

INFLUENCE OF DAMPING AND FLUID LOADING ON THE PLATE VIBRATION CONTROL

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The aim of this paper is to present the effect of acknowledging viscous damping, structural internal damping and fluid loading on active vibration control of a circular plate in case of axially symmetrical vibrations. It was assumed that a planar vibrating structure located in a finite baffle and interacting with fluid is driven by a periodic force with constant amplitude, so the structure radiates the acoustic waves into a surrounding fluid and the axially-symmetrically located circular piezoelectric actuators are used to reduce its vibrations. For the purpose of control strategy designing process the continuous model derived from physical principles for the system under consideration has been developed. Next, the linear state model was obtained by reduction and approximation of the continuous model using the orthogonal series method. After that, the acquired model was used to produce body plots and to solve corresponding Riccati equation. Obtained control forces led to significant reduction of the plate vibration and attenuation of accompanied acoustic waves. The simulations of the active cancellation of the plate vibrations were made with a Simulink/Matlab computer program.

Keywords: fluid loading, viscous damping structural internal damping, plate vibration control.

1. Introduction

The vibration and sound radiation of a circular plates have been studied by many researchers since it is a significant structural element in many industrial fields. Lord Rayleigh was the first who analysed the “reaction of the air on a vibrating circular plate” [14], showing that reaction to be equivalent to a virtual mass and radiation damping to be added to the plate mass and the mechanical damping. For the design of an effective kind of control suppressing plates vibrations and related acoustic radiation, the accurate modeling of the acoustic structural and coupling components is necessary. Except of internal and viscous damping phenomenon, the major difficulty when treating acoustic radiation into fluid medium is the inclusion of the fluid structure coupling.

As pointed out by JUNGER and FEIT [5] it should be noted that the structure/fluid interaction in case of plates, where all plate modes are coupled by the fluid, may change plates responses in a significant way. The effect of the fluid loading as well as the influence of viscous and internal damping needs to be carefully examined if one wants to design an effective kind of control.

Early solutions of the radiation of plates with fluid/structure coupling were given by DAVIES [3] in case of a simply supported rectangular plate. He proposed to use a wavenumber transformation of the fluid and structure governing equations and to express the radiation impedance in terms of the *in vacuo* modes of the plate. The classical methods using *in vacuo* plate eigenfunctions to formulae fluid/structure coupling, which essentially involves the calculation of the radiation impedance, have been reported also for a circular plate in an infinite baffle (LEVINE and LEPPINGTON [13], KWAK and KIM [8], RDZANEK [16], more recently by DINGGUO and CROCKER [4]) and also for the plate located in a finite baffle (LENIOWSKA [9]).

Furthermore, the problem of control of fluid-loaded circular plate has been examined by several authors FULLER [5], GU and FULLER [6], MEIROVICH [14]. It was also solved by the author of this paper (LENIOWSKA [10–12]) for point and distributed actuators and for different boundary conditions of the circular plate located in a finite baffle.

The present work is a continuation of a previous study that considered active control of vibration and sound radiation from a clamped circular plate which radiates into a “light” fluid medium. It is focused on the investigation how much the damping and modal coupling may affect control performance. This study adds new understanding to research in controlling the vibration and sound radiation from objects in questions. It is demonstrated how the poles and zeros of object model can migrate on the complex plane in dependence of changing parameters and what are the limits of a such moving. This information is very useful for finding basic object properties as stability, etc. and to design an effective kind of control. Moreover, on the base of this information the feedback control law with feedforward correction is developed for sound and vibration cancellation of the considered plate.

In this approach the solution is expressed in terms of *in vacuo* plate eigenfunctions. The resulting integral expressions can be calculated numerically and they are used to form the state-space equation. The formal solution of the fluid-plate coupled equation is presented in case of axially symmetrical vibrations, for a plate driven by a uniform harmonic primary force and controlled by a distributed secondary forces generated by piezodisks. Three parameters which characterize fluid density, plate material internal damping and viscous fluid damping are included in the considered model. It is well-known that the dynamic behavior of the linear systems depends strongly on the location of the models’ roots (zeros and poles). To examine the stability of the system the roots of the system with assumed parameters were plotted on a complex plane. The effects of fluid-loading, internal and viscous damping on the system response are observed and presented graphically. Finally, the feedback control law is developed for the plate vibration cancellation.

2. State-space system model

The structure under study is a vibrating circular plate of radius a , having a constant thickness h , surrounded by a lossless medium with the rest density ρ_0 . In the case being considered, the applied loading and end restraints of the circular plate are independent of the angle φ , (axially symmetrical vibrations), thus we can write the governing differential equation of the forced motion of the plate as follows [11]:

$$B\nabla^4 w(r, t) + R\frac{\partial}{\partial t}[\nabla^4 w(r, t)] + \gamma\frac{\partial}{\partial t}w(r, t) + \rho h\frac{\partial^2}{\partial t^2}w(r, t) = f_w(r, t) + f_s(r, t) + f_p(r, t), \quad (1)$$

where $B = Eh^3/12(1 - \nu^2)$ is the bending stiffness of the plate, E , ν and R are the Young's modulus, Poisson's ratio and Kelvin–Voigt damping coefficient for the plate, ρ is density for the combined structure, and γ is viscous fluid damping coefficient, r is the radial variable. It is assumed that the plate, clamped in a flat, rigid and finite baffle of radius b , ($b > r > a$, $z = 0$), is excited on one side by a uniform periodic force with constant amplitude F_0 : $f_w(r, t) = F_0 e^{-i\omega t}$ for $0 \leq r \leq a$ and it radiates into free space filled with fluid of density ρ_0 . The system model is formulated when taking into account the coupling effect between the structure and the acoustic medium, so the third component of the right hand side of Eq. (1), $f_p(r, t)$, represents the acoustic fluid-loading acting on the plate as an additional force. The goal in the control problem is to determine a control force $f_s(r, t)$ which, when applied to the plate, leads to reduced level of vibrations. The second component of the right hand side of Eq. (1) represents such a wanted control force, $f_s(r, t)$, which will cancel the plate vibrations. The model presented above can be expressed in the state-space format [11]:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{V}\mathbf{z}(t), \quad (2)$$

where the dot denotes differentiation with respect to time, \mathbf{x} is the $(n \times 1)$ state vector, \mathbf{u} is $(m \times 1)$ control vector, and \mathbf{A} is $(n \times n)$ state matrix, \mathbf{B} is the $(n \times m)$ control input matrix, \mathbf{V} is $(1 \times n)$ disturbance matrix, described as follows:

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ -(\mathbf{I} + \mathbf{E})^{-1}\mathbf{\Omega}^2 & -(\mu_2 + \mu_1\mathbf{\Omega}^2)(\mathbf{I} + \mathbf{E})^{-1} \end{bmatrix}, \quad (3)$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ (\mathbf{I} + \mathbf{E})^{-1}\mathbf{K}_s \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} \mathbf{0} \\ (\mathbf{I} + \mathbf{E})^{-1}\mathbf{K}_w \end{bmatrix}.$$

In above expression \mathbf{I} denotes identity matrix, \mathbf{K}_s and \mathbf{K}_w are the coefficient vectors, \mathbf{E} represents fluid-plate interaction matrix, $\mathbf{\Omega} = \mathbf{diag}[\omega_1, \omega_2, \dots, \omega_N]$, $\mu_1 = R/B$ and $\mu_2 = \gamma/\rho h$.

3. System dynamics and damping effects

It is convenient to plot the roots of the system for assumed parameters on a complex plane to examine the possibility of instability of the system and the influence of fluid loading as well as structural internal damping and viscous damping. The plate is made of aluminum. The surrounding fluids were water, propane and air representing respectively strong, moderate and light coupling between the structural and acoustic response.

The location of poles and zeros of the eight-order system has been presented in air, for two values of parameter μ_1 and constant μ_2 (Fig. 1). It can be seen that the values of internal damping coefficient μ_1 have considerable influence on systems' root locations.

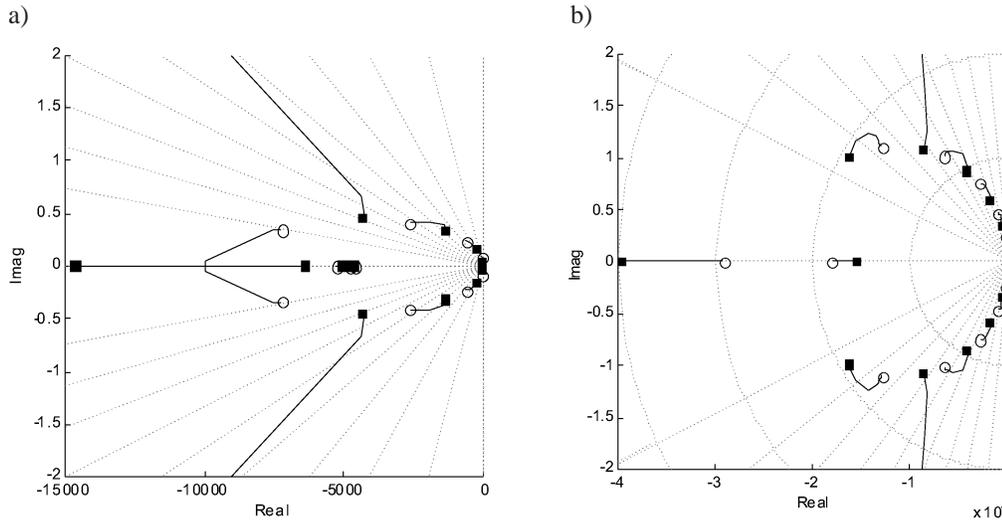


Fig. 1. Distribution of poles and zeros of the considered system in air for $\mu_2 = 0.5$: a) $\mu_1 = 0.0001$, b) $\mu_1 = 0.00002$; \circ – zeros, \blacksquare – poles.

On the basis of the pole locations we can make a conclusion about the system dynamics which may be observed on the Bode diagram (Fig. 2). It is worth to note that the modification of parameter μ_2 , assuming the linear range 0.1–100 [sN/m³], does not change root locations noticeably and can be observed on the Bode diagram in the vicinity of the resonance peaks. The third parameter included in the model derived, the fluid-loading term [2]: $\varepsilon_0 = \rho_0/\rho h k_0$, in which k_0 is the acoustic wave number, is helpful for examining the influence of the fluid surrounding the considered plate.

As the density of the surrounding fluid medium increases, the roots of the system move to point (0, 0) on the complex plane (Fig. 3).

The Bode diagram reveals an additional feature which is important for the correct design of controller transfer function, namely a phase shift, especially for low frequency radiated acoustic waves. As a result of the fluid coupling the response of the plate in fluid can be significantly different from responses *in vacuo*. It can also be observed, that the effect of the fluid-loading on the considered plate is dependent on the excitation

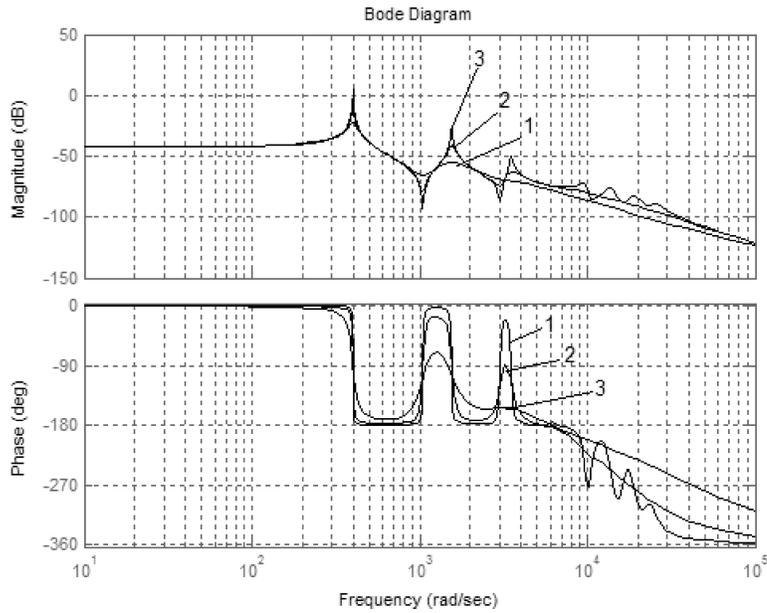


Fig. 2. Bode diagram of the fluid-plate system in air: 1 - $\mu_1 = 0.0001122$, 2 - $\mu_1 = 0.000022$, 3 - $\mu_1 = 0.000044$.

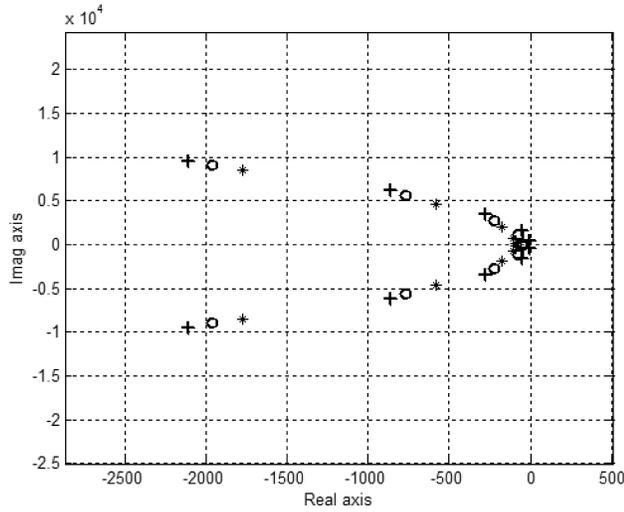


Fig. 3. Distribution of poles of the considered system for $\mu_1 = 0.00002$, $\mu_2 = 0.5$ and three kinds of the fluid density: * - $\rho_0 = 1000$, \circ - $\rho_0 = 500$, + - $\rho_0 = 1.2$.

frequency. In the case of lower frequencies, the shift of the resonance peaks is greater and when the operating frequency increases it diminishes.

To illustrate this effect let us consider an example where the fluid is water, so the strong coupling can be theoretically assumed. The Bode diagrams of the system in ques-

tion for the “lower” (Fig. 4) and “higher” (Fig. 5) frequencies of acoustic waves presents as follows:

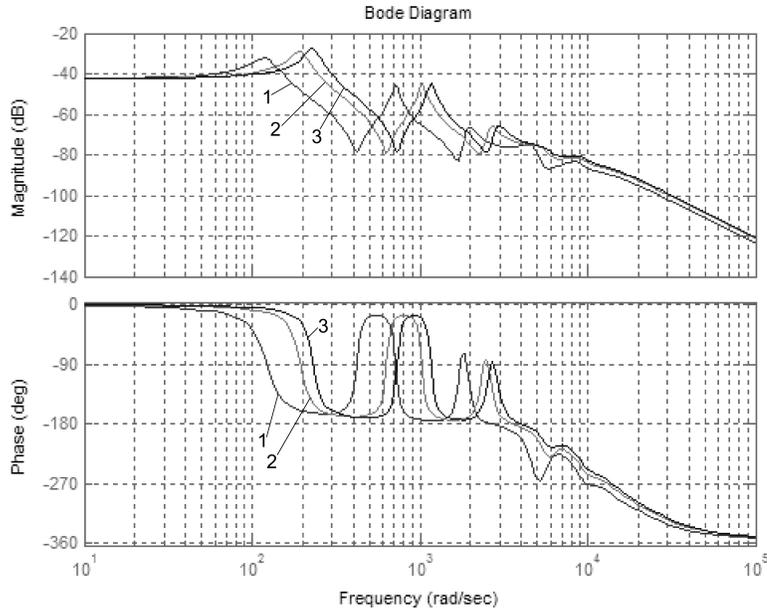


Fig. 4. Bode diagram of the fluid-plate system in water, for $\mu_1 = 0.00002$, $\mu_2 = 0.1$ and for the “lower” frequencies: 1 – $k_0 = 0.42$ (100 Hz), 2 – $k_0 = 1.26$ (300 Hz), 3 – $k_0 = 2.09$ (500 Hz).

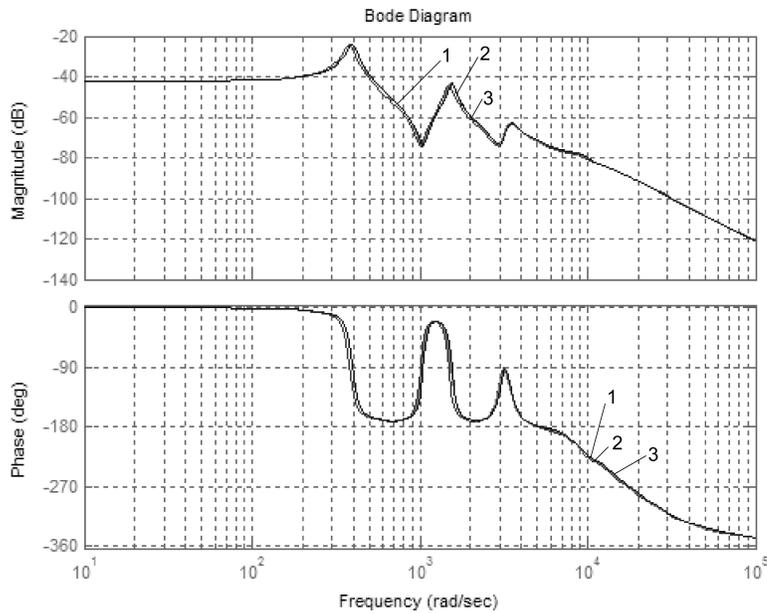


Fig. 5. Bode diagram of the fluid-plate system in water, for $\mu_1 = 0.00002$, $\mu_2 = 0.1$ and for the “higher” frequencies: 1 – $k_0 = 25.13$ (6000 Hz), 2 – $k_0 = 41.88$ (10000 Hz), 3 – $k_0 = 62.83$ (15000 Hz).

It can be observed that for higher frequencies the characteristics have not shifted. It leads into conclusion that the frequency spectrum is not an uniform domain. A further examination of the system dynamics shows that it divides into two regions. Below the frequency defined as [2]: $f_n = \frac{\rho_0 c_0}{2\pi\rho h}$, the surrounding fluid mass-loads the plate. In this case the resonances peaks and corresponding phases are shifted towards lower frequencies on the Bode diagram (Fig. 4). On the other hand, for frequencies of acoustic response, the fluid acts as dampener – the effect of the fluid-loading is small and curves on Bode diagram cover each other (Fig. 5).

4. Plate vibration control

The aim of the project is to design a control system to modify the response of the plant in some desired fashion. The closed-loop setup is sketched in Fig. 6.

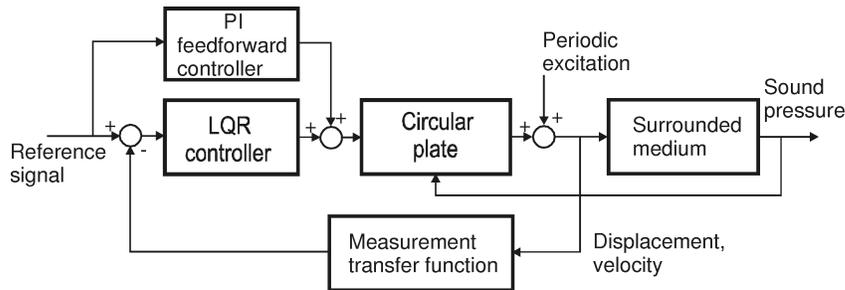


Fig. 6. Block diagram of the simulated system.

The object is to minimise the plate vibrations and the far-field acoustic pressure $p(P_o, t)$, in plate axis. This is to be achieved by the control force $\mathbf{f}_s(t) = \mathbf{u}(t)$ acting on the plate surface. For the system modelled as (2)–(3) the problem is to determine the necessary control $\mathbf{u}(t)$ which will minimise in time t_k the following performance index [11]:

$$J = \frac{1}{2} \int_0^{t_k} [\mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{u}^T \mathbf{R} \mathbf{u} + \vartheta \dot{\mathbf{x}}^T \mathbf{P} \dot{\mathbf{x}}] dt, \quad (4)$$

where \mathbf{Q} , \mathbf{R} denote weighting matrices and \mathbf{P} is the far-field acoustic pressure matrix corresponding to $p(P_o, t)$ [11]. The control input that minimises this performance index is derived by applying Hamilton's principle and by solving Riccati equation using Shur tuning technique [1]. Finally we obtain the following control law [11]:

$$\mathbf{u} = -\mathbf{R}^{*-1} [(\mathbf{Q}_{AB}^{*T} + \mathbf{B}^T \mathbf{K}) \mathbf{x} + \mathbf{Q}_{BV}^* \mathbf{z} + \mathbf{B}^T \mathbf{v}]. \quad (5)$$

The obtained control contains three components. The first one is a matrix coefficient $\mathbf{B}^T \mathbf{K}$ with an additional weighting matrix \mathbf{Q}_{AB}^{*T} multiplied by the state vector. The next two components make additional feedforward correction with PI-structure.

In the case of feedforward controller the gain coefficients are achieved from the analytical computations according to the expression [11]:

$$\mathbf{v}(t) = \int_0^{t_k} e^{\mathbf{A}^*(t-\tau)} \mathbf{B}^* \mathbf{u}(\tau) d\tau. \quad (6)$$

The integral and the proportional part of the wanted control $\mathbf{u}(t)$ in (5) is calculated directly from the excitation signal.

In simulations the model including the eight modes of the aluminum plate of a 0.4 m diameter and 1 mm thickness was applied. The time response of the system on sinusoidal disturbance of 100 Hz has been obtained using the Simulink/Matlab computer program.

The plate displacement (sum of eight modes) is plotted in Fig. 7a. It can be seen that the uncontrolled plate response vibrates significantly while the switching on the LQR controller (after 0.4 sec) causes that plate vibrations have been reduced about 50%. The acoustic pressure generated by the plate in z axes at Fraunhofer's zone is plotted in the Fig. 7b. It is attenuated after 0.4 sec about 40%, except of the points where the controller starts. The feedforward corrector which starts after 1.2 seconds improves both results significantly.

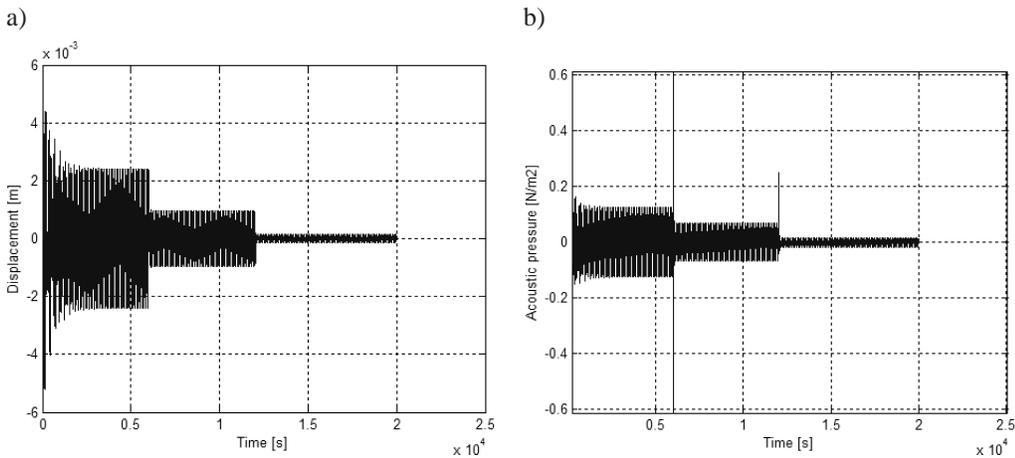


Fig. 7. The time response of the open-loop system (0–0.4 s), the close-loop system with LQR controller (0.4–1.2 s) and the close-loop system with LQR controller and feedforward corrector (1.2–2 s) to the sinusoidal signal of 100 Hz for $a_1/a = 0.07$: a) the plate displacement (sum of eight modes), b) the acoustics pressure generated by the plate at Fraunhofer's zone.

5. Concluding remarks

This paper is focused on the investigation how much the damping and modal coupling may affect control performance. The structure under consideration was a thin circular plate with distributed control forces located in its center. The application of

optimal linear quadratic theory to the problem of plate vibrations, inducing acoustic noise in the audible frequency range, shows that this control technique led to a very good reduction of plate vibration, but the response of the plate in fluid can be significantly different from responses *in vacuo*. It can be observed, that the effect of the fluid-loading on the considered plate is dependent on the frequency of the vibration. In the case of lower frequencies, the shift of the resonance peaks and corresponding phases is greater and when the operating frequency increases it diminishes. A further examination of the system dynamics shows that the frequency spectrum can be divided into two regions. Below the border frequency: $f_n = \frac{\rho_0 c_0}{2\pi \rho h}$, the surrounding fluid mass-loads the plate, and above – the fluid acts as dampener. It can be also stated that the values of internal damping coefficient μ_1 have considerable influence on systems' root locations.

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