ARCHIVES OF ACOUSTICS
33, 4, 521–530 (2008)

IMPROVEMENT OF EFFECTIVENESS IN ACTIVE TRIANGULAR PLATE VIBRATION REDUCTION

Adam BRAŃSKI(1), Stanisław SZELA(2)

(1) Technical University of Rzeszów
Laboratory of Acoustic, WEiI
Wincentego Pola 2, 35-959 Rzeszów, Poland
e-mail: abranski@prz.rzeszow.pl

(2) University of Rzeszów
Institute of Technology
Al. Rejtana 16c, 35-959 Rzeszów, Poland

(*received March 15, 2008; accepted September 16, 2008*)

In the paper, a new method of the effectiveness increase of the active asymmetrical structure vibration reduction via PZTs is proposed. The new method is based on adding successive PZTs to quasi-optimal sub-areas (QO) in the determined order. A triangular plate with the C-F-F boundary conditions is chosen for this purpose. Only the second mode of the plate is taken into consideration in this study. To realise the aim, first the QO are determined. To each QO, PZTs are bonded from one to seven according to the following rule; namely, the consecutive PZT takes up free surface with locally maximal curvature on the QO. The effectiveness of the vibration reduction is measured in particular sense. Numerical experiments are performed to confirm the validity of this idea.

Keywords: triangular plate, PZT (Piezoelectric Zirconate Titanate), C-F-F (Clamped-Free-Free) boundary condition, active vibration reduction.

1. Introduction

One of the most important technical problems is an active reduction of the transverse vibration of structures. The problem may be realised with PZTs (piezoelectric elements) [4–6, 9, 12, 13]. There are many different parameters of PZTs having an influence on the efficiency of active vibration reduction; they are clearly described in [9]. But not only parameters of PZTs, but also their distribution has a great influence on the effectiveness of the reduction. The way of the PZTs distributing has the particular meaning, if the structure has got neither an axis nor a point of symmetry, such as the triangular plates. It was pointed out in [1] and [2], that the appropriate distribution of the PZTs is at the surface places with maximum curvatures; they are the so-called quasi-optimal sub-areas (QO). Consequently, the distribution of the PZTs on these places was called the quasi-optimal ones. The search of the QO was substantiated mathematically and numerically. To do that, the following procedure was realised. The triangular plate with clamped-free-free (C-F-F) boundary conditions was considered [8, 10]. The only second mode of the plate was excited with harmonic acoustic wave. The sub-areas with maximum curvatures were determinate i.e. QO. Next, two cases were considered. In the first case, the PZTs were bonded at the QO exactly and at the second case, they are somewhat shifted outside these sub-areas. As a measure of effectiveness of the vibration reduction, for these two cases, the difference (in some sense) of some quantities are determined. They are the vibration amplitudes on the plate surface, bending moments and shearing forces at the clamped edge (side) [2]. Furthermore in paper [3] it was pointed out that the acoustic waves emitted by the plate may always be a measure of the vibration reduction. All results confirmed the validity of the idea of quasi-optimal distribution of PZTs in active vibration reduction of the triangular plate.

In this paper is proposed an improvement of the effectiveness via adding successive PZTs, not at arbitrary places but at the QO. To realize the goal of the paper, at each QO from one to seven, the PZTs are bonded. Doing it, it was kept to the following rule: at each QO the successive PZTs take up free space of the surface with locally maximal curvature. All PZTs, on each QO, are excited by the same signal. The effectiveness of the vibration reduction is measured by the same quantities as in [2], i.e. vibration amplitudes on the plate surface, bending moments and shearing forces at the clamped edge. After adding one PZT on each QO, the effect of the reduction is measured. The proposed idea is substantiated numerically.

2. Formulation of plate vibration theory

To date, applying different techniques, much of the attention has been focused on free vibrations of thin triangular plates [7, 11]. The governing equation of transverse vibration of the plate, on the basis of Kirchhoff's classical small deflection theory, has the form [4, 5],

$$N\Delta^2 w + \rho h \frac{\partial^2 w}{\partial t^2} = F, \tag{1}$$

where $w = w(x,y,t)$ – transverse displacement, $F = F(x,y,t)$ – external surface force, ρ – mass density, h – thickness, $N = Eh^3/12(1-\nu^2)$ – flexural rigidity, E – Young's modulus, Δ – Laplace'a operator, $\Delta w = \partial^2 w/\partial x^2 + \partial^2 w/\partial y^2$, ν – Poisson's ratio.

The external surface force consists of two components,

$$F = F_E + \sum_j \left(\sum_s F_{js} \right), \qquad s = 1, 2, ..., n_s, \qquad j = 1, 2, ..., n_j, \tag{2}$$

where $F_E = F_E(x, y, t)$ – external surface excitation forced by plane acoustic wave, $F_{js} = F_{js}(x, y, t)$ – surface control force due to one PZT, n_j – number of areas with maximum curvature, n_s – number of PZTs in one sub-area.

The C-F-F boundary conditions imposed on the plate are defined [8, 10], Fig. 1, on the clamped edge (b-edge):

$$w = 0, \qquad \theta = \frac{\partial w}{\partial x} = 0, \tag{3}$$

on the free edge (a-edge):

$$\begin{aligned} Q_y &= -N\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial y^2} + (2-\nu)\frac{\partial^2 w}{\partial x^2}\right) = 0, \\ M_{yy} &= -N\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) = 0, \end{aligned} \tag{4}$$

on the free edge (c-edge);

$$\begin{aligned} Q_n &= -N\left[\frac{\partial}{\partial \mathbf{n}}D^+w + (1-\nu)\frac{\partial}{\partial \mathbf{s}}\left(D^-w\cos\alpha - \frac{\partial^2 w}{\partial x\partial y}\cos(2\alpha)\right)\right] = 0, \\ M_{nn} &= -N\left[\nu D^+w + (1-\nu)\left(\frac{\partial^2 w}{\partial x^2}\cos^2\alpha + \frac{\partial^2 w}{\partial y^2}\sin^2\alpha\right) + \frac{\partial^2 w}{\partial x\partial y}\sin(2\alpha)\right] = 0, \end{aligned} \tag{5}$$

where $D^{\pm}w = \partial^2 w/\partial x^2 \pm \partial^2 w/\partial y^2$, θ – torsion angle, Q_n, Q_y – equivalent shear forces, M_{nn}, M_{yy} – bending moments, **n** – outside normal to the boundary, **s** – outside tangential to the boundary, α – angle between the normal **n** and x-axis.

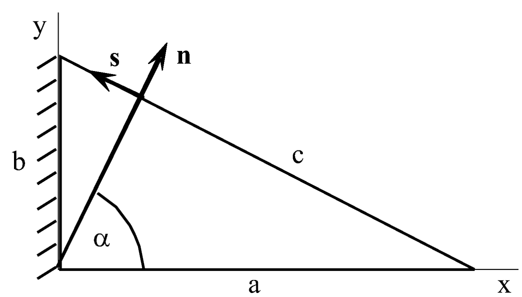

Fig. 1. Triangular plate with C-F-F boundary conditions.

Approximate solution to the vibration equation of the triangular plate relays on FEM. This solution method is discussed in detail in [8] and therefore will not be repeated here. The FEM is the base of ADINA computer code and this program is applied in all calculations.

The system considered herein consists of triangular plate with PZTs perfectly bonded to the top surface. The masses of PZTs are omitted, as they are far less than the mass of the plate. Due to the same reasons, the stiffness is omitted. If the PZT is excited by the voltage applied in the poling direction, the interaction between PZT and the plate appears. The way of this interaction is presented in [2, 4, 5] and in the references cited therein.

3. Curvature of the surface and quasi-optimal places

The problem of maximum surface curvature of the triangular plate with C-F-F boundary conditions was discussed in [1] and [2] in details. In the case of such boundary conditions, the displacement transverse to the direction perpendicular to the clamped edge is greater than the one parallel to the direction of this edge. As it was pointed out in [1, 2], the former curvature is higher (about three times) than the latter one. It substantiates for the consideration of the problem to x-direction only and it is sufficient to analyse the curvatures of the lines perpendicular to the clamped edge. This curvature is described by

$$\kappa = \kappa_x = -\frac{\partial^2 w}{\partial x^2}. \tag{6}$$

In numerical calculations one obtains the sets of discrete values $\{x_i\}$ and $\{w_i\} = \{w(x_i)\}$ rather than the function $w = w(x)$, x_i – nodes. Therefore, the differentiation in (6) is replaced by finite differences

$$\frac{\partial^2 w}{\partial x^2} = \left(w_{i+1} + w_{i-1} - 2w_i\right)/h_s^2, \tag{7}$$

where h_s – distance between the nodes.

The points in which the curvatures take locally maximum successively, determine the quasi-optimal sub-area (QO). More precisely, the surrounding of these points is called the QO. Consequently, the distribution of PZTs on the plate surface at the QO is called the quasi-optimal distribution.

To exemplify the determination of the QO, tracing the papers [1] and [2], the following assumptions are made. The plate made of steel with thickness $h = 1.59$ mm is taken into account. The length b, which is measured along clamped edge is taken as $b = 254$ mm and free edge $a = 381$ mm. The material constants used for calculations are assumed as [10]: the density $\rho = 7169$ kg/m^3, Young's modulus $E = 71.7$ GPa, Poisson's ratio $\nu = 0.32$, respectively. For the PZTs, the piezoelectric strain constant $d_{31} = -190 \times 10^{-12}$ m/V and thickness $h_a = 0.48$ mm, density $\rho_a = 7800$ kg/m^3, Young's modulus $E_a = 66$ GPa and Poisson's ratio $\nu_a = 0.34$, respectively.

The excitation is provided by the loudspeaker in form of a plane acoustic wave, harmonic in time. The plate is forced to the vibration with the second mode and frequency $f = 73.1$ Hz. Such frequency is chosen, because the first mode (for the first frequency $f = 17.4$ Hz) has only one global QO, hence this case is less interesting. In turn, the third and next modes are more complicated and the quasi-optimal distribution of PZTs should have been med. Furthermore, the higher modes play a minor part in the vibration.

There are two QO calculated to x-direction. They are clearly depicted in Fig. 2.

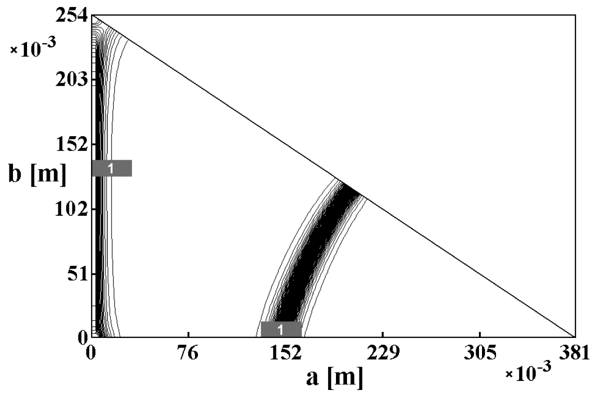

Fig. 2. Two quasi-optimal sub-areas (QO).

In the paper [1–3] only one PZT on each QO was bonded. In this paper the successive PZTs, on QO, are added, according to the following rule (not arbitrarily!). On each QO, the successive PZT takes this part of the free surface which has a maximum curvature. Clearly, the first PZT (marked by number one, Fig. 2) takes up space with the greatest curvature. On the remaining surface, the next part of the surface with the greatest curvature is determined. In this place, the successive PZT (marked by number two, Fig. 3) is bonded. The procedure outlined previously is repeated.

4. Quasi-optimal distribution of higher number of the PZTs

In the paper seven PZTs are bonded on each QO, they are depicted in Fig. 3. The number of the PZT means the succession in which the particular PZTs were added to the system.

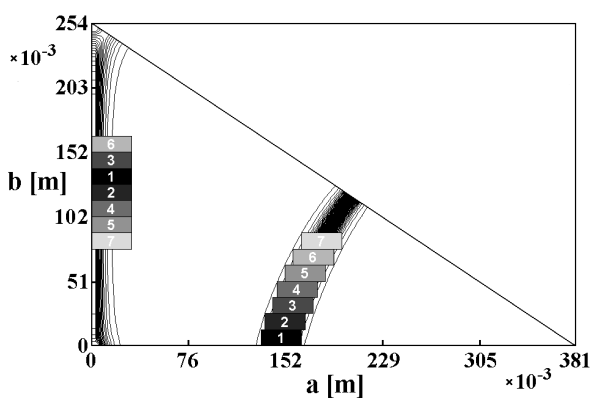

Fig. 3. The distribution of the PZTs on QO.

5. Numerical calculations, results and discussion

The effect of acting of the successive PZTs is measured by the reduction of:
- the vibration amplitude on the surface plate,
- the bending moment at the clamped edge,
- the shearing force at the clamped edge.

In practice, two last quantities play the principal role. But the reduction of the first one is transposed on the remaining ones. Hence the reduction of the amplitude has an essential meaning in the vibration reduction problem. All numerical experiments were performed for 4, 6, 8, 10, 12, 14 PZTs and they are related to the ones performed for 2 PZTs [1, 2]. But only some more interesting results are presented hereafter.

To determine the reduction of the amplitude, the reduction amplitude factor is formulated [1–3],

$$R = \frac{A - A_P}{A} \cdot 100\%, \tag{8}$$

where A – vibration amplitude without PZTs, A_P – vibration amplitude with PZTs.

Based on Eq. (8), the results of the numerical experiments are presented in Fig. 4 for only 2 and 8 PZTs.

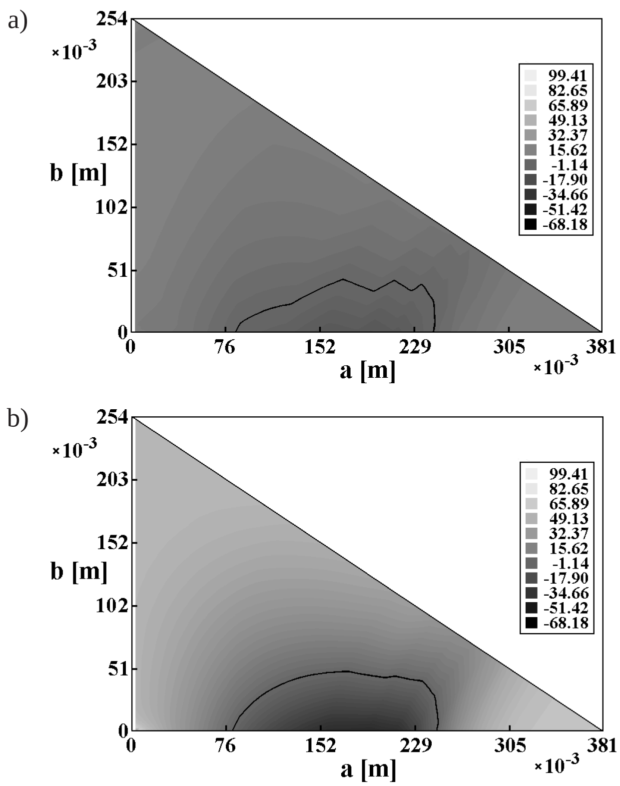

Fig. 4. Reduction amplitude factor for: a) 2 PZTs, b) 8 PZTs.

Examining the Fig. 4a, b considerable decreases of vibration amplitude are observed in almost nearly whole surface. The decrease, for the successive PZTs, is larger than for the previous ones if the number of PZTs does not exceed any boundary; all results in compact form are presented in Fig. 7. Only in a certain area (marked inside the dark line), the amplitude somewhat increases. One way or another, globally (on the whole surface) in all cases, the decrease of amplitude is bigger than its increase.

The reduction of the vibration amplitude is transposed to the reduction of the bending moment and shearing force at the clamped edge. These two quantities are particularly interesting from the mechanical point of view. They are calculated assuming the same conditions as those given above for reduction of the vibration amplitude. The results are presented in Figs. 5 and 6 respectively.

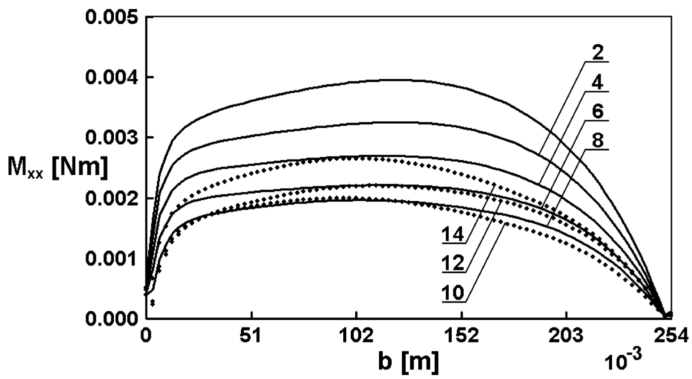

Fig. 5. Bending moment at the clamped edge; solid lines – without PZTs, 2, 4, ... – with 2, 4, ... PZTs respectively.

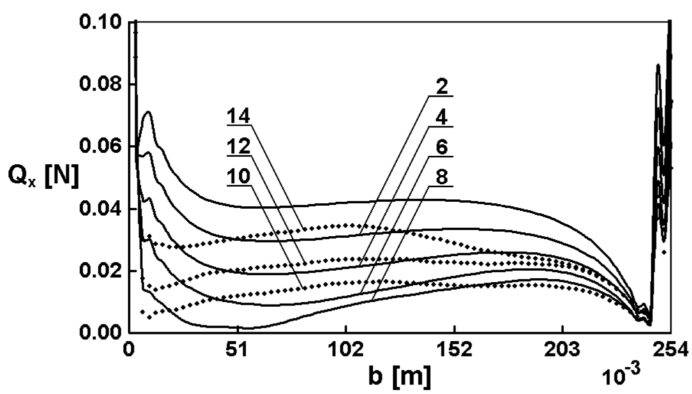

Fig. 6. Shearing force at the clamped edge; solid lines – without PZTs, 2, 4, ... – with 2, 4, ... PZTs respectively.

As it can be seen from the last figures, both quantities decrease in relation to the number of PZTs provided that their number does not exceed any boundary.

All results are presented in Fig. 7 in compact form. Namely, the algebraic mean values of the reduction of above quantities (in percent) are plotted against the number of the PZTs. They are computed from the following general formula:

$$R_\alpha = \frac{1}{n_i} \sum_i \frac{\alpha_i - \alpha_{Pi}}{\alpha_i} \cdot 100\%, \qquad i = 1, 2, ..., n_i, \tag{9}$$

where α_i and α_{Pi} – values of the quantity without and with PZTs, respectively, at the calculated i-point, n_i – number of calculated points.

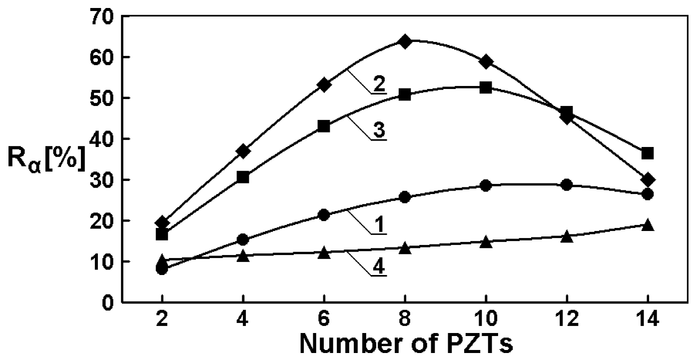

Fig. 7. Mean values of the reduction vs. PZTs number: 1 – vibration amplitude, 2 – bending moment, 3 – shearing force, 4 – surface of the increasing vibration amplitude.

These quantities are the vibration amplitude, amplitude of the bending moment and shearing force at the clamped edge and the magnitude of the surface on which the vibration amplitude increases.

Based on the Fig. 7, the numerical experiments show that if the number of PZTs increases, the effectiveness of the vibration reduction also increases. But for the number of PZTs larger than eight (approximately), the conclusion is contrary. As it can be seen, the mean value of the shearing force reduction is somewhat worse. In the circumstances described above, the optimal number of PZTs is about eight. Exceeding this (optimal) value, the PZTs act as actuators and the vibrations are intensified. Until now, no objective criterion of the optimal number of PZTs has been proposed.

6. Conclusions

Theory and numerical calculations lead to the following conclusions:

1. The quasi-optimal sub-areas are the best places to attach some PZTs in active vibration reduction of triangular plates. This conclusion comes from considerations performed in this paper and the other ones cited in references.

2. The increase of the PZTs number, to any boundary by one, assures a better effect, i.e. better vibration reduction. The effect may be measured, in some sense, with the vibration amplitude on the whole plate surface, the bending moment and shearing force at the clamped edge. Exceeding any boundary number of the PZTs, they are used as actuators and their effectiveness is worse.
3. Using these results based on numerical calculations, the nearly optimal number of PZTs may be determined.
4. It is possible to improve the effectiveness of the active triangular plate vibration reduction by increasing the number of the PZTs bonded on quasi-optimal sub-areas.

The conclusions confirm the validation of an idea given in the title.

Despite the researches provided here are limited to one mode and C-F-F boundary conditions, it seems that the presented method of effectiveness improvement can be applied to other cases.

References

[1] BRAŃSKI A., SZELA S., *The quasi-optimal distribution of PZTs in active reduction of triangular plate vibration*, Proceedings 8th Conference on Active Noise and Vibration Control Methods, pp. 60–61, AGH, Kraków 2007.

[2] BRAŃSKI A., SZELA S., *On the quasi-optimal distribution of PZTs in active reduction of the triangular plate vibration*, Archives of Control Sciences, **17**, 4, 427–437 (2007).

[3] BRAŃSKI A., SZELA S., *Evaluation of active vibration reduction of the triangular plate via acoustic field parameter*, Proceedings LIV Open Seminar on Acoustics, pp. 162–163, University of Rzeszów, Rzeszów 2007.

[4] FULLER C.R., ELLIOT S.J., NIELSEN P.A., *Active Control of Vibration*, Academic Press, London 1997.

[5] HANSEN C.H., SNYDER S.D., *Active control of noise and vibration*, E&FN SPON, London 1997.

[6] KOZIEŃ M.S., WICIAK J., *Influence of the piezoelectric elements configuration on the lowering of plate vibrations*, Proceedings of the Conference: Structural Acoustics – Waves – Biomedical Engineering, pp. 47–54, Polish Acoustical Society, Kraków–Zakopane 2003.

[7] LEISSA A.W., *Vibration of plates*, NASA SP-160, D.C.: Office of Technology Utilization, NASA, Washington 1969.

[8] MIRZA S., ALIZADEH Y., *Free vibration of partially supported triangular plates*, Computers & Structures, **51**, 2, 143–150 (1994).

[9] PIETRZAKOWSKI M., *Active damping of transverse vibration using distributed piezoelectric elements*, Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa 2004.

[10] SINGHAL R., REDEKOP D., *Vibration of right-angled triangular plates partially clamped on one side*, Journal of Sound and Vibration, **251**, 2, 377–382 (2002).

[11] TIMOSHENKO S.P., WOINOWSKY–KRIEGER S., *Theory of plates and shells*, McGraw-Hill, New York 1959.

[12] TYLIKOWSKI A., *Control of circular plate vibrations via piezoelectric actuators shunted with a capacitive circuit*, Thin-Walled Structures, **39**, 83–94 (2001).

[13] WICIAK J., *Investigations of a noise control of a plate with four pairs of piezoelectric elements*, Archives of Acoustics, **31**, 503–512 (2006).