

## ACOUSTIC CAVITATION AND BUBBLE DYNAMICS

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Acoustic cavitation is investigated experimentally and theoretically starting with a single bubble in an acoustic field. The single bubble is trapped in an acoustic resonator and observed by high-speed imaging and acoustic measurements following its transient dynamics to a possible steady state after generation by laser light. Numerical calculations are done to explore the regions of survival for acoustically driven bubbles with respect to dissolution or growth, surface oscillations and positional stability in a standing sound field. The interior gas dynamics is explored via molecular dynamics simulations. A glance is thrown at the complex dynamics of multi-bubble systems.

**Keywords:** acoustic cavitation, bubbles.

### 1. Introduction

Intense sound fields in liquids generate cavities or bubbles that start to oscillate and move around in the liquid, a phenomenon called acoustic cavitation [2, 11, 13]. The basic element to start an investigation with is the single, spherical bubble in an infinite liquid subject to a sinusoidal sound field. The best experimental approximation so far is realized in the acoustic bubble trap as mainly used for sonoluminescence studies [5], and in bubbles generated by focused laser light [9]. Both cases have been combined to investigate transient single bubble dynamics and its sonoluminescence in an acoustic field [8]. Theoretical bubble models have been conceived with different states of sophistication to include different influences and phenomena as, for instance, liquid compressibility, heat and mass transfer across the bubble wall or stability of the surface. Noteworthy is the strong collapse of a bubble at high sound pressure amplitudes leading to shock wave and light emission and high speed liquid jet formation under appropriate conditions [11]. The interior of a bubble is usually treated as homogeneous with respect to temperature and pressure. However, this may not be true in the last phase of

a strong collapse [21]. This problem is not only tackled by continuum mechanics via partial differential equations but also in a molecular dynamics approach [10, 15].

The next step in bubble dynamics investigations is given by bubble interaction. It leads to complicated forms of attraction and repulsion already in the case of two spherical, interacting bubbles. A survey of bubble structures as they appear in acoustic cavitation is given in [16]. A particle model has been formulated to address the problem of multi-bubble systems in an ultrasonic field as a way to cope with acoustic cavitation bubble ensembles [17, 19]. This paper is a kind of progress report in the field of bubble dynamics in acoustic fields.

## 2. Single bubble dynamics

From the many models to describe the oscillation of bubbles in an acoustic field the model of KELLER and MIKSYS [6] is adopted:

$$\left(1 - \frac{\dot{R}}{C_l}\right) R\ddot{R} + \frac{3}{2} \left(1 - \frac{\dot{R}}{3C_l}\right) \dot{R}^2 = \left(1 + \frac{\dot{R}}{C_l}\right) \frac{p}{\rho_l} + \frac{R}{\rho_l C_l} \frac{dp}{dt}. \quad (1)$$

Here,  $R$  is the bubble radius,  $C_l$  the sound velocity of the liquid at the bubble wall,  $\rho_l$  the density of the liquid, and  $p$  the difference between external and internal pressure terms at the bubble wall, as given in the following equation.

$$p = \left(p_{\text{stat}} + \frac{2\sigma}{R_n}\right) \left(\frac{R_n}{R}\right)^{3\kappa} - p_{\text{stat}} - \frac{2\sigma}{R} - \frac{4\mu}{R} \dot{R} - p_s. \quad (2)$$

The quantities introduced are:  $p_{\text{stat}}$  the static ambient pressure,  $\sigma$  the surface tension,  $R_n$  the radius of the bubble at rest,  $\kappa$  the polytropic exponent of the gas inside the bubble, and  $\mu$  the dynamic viscosity.  $p_s$  is an additional external pressure, for instance the pressure pulse of a lithotripter.

The Keller-Miksis bubble equation has been solved for various parameter configurations with a sinusoidal sound field  $p_s$ :

$$p_s = \hat{p}_A \sin(2\pi\nu t). \quad (3)$$

Figure 1 gives a typical set of steady-state bubble oscillations with increasing sound pressure amplitude from  $\hat{p}_A = 100$  kPa to 200 kPa in steps of 20 kPa at a frequency of  $\nu = 20$  kHz for a bubble with a radius at rest of  $R_n = 5$   $\mu\text{m}$ . After a long expansion phase a steep collapse occurs with some afterbounces until the cycle repeats. Furthermore, the maximum of the bubble oscillation shifts from the tension phase of the driving sound field to the pressure phase leading to a shift in the stability position of the bubble in a standing sound field [1, 14].

For a more thorough description of the oscillation properties of a bubble condensed forms of diagrams have been developed to combine special features as in bifurcation diagrams or in phase diagrams [12, 18]. Figure 2 gives a phase diagram of the periodicity of the oscillation with respect to the forcing sound oscillation for bubbles in the range

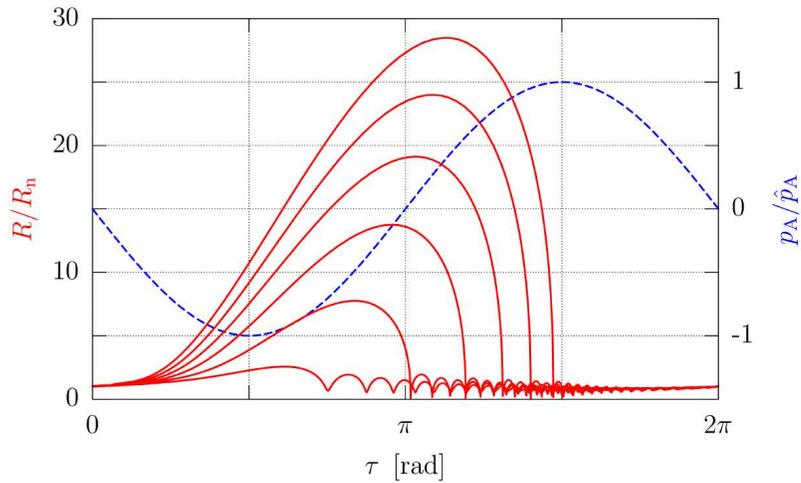


Fig. 1. Typical steady-state bubble oscillations with steep collapse and afterbounces. Shift of the relative phase of the bubble oscillation with respect to the phase of the sound field with increasing sound pressure amplitude;  $\hat{p}_A = 100 \dots 200$  kPa,  $\nu = 20$  kHz,  $R_n = 5$   $\mu\text{m}$ , isothermal.

of 10  $\mu\text{m}$  to 100  $\mu\text{m}$  and for driving pressure amplitudes from 0 to 300 kPa for a sound field frequency of  $\nu = 20$  kHz. It is seen that specific ranges with repeated structures of interlaced periodicity appear that are similar for other and presumably all bubble models [18] and even a larger class of nonlinear oscillators [20].

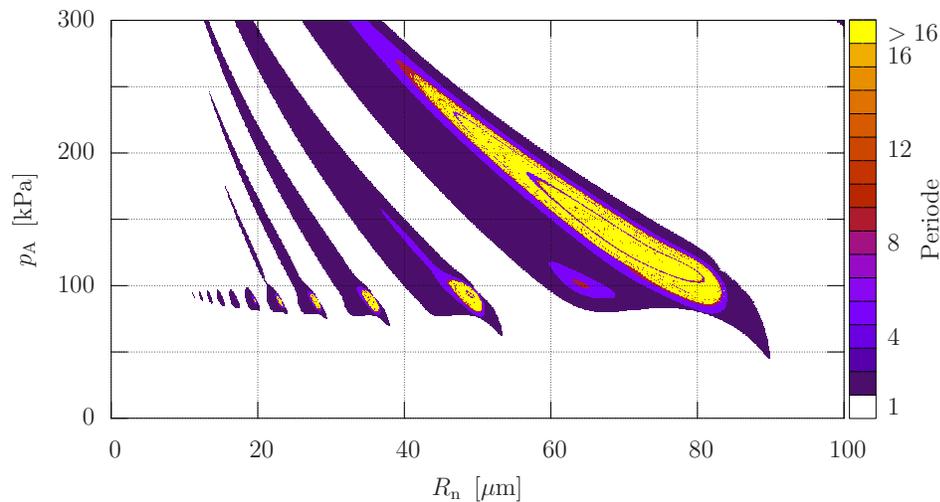


Fig. 2. Phase diagram giving the periodicity of the bubble oscillations with respect to the driving sound oscillation;  $\nu = 20$  kHz, isothermal.

The bubble model can be extended to include diffusion processes [3, 4]. Then bubbles may dissolve or even grow due to a process called rectified diffusion. Also the

bubble may become positional unstable in the sound field due to acoustic radiation forces (Bjerknes forces). Moreover, when additionally surface oscillation of the bubble are included [7] a restricted survival region results in the sound pressure amplitude – equilibrium bubble radius plane. In Fig. 3 this stability region is shown shaded for bubbles with rest radii between 0 and 30  $\mu\text{m}$  and for pressure amplitude between 0 and 300 kPa at a fixed frequency of the sound field of 20 kHz. Below the shaded region the bubble is diffusion unstable, above the phenomenon of rectified diffusion prevents the dissolution of the bubble. Above the shaded region the bubble gets unstable due to surface oscillations that lead to a break-up of the bubble. There is a dotted line running through the middle of the figure. This dotted line separates the bubbles being attracted to the pressure antinode in the standing sound field (in a resonator or bubble trap) and thus can be stably trapped and the bubbles above that are driven away from the pressure antinode and are sentenced to move to and on a surface of lower pressure in the standing sound field.

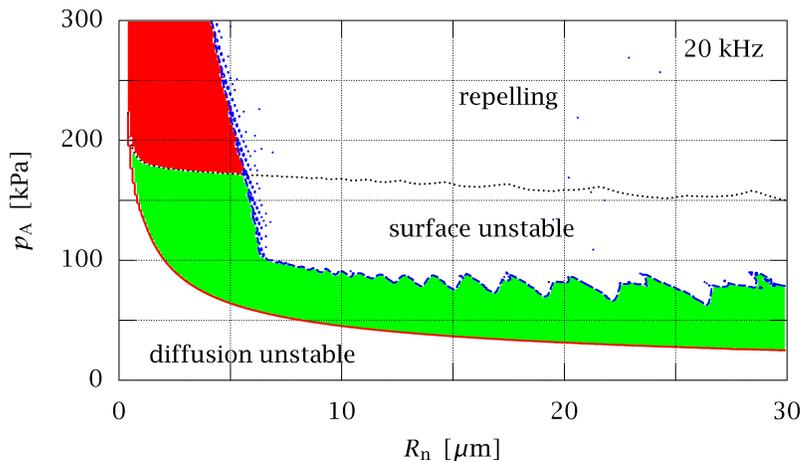


Fig. 3. Region of stable bubble oscillations in a standing sound field of  $\nu = 20$  kHz (shaded area).

This type of diagram can be calculated for other frequencies and for other ranges of sound pressure and bubble radii. Figure 4 gives an example for  $\nu = 40$  kHz in an extended bubble range normalized to the linear resonance radius  $R_n^{\text{lin}} = 69 \mu\text{m}$  and for the acoustic pressure range 0 to 500 kPa. It can be noticed that the positionally stable region (lightly shaded) is a rather small rugged stripe and located below the linear resonance radius. Generally it can be stated from calculations for higher frequencies that the survival space shrinks for higher frequencies to a small region in this parameter space.

When the interior of the bubble is closer looked at, additional phenomena are observed. The interior contents may not be homogeneously distributed, but pile up at the bubble boundary or in the center, compression waves may run towards the center and chemical reactions may take place. This problem has been addressed by molecular dy-

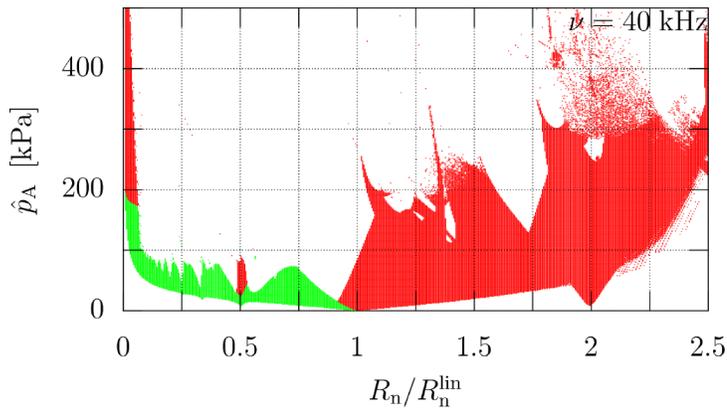


Fig. 4. Region of stable bubble oscillations in a standing sound field of  $\nu = 40$  kHz (shaded area).

namics calculations where the interior contents are modelled by a number of particles that move around in the interior according to the laws of mechanics. They transfer impulse and energy when they hit each other and the bubble wall. The interior dynamics is calculated when the bubble is subject to a collapse in a sound field [10]. Figure 5 shows the interior temperature distribution in a bubble for 800 ps around its strong collapse. In this case with nonreacting particles (Argon) and reacting particles (water molecules) maximum temperatures of about 18,000 K are reached in the center.

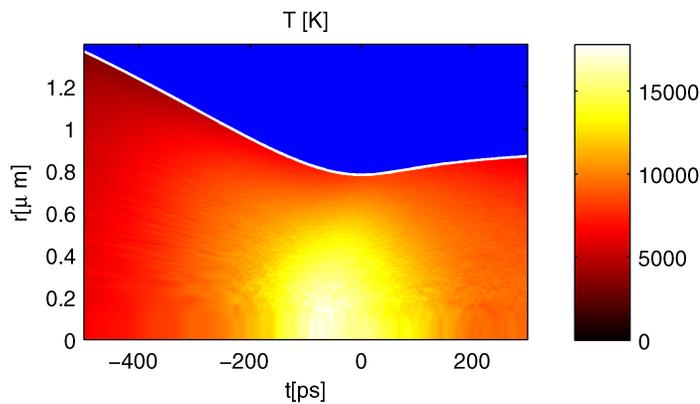


Fig. 5. The distribution of temperature inside a bubble versus time upon collapse. Rest radius of the bubble  $R_n = 4.5 \mu\text{m}$  in a sound field of  $\nu = 26.5$  kHz at a pressure amplitude of 1.3 bar. Argon–vapour bubble including chemical reactions.

### 3. Experiments

To study single bubble dynamics in a controlled way a method to prepare a bubble and insert it in a sound field without negligible disturbance of the field must be

conceived. One method is to produce a bubble by laser light at a pressure antinode of a standing wave in an acoustic resonator [8]. Figure 6 shows an arrangement for this purpose. The bubbles are generated by focusing a laser light pulse with a duration of about 130 fs and an energy of a few  $\mu\text{J}$  into a water filled cuvette that is excited by a transducer to its (1,1,1)-mode at about 44 kHz. That way, a tiny bubble is placed at the pressure antinode where it is kept by Bjerknes forces during its oscillations when being able to reach its survival zone (compare Fig. 3). The bubble is photographed by a CCD camera through a long distance microscope. Also the light emitted by the bubble can be registered.

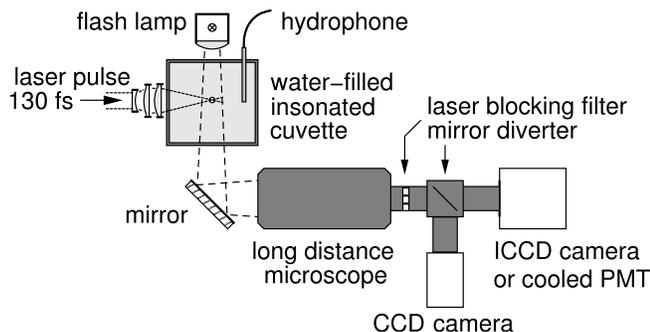


Fig. 6. Experimental arrangement for the investigation of laser generated bubbles in an ultrasonic field.

Figure 7 shows the evolution of a bubble inserted into the sound field versus the number of acoustic cycles from one up to a million cycles. The conditions are set to

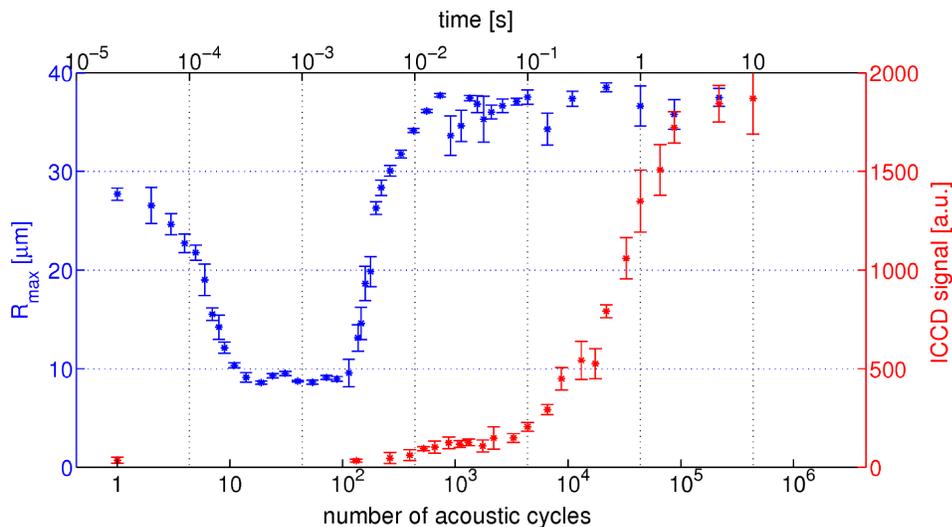


Fig. 7. Long-term evolution of the maximum bubble radius (left curve) and emitted light as captured by an ICCD camera (right curve). The acoustic frequency is 43.68 kHz, and the driving pressure 1.55 bar (Courtesy of T. Wilken).

typical single bubble sonoluminescence conditions with an acoustic pressure of 1.55 bar at 43.68 kHz and 200 mbar partial pressure of dissolved air. In this case the bubble survives for long time undergoing shrinking and growth presumably by diffusion processes until it settles at a large maximum bubble radius and the maximum light emission during its development. Before the onset of bright light output, but already at large maximum bubble radius, the bubble experiences a jittering motion presumably by surface instabilities leading to the shedding of microbubbles. After these adjustments of the bubble the light emission grows strongly to a long-term stable state.

#### 4. Acoustic cavitation bubble clouds

When many bubbles are present simultaneously as normally in acoustic cavitation, the bubbles interact and form structures [16, 17, 19]. These highly dynamic structures are not easily captured, but new technological developments in high speed videography open up new possibilities. Figure 8 shows an arrangement where two high speed CCD cameras are combined to capture a stereoscopic image of the bubble cloud to locate the individual bubbles in three-dimensional space. Figure 9 gives an example of the spatial distribution of bubbles as taken from 75 consecutive frames. The sound field frequency is 22.8 kHz and the sound pressure amplitude is 132 kPa. It is seen that the bubbles aggregate on a filamentary structure. This is due to attractive forces (besides repulsive ones) among bubbles and acoustic radiation forces that drive small bubbles to the pressure antinode.

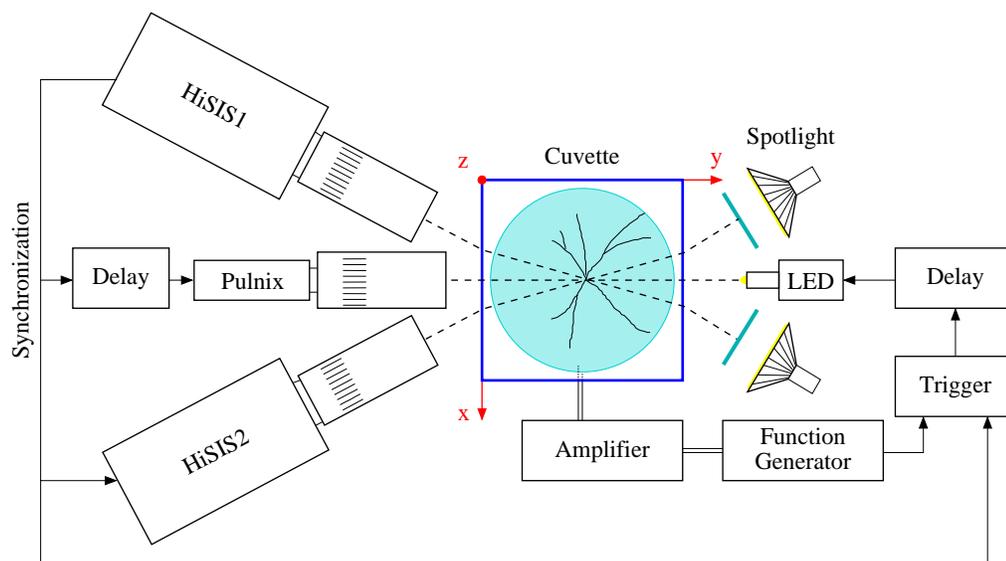


Fig. 8. Experimental arrangement for stereoscopic high speed videography of bubble clouds in a sound field.

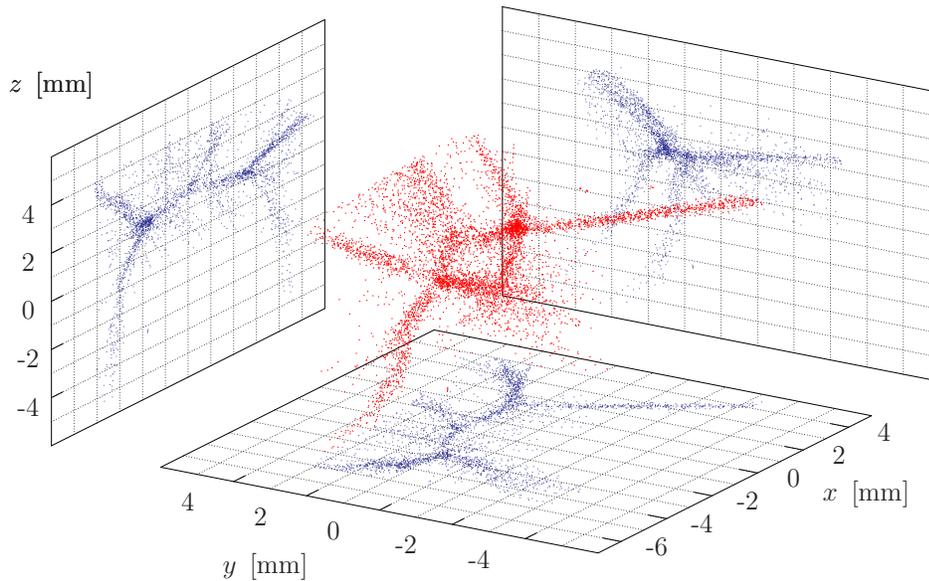


Fig. 9. Spatial distribution of bubbles from stereoscopic high speed videography at 2250 frames/s.

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