tag{2}$$

where $e_r(r,t)$, $e_\theta(r,t)$ are the radial and hoop components of the tensor of deformation, respectively:

$$e_r = \frac{\partial u}{\partial r}, \qquad e_\theta = \frac{u}{r}, \qquad e = e_r + e_\theta. \tag{3}$$

The pressure amplitude in acoustic fluid $p(r,t)$ is determined by the wave equation [17]

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \qquad (a < r < \infty), \tag{4}$$

where c is the sound velocity. The pressure $p(r,t)$ is connected with radial displacement of the particles $w(r,t)$ in acoustic medium by relation

$$\frac{\partial^2 w}{\partial t^2} + \frac{1}{\rho^0}\frac{\partial p}{\partial r} = 0 \qquad (a \le r < \infty), \tag{5}$$

where ρ^0 is the equilibrium density of liquid.

On the interface of two media, the solutions must satisfy the boundary conditions:

$$\begin{aligned} \sigma_r + p &= 0 \qquad (r = a), \\ \frac{\partial u}{\partial t} &= \frac{\partial w}{\partial t}, \qquad (r = a), \\ \sigma_r &= 0 \qquad (r = b). \end{aligned} \tag{6}$$

The integral Fourier transformation with respect to time is used for the solution of problem [18]

$$f^F(\omega) = \int\limits_{-\infty}^{\infty} f(t) e^{i\omega t} dt \quad (-\infty < \omega < \infty), \tag{7}$$

where ω is the integral transform parameter (the circular frequency). So, taking into account the causality principle [17] and the fact that $\Omega(t) = 0$ for $t \le t_1$ $(t_1 > -\infty)$, the initial conditions must be satisfied:

$$u = 0 = p, \qquad t \le t_1. \tag{8}$$

Applying the Fourier-transform (7) to Eqs. (1) and (4), the following ordinary differential equations for the F-transforms are obtained:

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + \left(k_L^2 - \frac{1}{r^2}\right)\right] u^F(r,\omega) + rK_L^2(\omega) = 0 \qquad (b < r < a), \tag{9}$$

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + k^2\right) p^F(r,\omega) = 0 \qquad (a < r < \infty). \tag{10}$$

Here $k = \omega/c$ is the wave number in acoustic medium, $k_L = \omega/c_L$ is the wave number in elastic body, $c_L = \sqrt{(\lambda + 2\mu)/\rho_s^0}$ is the velocity of longitudinal wave in the cylinder, $K_L(\omega) = \sqrt{\Omega^F(\omega)}/c_L^2$ is the additional "longitudinal wave number", connected with time modulation of the angular velocity of cylinder rotation, where

$$\Omega^F(\omega) = \int_{-\infty}^{\infty} \Omega^2(t) e^{i\omega t} dt. \tag{11}$$

The boundary conditions (6) in the Fourier-transforms have the forms:

$$\begin{aligned} \sigma_r^F + p^F &= 0 \qquad (r = a), \\ u^F &= w^F \qquad (r = a), \\ \sigma_r^F &= 0 \qquad (r = b). \end{aligned} \tag{12}$$

Here the functions $p^F(r, \omega)$ and $u^F(r, \omega)$ must be analytic in the upper half-plane Im $\omega > 0$ and singular, but integrated on Im $\omega = 0$ [17].

Then in the space of Fourier-transforms, the exact solutions of Eqs. (9) and (10) are found in the following form:

$$p^F(r, \omega) = (\lambda + 2\mu) X_L^2(\omega) P(r, \omega) \qquad (a \leq r < \infty), \tag{13}$$

$$u^F(r, \omega) = a X_L^2(\omega) U(r, \omega) \qquad (b \leq r \leq a), \tag{14}$$

where

$$P(r, \omega) = B H_0^{(1)}(kr), \qquad X_L^2(\omega) = K_L^2(\omega) a^2, \tag{15}$$

$$U(r, \omega) = A_1(\omega) J_1(k_L r) + A_2(\omega) N_1(k_L r) - (r/a) x_L^{-2}, \tag{16}$$

A_1, A_2, B are unknown constants of integration. Here and below $J_n(z)$ and $N_n(z)$ ($n = 0,\ 1,\ 2$) are the Bessel and Neumann functions, respectively, $H_n^{(1)}(z)$ ($n = 0,\ 1$) are the Hankel functions of the first kind.

Then from the transformed relations (2), (3), (5), the Fourier-transforms of the radial displacement in acoustic medium and stresses in the cylinder are also obtained:

$$w^F(r, \omega) = a X_L^2(\omega) W(r, \omega) \quad (a \leq r < \infty), \tag{17}$$

$$\sigma_{\theta,j}^F(r, \omega) = (\lambda + 2\mu) X_L^2(\omega) \Sigma_j(r, \omega) \quad (b \leq r \leq a,\ j = r, \theta, z), \tag{18}$$

where

$$\begin{aligned} W(r,\omega) &= -(x_L\varsigma)^{-1}BH_1^{(1)}(kr),\\ \Sigma_{r,\theta}(r,\omega) &= x_L[A_1(\omega)J_{02}^{\mp}(k_Lr)+A_2(\omega)N_{02}^{\mp}(k_Lr)]-2(1-\alpha)x_L^{-2},\\ \Sigma_z(r,\omega) &= \frac{1-2\alpha}{2(1-\alpha)}\left[\Sigma_r(r,\omega)+\Sigma_\theta(r,\omega)\right], \\ J_{02}^{\mp}(k_Lr) &= (1-\alpha)J_0(k_Lr)\mp\alpha J_2(k_Lr),\\ N_{02}^{\mp}(k_Lr) &= (1-\alpha)N_0(k_Lr)\mp\alpha N_2(k_Lr). \end{aligned} \tag{19}$$

Here $\varsigma = \kappa/\kappa_s$, $\kappa = \rho^0 c$ and $\kappa_s = \rho_s^0 c_L$ are the wave resistances in acoustical medium and solid, respectively, $\alpha = c_T^2/c_L^2$, $c_T = \sqrt{\mu/\rho_s^0}$ is the velocity of the shear wave in elastic material, $x_L = k_L a$.

If the boundary conditions (12) are satisfied, then the constants of integration are determined, in particular:

$$A_n(\omega) = (-1)^n\Delta_n/(x_L^3\Delta)\ (n=1,\ 2), \tag{20}$$

where

$$\begin{aligned} \Delta\ &= H_1^{(1)}(x)[J_{02}^-(x_L)N_{02}^-(y_L)-N_{02}^-(x_L)J_{02}^-(y_L)]\\ &\qquad -\varsigma H_0^{(1)}(x)[J_1(x_L)N_{02}^-(y_L)-N_1(x_L)J_{02}^-(y_L)],\\ \Delta_1\ &= 2(1-\alpha)H_1^{(1)}(x)[N_{02}^-(y_L)-N_{02}^-(x_L)]\\ &\qquad +\varsigma H_0^{(1)}(x)[2(1-\alpha)N_1(x_L)-x_LN_{02}^-(y_L)],\\ \Delta_2\ &= 2(1-\alpha)H_1^{(1)}(x)[J_{02}^-(y_L)-J_{02}^-(x_L)]\\ &\qquad +\varsigma H_0^{(1)}(x)[2(1-\alpha)J_1(x_L)-x_LJ_{02}^-(y_L)],\\ x\ &= ka, \qquad y_L = k_Lb. \end{aligned} \tag{21}$$

Thus, using the Eqs. (14), (16), (18), (19), we can obtain the formulae for complex amplitudes of the displacement and stresses in the cylinder.

3. Regularized form for the solution of problem

The analysis of spectral characteristics in a wide frequency range is connected with the difficulties, which appear in the numerical calculations for $\omega \approx 0$ [13]. In particular, it is visible during the numerical realization of the inverse Fourier-transforms by replacement of a semi-infinite interval of integration by a finite interval, which includes the value $\omega = 0$ [15, 19]. In consequence of the indeterminate expression of "zero divided by zero", the integrands have very large values near $\omega = 0$, even if the calculation is made with double precision. Meanwhile, it is easy to make sure that functions

$U(r,\omega)$, $\Sigma_j(r,\omega)$ $(j=r,\theta,z)$ for $\omega\to 0$ approach the expressions which correspond to the solutions for the case of constant angular velocity of cylinder rotation [20, 21]:

$$\begin{aligned}
U_0(r) &= 0.125\left[\frac{2-\alpha}{\alpha}\varepsilon^2\frac{a}{r}+\frac{2-\alpha}{1-\alpha}(1+\varepsilon^2)\frac{r}{a}-\left(\frac{r}{a}\right)^3\right],\\
\Sigma_{r0}(r) &= 0.25(2-\alpha)\left[-\varepsilon^2\left(\frac{a}{r}\right)^2+(1+\varepsilon^2)-\left(\frac{r}{a}\right)^2\right],\\
\Sigma_{\theta 0}(r) &= 0.25\left[(2-\alpha)\varepsilon^2\left(\frac{a}{r}\right)^2+(2-\alpha)(1+\varepsilon^2)-(2-3\alpha)\left(\frac{r}{a}\right)^2\right],\\
\Sigma_{z0}(r) &= 0.25(1-2\alpha)\left[\frac{2-\alpha}{1-\alpha}(1+\varepsilon^2)-2\left(\frac{r}{a}\right)^2\right],
\end{aligned} \tag{22}$$

where $\varepsilon = b/a$.

Then for regularization of the numerical calculations of complex amplitude module of the displacement and stresses including point $\omega = 0$, we must write the functions $U(r,\omega)$, $\Sigma_j(r,\omega)$ $(j=r,\theta,z)$ from Eqs. (16) and (19) as the sums:

$$\begin{aligned}
U(r,\omega) &= U_0(r)+U_1(r,\omega),\\
\Sigma_j(r,\omega) &= \Sigma_{j0}(r)+\Sigma_{j1}(r,\omega)\ (j=r,\theta,z),
\end{aligned} \tag{23}$$

where

$$\begin{aligned}
U_1(r,\omega) &= \{[\widetilde{\Delta}_1 J_1(k_L r)-\Delta_2\psi_1(k_L r)]-x_L[(r/a)+x_L^2 U_0(r)]\widetilde{\Delta}\}/(x_L^3\widetilde{\Delta}),\\
\Sigma_{r,\theta 1}(r,\omega) &= \{[\widetilde{\Delta}_1 J_{02}^{\mp}(k_L r)-\Delta_2\psi_{02}^{\mp}(k_L r)]\\
&\quad -\ [2(1-\alpha)+x_L^2\Sigma_{r,\theta 0}(r)]\widetilde{\Delta}\}/(x_L^2\widetilde{\Delta}),\\
\Sigma_{z1}(r,\omega) &= \frac{1-2\alpha}{2(1-\alpha)}\left[\Sigma_{r1}(r,\omega)+\Sigma_{\theta 1}(r,\omega)\right],
\end{aligned} \tag{24}$$

and

$$\begin{aligned}
\widetilde{\Delta} &= J_{02}^{-}(x_L)-\psi_{02}^{-}(x_L)J_{02}^{-}(y_L)+\varphi_0(x)[J_1(x_L)-\psi_1(x_L)J_{02}^{-}(y_L)],\\
\widetilde{\Delta}_1 &= 2(1-\alpha)[1-\psi_{02}^{-}(x_L)]-\varphi_0(x)[2(1-\alpha)\psi_1(x_L)-x_L],\\
\varphi_0(kr) &= -\varsigma\frac{H_0^{(1)}(kr)}{H_0^{(1)}(x)}, \qquad \varphi_1(kr) = \frac{H_1^{(1)}(kr)}{H_0^{(1)}(x)},\\
\psi_n(k_L r) &= \frac{N_n(k_L r)}{N_{02}^{-}(y_L)}\ (n=1,\ 2), \qquad \psi_{02}^{\pm}(k_L r) = \frac{N_{02}^{\pm}(k_L r)}{N_{02}^{-}(y_L)}.
\end{aligned} \tag{25}$$

In the formula $(24)_2$ sign "minus" corresponds to index "r", and sign "plus" – to index "θ".

The significant characteristic of wave process in the elastic body is also the variable density of material $\rho_s(r,t)$, which is connected with the vector of particle velocity $\mathbf{v}(r,t)$ by the linearized mass balance equation [22]:

$$\frac{\partial \rho_s(r,t)}{\partial t} + \rho_s^0 \mathrm{div}\mathbf{v}(r,t) = 0, \qquad \rho_s^0 = \rho_s(r,0). \tag{26}$$

In this case the Fourier-transform for function $\rho_s(r,t) - \rho_s^0$ is obtained as

$$\rho_s^F(r,\omega) = \rho_s^0 X_L^2(\omega) R_s(r,\omega), \tag{27}$$

where

$$R_s(r,\omega) = R_{s0}(r) + R_{s1}(r,\omega) \tag{28}$$

and

$$R_{s0}(r) = 0.125\left[\frac{2-\alpha}{\alpha}\varepsilon^2\left(\frac{a}{r}\right)^2 - \frac{2-\alpha}{1-\alpha}\left(1+\varepsilon^2\right) + 3\left(\frac{r}{a}\right)^2\right] \tag{29}$$

is the relative density corresponding to the case of $\Omega(t) = \Omega_0$, i.e. if

$$\rho_s(r,t) = \rho_s(r) = \rho_s^0[1 + X_{L0}^2 R_{s0}(r)], \qquad X_{L0} = \Omega_0 a \,/c_L. \tag{30}$$

The second term on the right-hand side of Eq. (28) is a regularized part of spectral function for a density $\rho_s(r,t)$:

$$R_{s1}(r,\omega) = -\{[\widetilde{\Delta}_1 J_1'(k_L r) - \Delta_2 \psi_1'(k_L r)] - [1 - x_L^2 R_{s0}(r)]\widetilde{\Delta}\}/(x_L^2\widetilde{\Delta}), \tag{31}$$

where

$$\psi_1'(k_L r) = \frac{N_1'(k_L r)}{N_{02}^-(y_L)}. \tag{32}$$

Here the prime denotes the derivative of function with respect to $k_L r$.

4. Time-average flux of elastic energy

For estimation of the strain energy transmission in elastic cylinder we consider the Umov-Poynting's vector of power [22, 23]. In the case of axisymmetrical plane problems this vector has only a radial non-zero component

$$P_r(r,t) = -\sigma_r(r,t)v_r(r,t), \qquad P_\theta(r,t) = 0, \qquad P_z(r,t) = 0. \tag{33}$$

In the conditions of time-harmonic excitation of cylinder vibrations:

$$\Omega(t) = \Omega_0(1 + \varepsilon_0 \sin\omega_0 t) \quad (-\infty < t < \infty), \tag{34}$$

the time average of power flow over a period $T_0 = 2\pi/\omega_0$ is

$$I_r(r,\omega_0) = \frac{1}{T_0}\int_0^{T_0} P_r(r,t)dt \ (b \le r \le a). \tag{35}$$

Here ε_0 is the small nondimensional parameter for determination of the amplitude disturbance of this velocity, ω_0 is the circular frequency. In this case

$$\begin{aligned} u(r,t) &= aU(r,t), \\ \sigma_\gamma(r,t) &= (\lambda+2\mu)\Sigma_\gamma(r,t) \quad (\gamma = r,\theta,z), \\ \rho_s(r,t) &= \rho_s^0[1+R_s(r,t)], \\ v_r(r,t) &= c_L V_r(r,t), \end{aligned} \tag{36}$$

where

$$\begin{aligned} \{U,\Sigma_\gamma,R_s,V_r\}(r,t) = (1+0.5\varepsilon_0^2)\{U_0,\Sigma_{\gamma 0},R_{s0},V_{r0}\}(r) \\ - \ 2\varepsilon_0 \mathrm{Im}\left[\{U_1,\Sigma_{\gamma 1},R_{s1},V_{r1}\}(r,\omega_0)e^{-i\omega_0 t}\right] \\ - \ 0.5\varepsilon_0^2 \mathrm{Re}\left[\{U_1,\Sigma_{\gamma 1},R_{s1},V_{r1}\}(r,2\omega_0)e^{-2i\omega_0 t}\right], \end{aligned} \tag{37}$$

$$V_{r0}(r) = 0, \qquad V_{r1}(r,\omega) = -ix_L U_1(r,\omega).$$

Then substituting $\sigma_r(r,t)$ and $v_r(r,t)$ in the Eq. (35), we obtain

$$\begin{aligned} I_r(r,\omega_0) = -2X_{L0}^4\kappa_s c_L^2\varepsilon_0^2 \mathrm{Re}[\Sigma_{r1}(r,\omega_0)V_{r1}^*(r,\omega_0) \\ + (\varepsilon_0/4)^2\Sigma_{r1}(r,2\omega_0)V_{r1}^*(r,2\omega_0)], \end{aligned} \tag{38}$$

where V_{r1}^* is the complex conjugate of V_{r1}.

5. Numerical results and discussion

Analysis of the spectral characteristics is carried out for the case of a rotating Armco iron cylinder [24, 25] surrounded by water. For the numerical calculations, the following physico-mechanical constants are used: for material of the cylinder – $\rho_s^0 = 7700$ kg/m^3, $c_L = 5960$ m/s, $c_T = 3240$ m/s [26] and for water – $\rho^0 = 1000$ kg/m^3, $c = 1493$ m/s [27].

Figure 1 shows the moduli of complex amplitudes $U(r,\omega)$, $\Sigma_\gamma(r,\omega)$ $(\gamma = r,\theta,z)$, $R_s(r,\omega)$ as functions of the nondimensional frequency $x = ka$ (the wave radius of cylinder) and the radial co-ordinate geometrical parameter r/a for the three values of relative tube thickness $\varepsilon = 0.25;\ 0.5;\ 0.75$. Since the amplitudes of high harmonic resonances are small in comparison with the amplitude of the basic tone, the decibel scale on the applied axis is used for proportional representation of spectral curves. In the selected range of frequencies ($0 \le x_L \le 25$), the resonance amplitudes of fundamental tone of vibrations are better observed and significantly exceed the corresponding static values. The whole frequency spectra have brightly expressed resonance character too as a consequence of the elastic waves reflections between the outer and inner cylindrical surfaces. Therefore, the resonance locations are dependent on the thickness

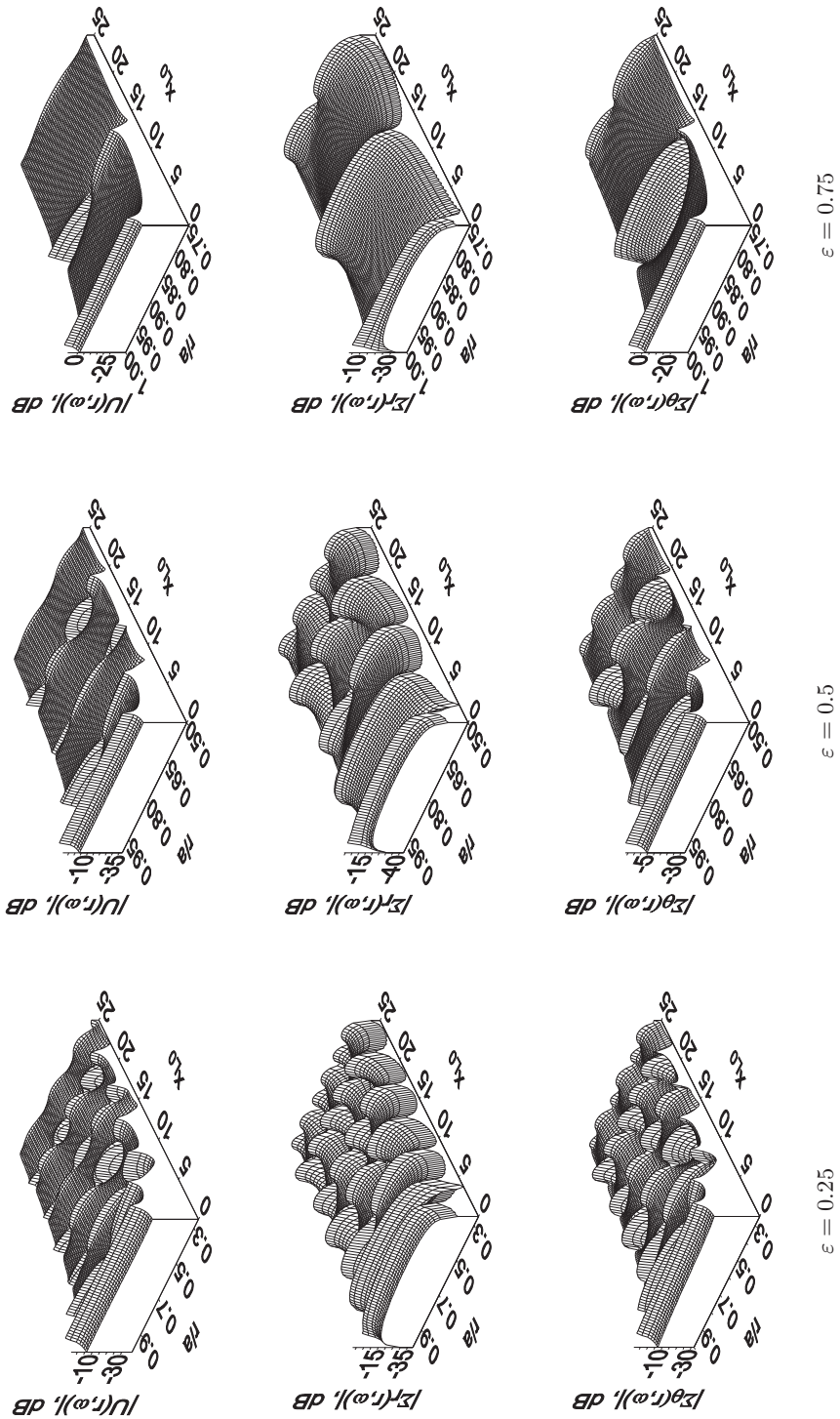
of cylindrical object. In particular, if the tube thickness decreases, then waves on resonance frequencies are subjected to the influence of geometrical dispersion phenomena, i.e. the resonance locations are non-monotonic functions of parameter thickness ε.

This effect is well illustrated in the Fig. 2, where the curves of identical levels of time-averaged power $|I_r(r,\omega_0)|$ are plotted. The cylinder of outer radius $a = 0.457$ m performs $N_0 = 50$ revolutions per second [20] ($\Omega_0 = 2\pi N_0$ rad/s), with the relative amplitude of angular velocity modulation $\varepsilon_0 = 0.1$. It may be noticed that because the amplitude falls sharply on high frequencies, module of this sign-changing characteristic is also calculated in dB. Here the power distributions are obtained along the radius of the shell and for fixed values of parameter ε. It is shown that the first, low-frequency resonance moves to lower frequency range if parameter ε decreases. At the same time, the resonances of higher orders shift to the side of high frequencies, and more quickly when shell thickness decreases. The thin spectral structure of the power flow at point $r/a = 0.8$ for three cases of tube thickness ($\varepsilon = 0.25,\ 0.5,\ 0.75$) and $\varepsilon_0 = 0.3$ is illustrated in the Fig. 3. We can observe spectral lines on the fundamental carrier frequencies ω_0 (the big picks and dips) and on doubled frequencies (the small picks and dips). In the Fig. 4 the spectral picture for these last resonances are demonstrated separately by calculation with taking into account only the second term of the Eq. (38). In the both Fig. 3 and Fig. 4 destructive resonances correspond to negative values of the power flow.

The Fig. 5 shows the distributions of same characteristic (in dB) as the function of new non-dimensional radial variable $y = (r/a - \varepsilon)/(1 - \varepsilon)$ and non-dimensional thickness of tube ε for several values of carrier frequency ω_0. Here the un-expected phenomenon is observed: diagonal line $r/a \approx r_*/a = \varepsilon(2 - \varepsilon)$ is as interface of two different parts of cylinder's cross-section. In the first part $b \leq r < r_*$ the power levels are low and wave modes are not clearly expressed. In the second part $r_* < r \leq 1$, the regularity of modal picture with essential amplitudes is noticed and amplitude levels decrease with frequency increasing.

The Fig. 6 illustrates three-periodic time amplitudes of non-dimensional displacement $U(r,t)$, radial, hoop and axial stresses $\Sigma_\gamma(r,t)$ ($\gamma = r, \theta, z$), and also the material density $R_s(r,t)$ on tube cross-section at resonance frequencies of the first three tones of vibrations, respectively, $x_{L0} = 1.6866;\ 4.7125;\ 8.6278$, and for $\varepsilon = 0.25$ ($\tau = c_L t/a$). These plots display smooth radial distributions of time-harmonic strain-stress state in a thick cylindrical tube. It is shown that for resonance frequencies of vibrations, the radial stress assumes its extremal values inside the cylinder and minimal values on the boundary surfaces. The hoop stress is extremal on inner "dry" surface of the cylinder and for absolute value it is three times as large as the maximal radial stress. The axial stress, which is the superposition of the radial and hoop stresses, may be significant on the inner surface and inside cylinder too.

The picture becomes absolutely different if the radial co-ordinate is fixed and thickness is variable. The same time characteristics as in the Fig. 6, but with $r/a = 0.8$ and $\varepsilon \in [0,\ 0.8)$ for two resonance frequencies $x_{L0} = 1.6866$ and 4.7125 are shown in Fig. 7. Here we observe the effect of the space-time coincidence, when the resonance



[Fig. 1]

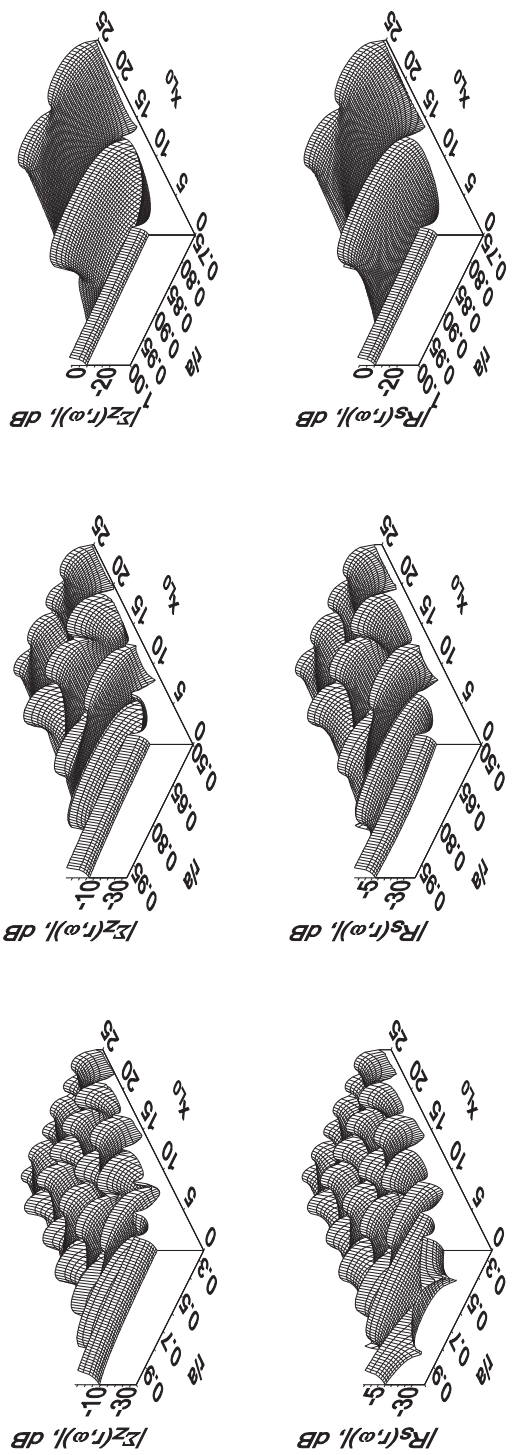


Fig. 1. The module of radial displacement $|U(r,\omega)|$, radial $|\Sigma_r(r,\omega)|$, hoop $|\Sigma_\theta(r,\omega)|$ and axial $|\Sigma_z(r,\omega)|$ stresses, and material density $|R_s(r,\omega)|$ (in decibels) for relative thickness of cylinder tube $\varepsilon = 0.25;\ 0.5;\ 075$ and for $\varepsilon < r/a < 1,\ 0 < x_{L0} \leq 25$.

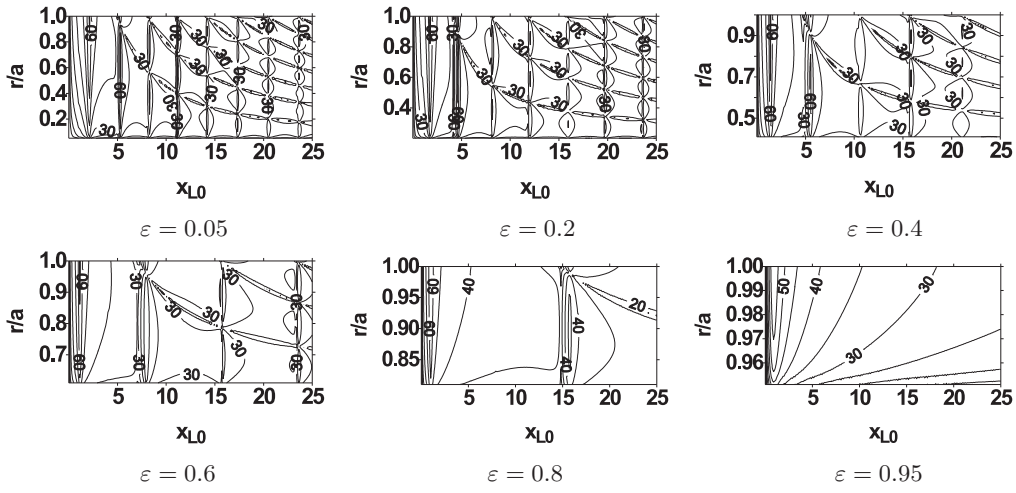


Fig. 2. The frequency-radial distribution of the modulus of time-average power flux $|I_r(r,\omega_0)|$ (in decibels) in an elasic hollow cylinder for $\varepsilon_0 = 0.1$.

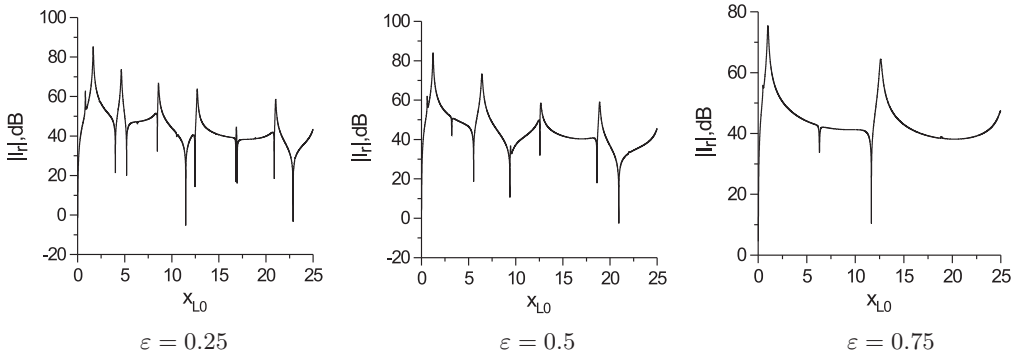


Fig. 3. The time-average power flux $|I_r(r,\omega_0)|$ (in decibels) at point $r/a = 0.8$ of elasic hollow cylinder for $\varepsilon_0 = 0.3$ and $\varepsilon = 0.25;\ 0.5;\ 075$.

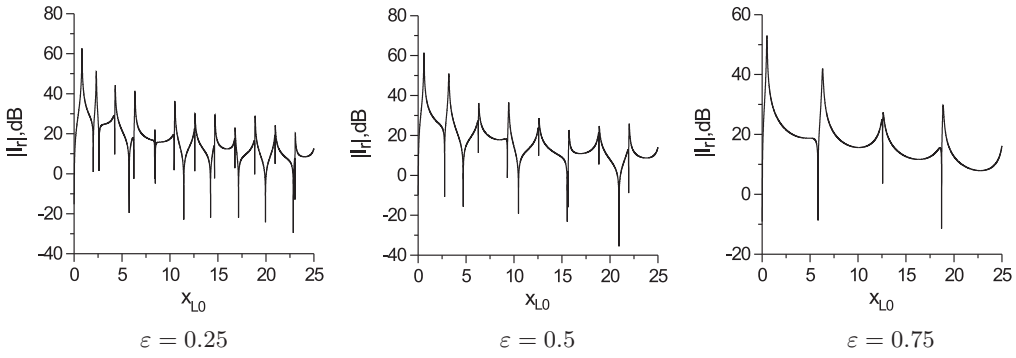


Fig. 4. The part of the time-average power flux $|I_r(r,\omega_0)|$ (in decibels) with taking into account only the second term of the Eq. (38) in a point $r/a = 0.8$ of an elastic hollow cylinder for $\varepsilon_0 = 0.3$ and $\varepsilon = 0.25,\ 0.5,\ 0.75$.

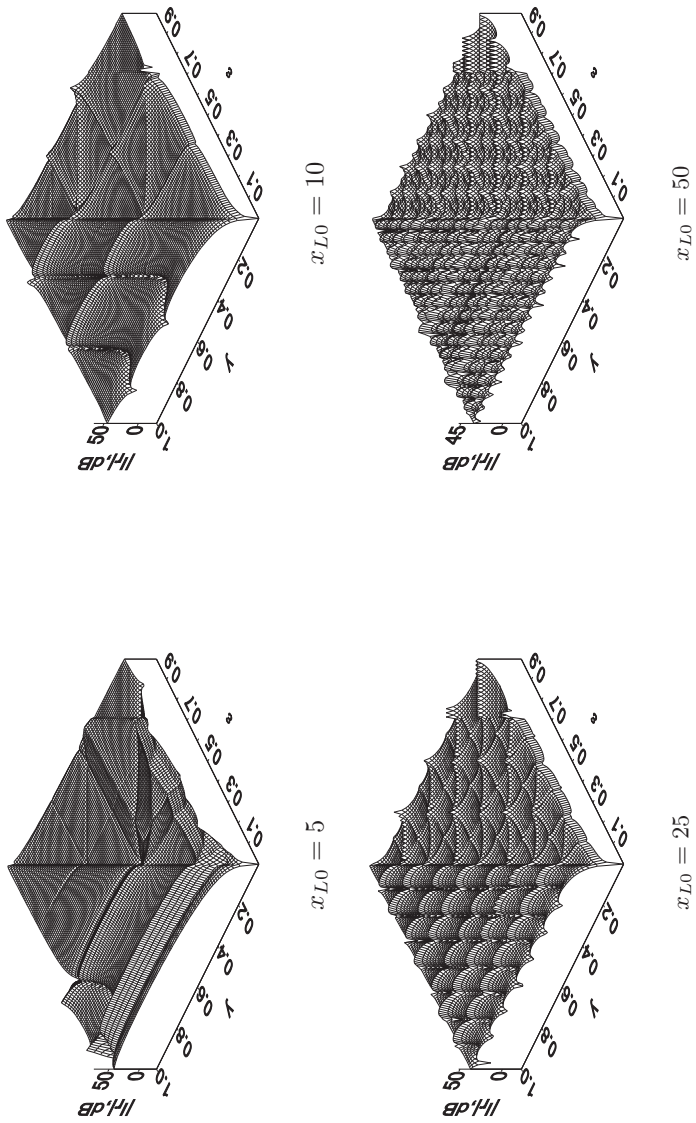
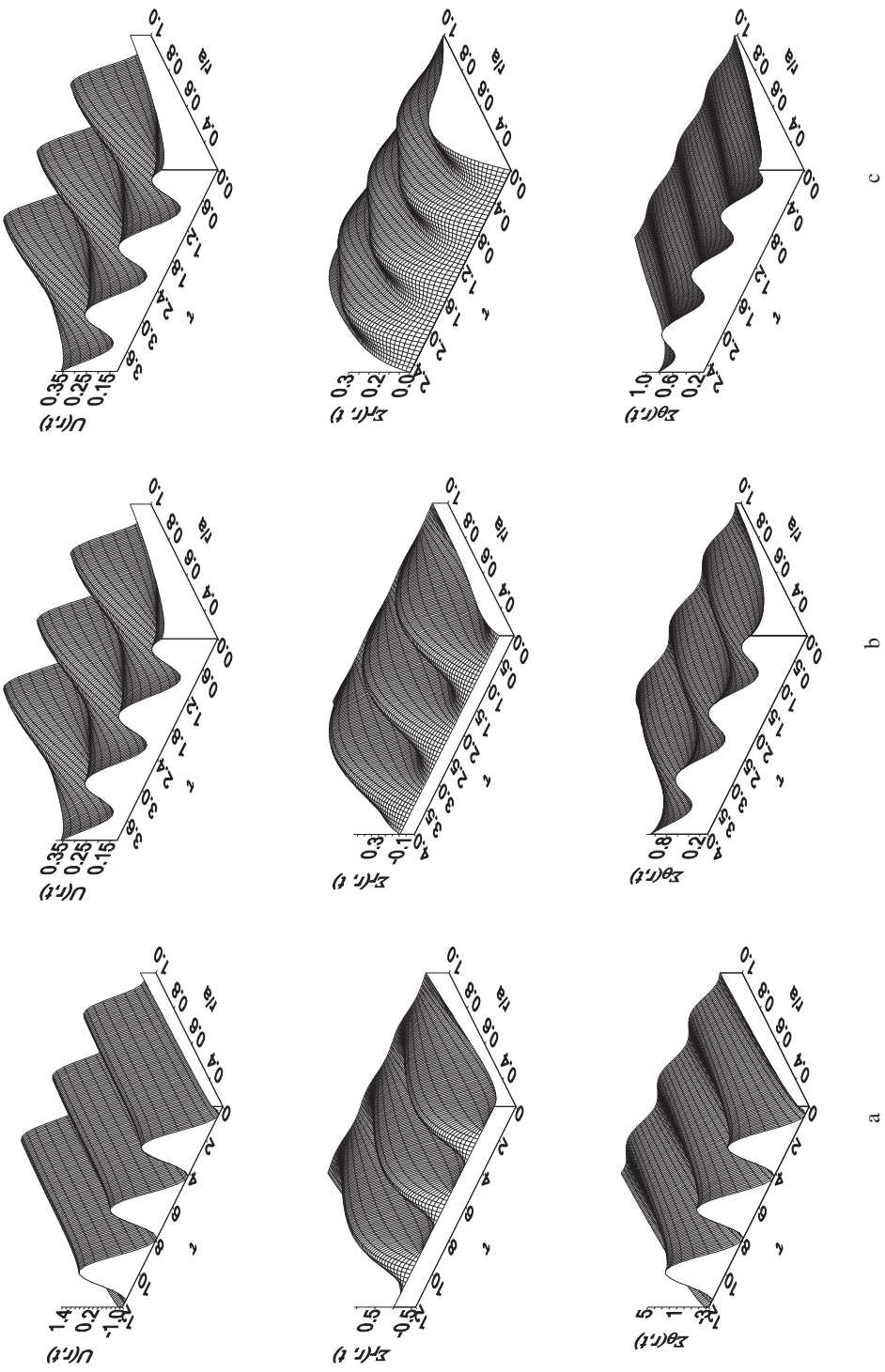


Fig. 5. The distribution of time-average power flux $|I_r(r, \omega_0)|$ (in decibels) as a function of $y = (r/a - \varepsilon)/(1 - \varepsilon)$ and ε for a fixed frequency ω_0.



[Fig. 6]

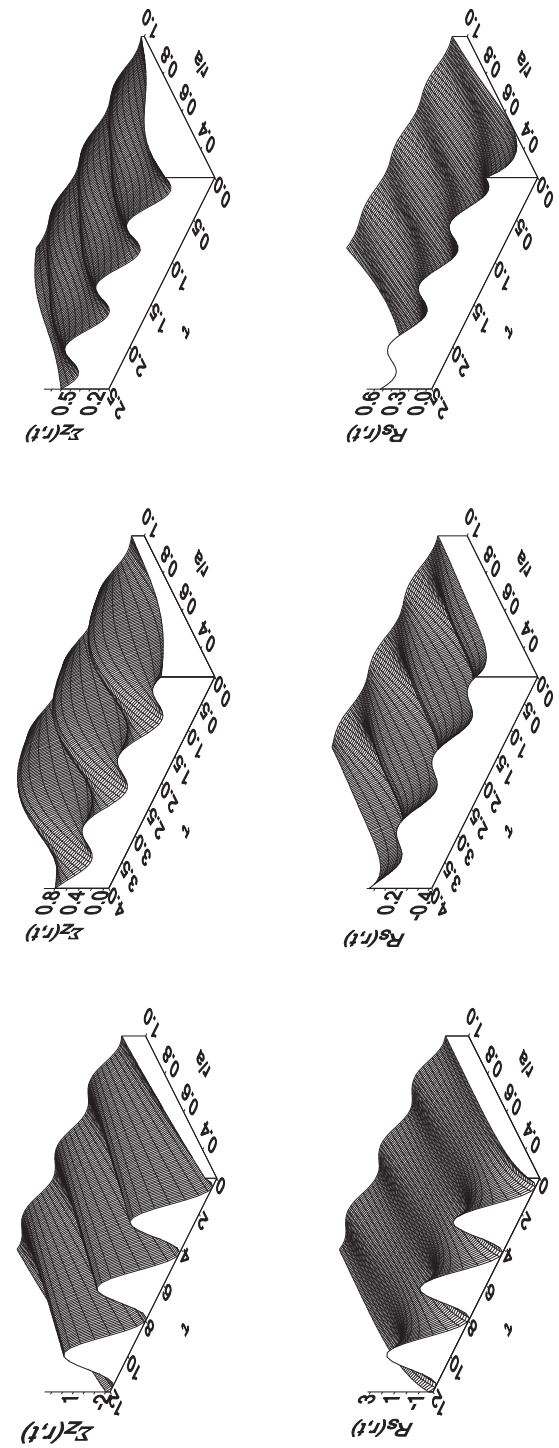
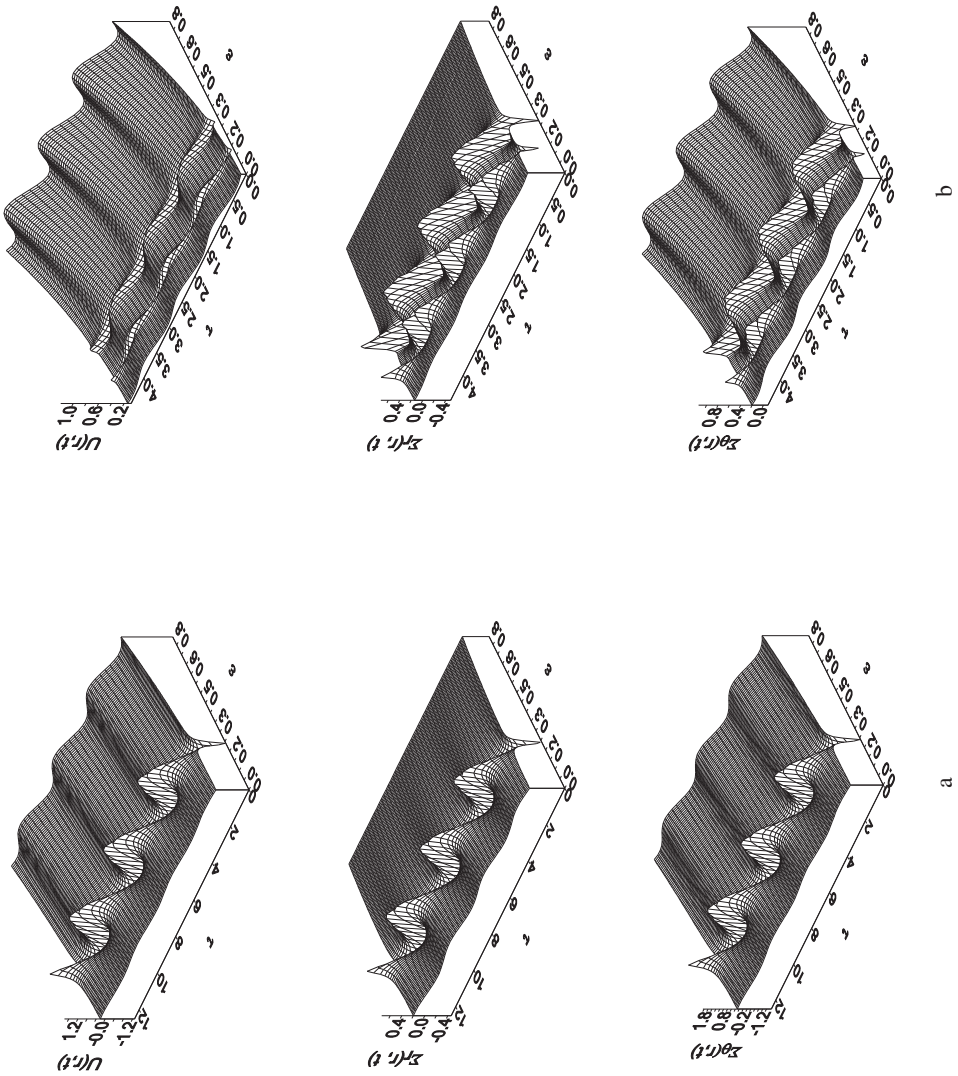
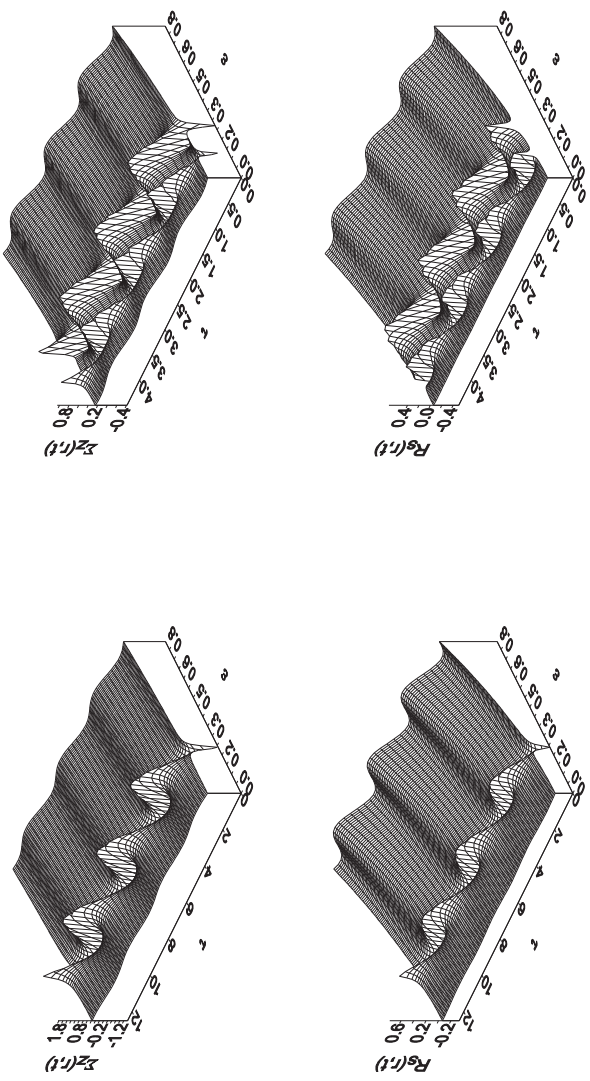


Fig. 6. The dependences of radial displacement $U(r,t)$, radial $\Sigma_r(r,t)$, hoop $\Sigma_\theta(r,t)$ and axial $\Sigma_z(r,t)$ stresses, and material density $R_s(r,t)$ on time, for relative tube thickness $\varepsilon = 0.25$ and $\varepsilon < r/a < 1$ for the resonant frequencies of the first three tones, respectively: a – $x_{L0} = 1.6866$; b – $x_{L0} = 4.7125$; c – $x_{L0} = 8.6278$.



[Fig. 7]



a b

Fig. 7. The transition through spatial resonances (with relative tube thickness changing) of the time dependencies of radial displacement $U(r,t)$, radial $\Sigma_r(r,t)$, hoop $\Sigma_\theta(r,t)$ and axial $\Sigma_z(r,t)$ stresses, and material density $R_s(r,t)$ at point $r/a = 0.8$ for the resonance frequencies of the first two tones, respectively: a – $x_{L0} = 1.6866$ and b – $x_{L0} = 4.7125$.

appears not only in consequence of wave excitation on resonance frequency, but also owing to the "resonance" values of tube thickness [28]. Note, that the similar effects are met in non-specular reflection of a bounded acoustic beam from an elastic object (see e.g. [29]), where angle of beam incidence is the "resonance" parameter, and in the theory of electrodynamics [30], where the transparency coefficient, dielectric constant, impedance etc. also can be used as spectral parameters. In our case it is shown that on spatial resonance, the absolute values of hoop and axial stresses are several times greater than radial stress (more than four times on frequency $x_{L0} = 1.6866$ and two times on $x_{L0} = 4.7125$).

Finally, Fig. 8 demonstrates the transition through resonance by changing of the frequency of the radial displacement harmonic signal $U(r,t)$ for $r/a = 0.8$ and $\varepsilon = 0.25$. Here the powerful amplitude is excited on resonance frequency of the first harmonic.

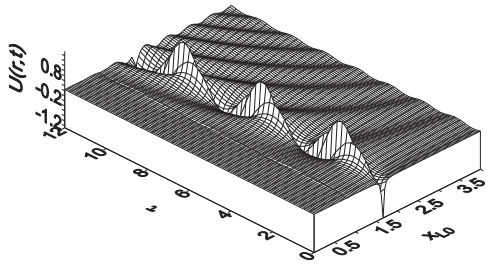


Fig. 8. The transition through resonance of base tone of vibrations by changing the frequency of radial displacement harmonic signal $U(r,t)$ for $r/a = 0.8$ and $\varepsilon = 0.25$.

6. Conclusions

The rotation of an elastic hollow circular cylinder with a non-constant angular velocity in a compressible fluid or gas is a cause of very complex wave processes in an elastic material. First of all the outside acoustic medium, contacting with rotating objects, has a large influence on formation of the dynamical strain-stress state in body, especially on resonance characteristics. Significant part of the elastic energy is transformed in sound, that is a cause of finiteness of width and amplitudes of the spectral lines of resonance excitation of displacement, stresses and material density in the tube. On the other hand, additional inner surface of object plays the role of a resonator for re-reflected elastic waves. On the basis of obtained results, we have arrived at the following conclusions:

1. The spectral complex amplitudes of radial displacement, components of the stress tensor and also density of the material have a resonance character. If the tube thickness decreases, then resonance lines of these amplitudes shift quickly into high frequencies.
2. The analysis of time-averaged power flux in the material of a cylinder shows that this characteristic has no constant direction. It appears as a resonance combination of constructive and destructive types in the spectrum. The fine structure of

power flow is formed with participation of fundamental (primary) and double frequencies.

3. The numerical results show two types of the power flow space distribution, each of them is characterized by different wave regularity. The key parameter of this phenomenon is relative thickness of shell.
4. The strain-stress characteristics in a hollow cylinder have visible values not only on the first, lowest resonance, but also on high resonances. On these resonances, non-homogeneity of radial distribution of these characteristics are displayed especially expressive. On the low resonance frequencies, the hoop stress concentration on inner cylindrical surface occurs. The resonance amplitudes of hoop and axial stresses exceed the relative values of the radial stress by 2–3 times.
5. When the hollow cylinder vibrations are excited on the fixed frequencies, spatial resonances appear if relative thickness parameter $\varepsilon = b/a$ $(0 < \varepsilon < 1)$ is considered as a variable value. On spatial resonances, the hoop and axial stresses are more dangerous than the radial stress.

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