Calculation Models for Acoustic Analysis of Sacral Objects

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Application of the technique of singular value decomposition (SVD) for analysis of acoustic properties of sacral objects has been presented in the paper. Previous researches by means of the SVD technique were related to the simplified calculation model basing on a three-index observation matrix. The next stage of researches presented in the paper is an attempt of decomposition of the observation matrix containing five partial indices of the index acoustic assessment of sacral objects. The matrices obtained through the SVD decomposition have been used for calculation of weights of partial indices applied for the global assessment of acoustic quality of the sanctuaries. The comparison of acoustic assessments of the sanctuaries by the global indices containing traditional weights and the weights obtained by means of the SVD technique has been performed.

Keywords: acoustic quality, index method, sacral objects, Singular Value Decomposition (SVD).

1. Introduction

The acoustic quality of a sacral object is characterised by various factors. They include the reverberation time, speech intelligibility, music sound quality, the uniformity of loudness, the external disturbances etc. [5]. Elaboration of the proposed index method for assessment of acoustic quality of sacral objects has been preceded by the series of researches and analyses by means of adapted methods for acoustic assessment of other types of rooms, such as audiences and concert halls [3, 4]. The index method is a comparative and approximate method, in which the measured values of acoustic parameters are compared with the preferred values by relations elaborated in the form of indices. Each acoustic property of every sanctuary interior is assessed, as well as an approximate evaluation

of acoustic quality by the global index is performed in a complex way [4, 5]. During further researches of acoustic properties of the sanctuaries, the singular value decomposition (SVD) has been applied to the above mentioned assessment [6]. The SVD method has many applications in various field of science, as well in diagnostics [1] and in investigations of vibroacoustic processes [2]. The observation matrix related to six objects and three partial indices has been analysed in order to obtain the hidden structure of sets [7]. The next stage of researches described in the paper is performance of the SVD analysis upon the observation matrix consisting of all five partial indices elaborated in the index method.

2. The observation matrix and singular value decomposition (SVD)

According to the proposed index method [4], the values of partial indices, i.e. reverberation index W_r, music sound quality index W_m, intelligibility of speech index W_{is}, external disturbances index W_{ed} and uniformity of loudness index W_{ul}, have been calculated on the basis of acoustic researches in every sacral object. The results are presented in Table 1. The reverberation index in Table 1 has been calculated according to the modified formula specified in [6] and [7], thereby the results are better than the ones obtained in accordance with the traditional relation for this index given in [4].

Table 1. The values of partial indices for examined sacral objects.

No	Sacral object	Volume, m^3	W_r	W_m	W_{is}	W_{ed}	W_{ul}
1	The St. Sebastian's Church in Strzelce Wielkie	1102	0.89	0.67	0.49	1.00	0.2
2	The Holiest Sacred Heart's Church in Kraków	2750	0.98	0.6	0.34	0.39	0.53
3	The St. Clemens Church in Wieliczka	6380	1	0.49	0.34	0.58	0.21
4	The Jesus Heart's Church in Kraków	9120	0.91	0.42	0.23	0.57	0.43
5	The Reformati Fathers Church in Wieliczka	4455	0.86	0.48	0.33	0.37	0.26
6	The St. Paul Apostle's Church in Bochnia	22000	0.11	0.21	0.21	1.00	0.78

The values of partial indices for six Roman Catholic churches have become the basis for forming the matrix observation A (of indices and sacral objects). The rows of the matrix A are the sacral objects listed in Table 1. The columns

of the matrix A are formed using the consecutive partial indices (Table 1). The matrix A of $m = 6$ and $n = 5$ sizes has been decomposed by the SVD technique into three specific matrices, i.e. U, Σ and V. The graphic interpretation of the SVD decomposition of the matrix of sacral objects and indices is shown in Fig. 1.
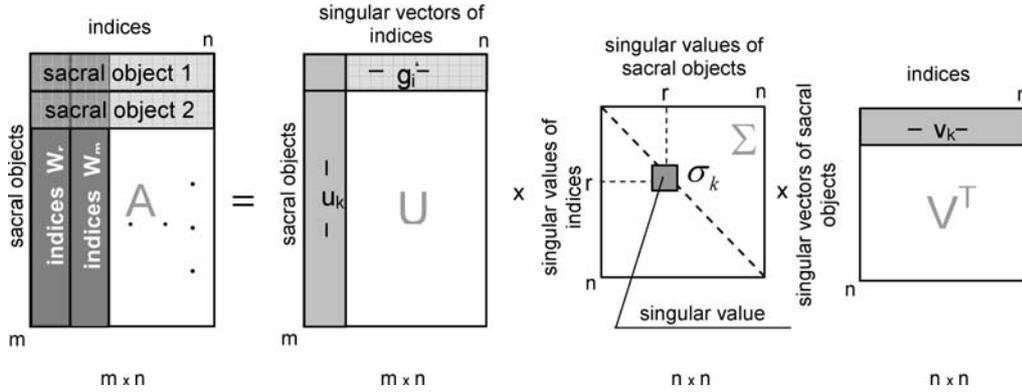

Fig. 1. The graphic interpretation of decomposition of the matrix of sacral objects and indices by the SVD technique [7].

The rows of the matrix A are vectors g_i that according to the following relation [7]:

$$A = U \cdot \Sigma \cdot V^{\mathrm{T}}, \tag{1}$$

where U – orthonormal matrix $m \times n$, Σ – diagonal matrix $n \times n$, V – orthonormal matrix $n \times n$, can be expressed as a linear combination of values of vector v_k:

$$g_i = \sum_{k=1}^{r} u_{ik} \sigma_k v_k \qquad i = 1, 2, \ldots, m. \tag{2}$$

Vector g'_i (Fig. 1) includes coordinates of i-th sacral object in the coordinate system (base) of weighted singular vectors of sacral objects $\sigma_k v_k$ [6, 7].

The matrix A can be deemed as the set of data forming a particular structure of relations. The observation matrix is subject to analysis by decomposition into singular values in order to obtain the hidden structure of sets.

3. The calculation model for acoustic analysis of the sanctuaries

Decomposition of the observation matrix A in relation to singular values has been performed in Matlab environment. The following calculation model has been obtained:

$$
A
$$
$$
\begin{bmatrix}
0.89 & 0.67 & 0.49 & 1 & 0.2 \\
0.98 & 0.6 & 0.34 & 0.39 & 0.53 \\
1 & 0.49 & 0.34 & 0.58 & 0.21 \\
0.91 & 0.42 & 0.23 & 0.57 & 0.43 \\
0.86 & 0.48 & 0.33 & 0.37 & 0.26 \\
0.11 & 0.21 & 0.21 & 1.0 & 0.78
\end{bmatrix}
$$

$$
U
$$
$$
= \begin{bmatrix}
-0.5020 & 0.0559 & 0.7457 & -0.2826 & 0.3179 \\
-0.4301 & -0.2083 & -0.5544 & -0.5215 & 0.2858 \\
-0.4195 & -0.2399 & 0.1537 & 0.4328 & -0.4278 \\
-0.4019 & -0.0683 & -0.2595 & 0.6456 & 0.4567 \\
-0.3581 & -0.2601 & -0.1088 & -0.2095 & -0.6176 \\
-0.3116 & 0.9074 & -0.1833 & 0.0006 & -0.2098
\end{bmatrix}
$$

$$
\Sigma
$$
$$
\times \begin{bmatrix}
3.0769 & 0 & 0 & 0 & 0 \\
0 & 0.9739 & 0 & 0 & 0 \\
0 & 0 & 0.4826 & 0 & 0 \\
0 & 0 & 0 & 0.17622 & 0 \\
0 & 0 & 0 & 0 & 0.0443
\end{bmatrix}
$$

$$
V^{\mathrm{T}}
$$
$$
\times \begin{bmatrix}
-0.6486 & -0.596 & -0.1569 & 0.4399 & -0.0760 \\
-0.3920 & -0.1726 & 0.0883 & -0.6781 & 0.5906 \\
-0.2635 & -0.0369 & 0.1970 & -0.5061 & -0.7963 \\
-0.5153 & 0.6240 & 0.5077 & 0.2818 & 0.0880 \\
-0.3008 & 0.4735 & -0.8191 & -0.1044 & -0.0587
\end{bmatrix}. \qquad (3)
$$

The SVD decomposition proved by relation (3) is a reduced decomposition after the zero row of matrix Σ and its corresponding last column of matrix U are deleted.

Through decomposition of the matrix A into singular values it is possible to reduce the calculation model by selecting the most informational singular components (singular values) characterising the state of the analysed object.

The values of percentage explanation of information about independent indices through consecutive singular components have been calculated according to formula (12) in [7] and presented in Fig. 2.

Figure 2 shows that the first singular component σ_1 is the most informational component. It contains 65% of total information (explanation of occur-
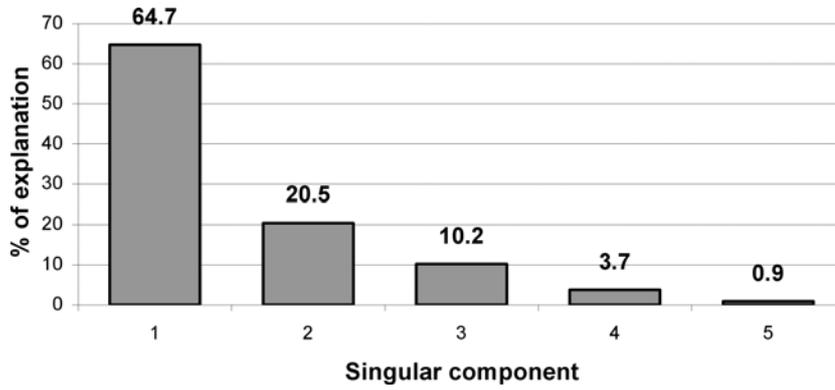
Fig. 2. Content of information about independent indices of the matrix A.

rence) of the observation matrix A. The second and the third singular component are 20% and 10% of total information, respectively. The last two components are of very low values in relation to the other ones and belong to the range of informational noise. Thereby, they can be discarded during further analysis. After summing, other components are about 95% of explanation of occurrence, therefore such calculation model can be reduced by using the approximation rank $r = 3$. Then, the following relation can be formulated [7]:

$$A_{r=3} = \sigma_1 u_1 v_1^{\mathrm{T}} + \sigma_2 u_2 v_2^{\mathrm{T}} + \sigma_3 u_3 v_3^{\mathrm{T}}. \tag{4}$$

The Frobenius norm is used during determining the approximation error, and the norm for the matrix $A = (a_{ij})_{m \times n}$ is defined as the root of the sum of squared matrix elements that can be expressed as:

$$\|A\|_F = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2}. \tag{5}$$

The difference between the Frobenius norm of the observation matrix and the Frobenius norm of the matrix obtained from the SVD decomposition for a relevant approximation rank shall be as low as possible, so that the approximation error is as low as possible. It can be formulated as:

$$\Delta_F = \min_{X:rank(X)=k} \|A - X\|_F = \|A - A_k\|_F = \sigma_{k+1}. \tag{6}$$

The graph of the approximation error as the function of approximation rank of the observation matrix A is presented in Fig. 3. The figure proves that for the accepted reduced calculation model characterised by relation (4) the approximation error is close to zero.
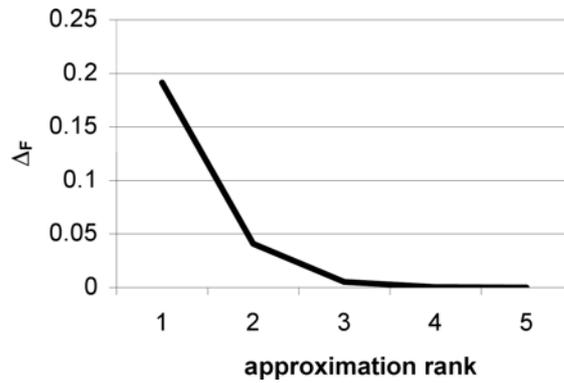
Fig. 3. The approximation error as the function of the rank r of the observation matrix A.

For the matrix A containing the structure of relations between partial indices, two indices, i.e. W_{ul} and W_{ed}, are weakly correlated with other indices, what can be deduced out of Fig. 2. Thereby, these indices can be rejected and other three can be selected for further analyses, and they shall be the base for forming the three-column observation matrix.

4. The simplified calculation model for acoustic analysis of the sanctuaries

Using three indices, i.e. reverberation index W_r, music sound quality index W_m and speech intelligibility index W_{is}, the observation matrix of indices and sacral objects denoted as A_3 has been created. The matrix has been analysed using singular value decomposition. The simplified calculation model is shown in [7] in Eq. (11).

In the simplified model the first singular value is the most informational value and contains 88% of total information of the matrix A (Fig. 8 in [7]). This first singular component is large as compared with the other components, thereby the calculation model can be reduced by using the approximation of the first rank. The obtained approximate model is burdened with an error (Fig. 4).
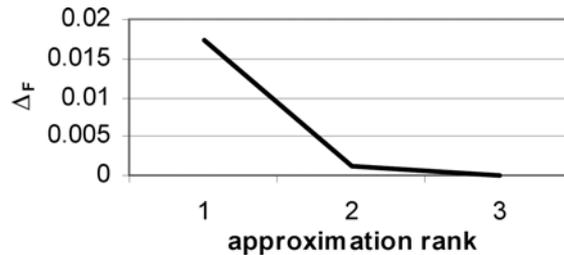

Fig. 4. The approximation error in relation to the rank r of the observation matrix A_3.

The error of the first rank approximation (Fig. 4) is a lot of smaller than the error for model based on five indices (Fig. 3).

5. Comparison of the acoustic assessment by application of elaborated calculation models

The method of calculating weight values by application of the SVD technique for three partial indices has been presented in [6] and [7]. The weight values have been used for the global assessment of acoustic quality of the sacral objects. In a similar way, the weight values for five partial indices (specified in Table 1) can be calculated using the calculation model according to Eq. (3). The components of the vector v_1 (of the matrix V^{T}) for the approximation of the first rank are identical for all sacral objects for every index. The vector components can be assigned to the indices, and the components can be accepted as weights for the global assessment. The global assessment of acoustic quality of a sacral object (proposed in [4]) shall have then the following form:

$$W_{AQS4} = \frac{0.6W_r + 0.4W_m + 0.3W_{is} + 0.5W_{ed} + 0.3W_{ul}}{2.1}. \tag{7}$$

The comparison of the global assessment of acoustic quality of a sacral object by application of three and five partial indices with traditional weights (described in [4]), and the indices with weights obtained from the SVD decomposition, has been presented in Table 2. The best results, i.e. the largest range of results for the global index W_{JAS} (Table 2), have been obtained during application of three partial indices (indices W_{JAS1} and W_{JAS2}). It can be observed that the values of the global index for three indices with traditional weights and weights obtained from the SVD technique are very similar.

The global assessment using five indices with traditional weights provides worse results in the form of the global index values of a smaller range, i.e. from 0.3 to 0.7. If the weights received from the SVD decomposition are applied, the range of the global assessment is even smaller, i.e. from 0.5 to 0.7. It is caused by the influence of weakly correlated indices of the sound system equability and external noises that are considered in the calculation model. In order to reproduce completely the structure of relations characterised by the matrix of objects and indices (formula (3)), the approximation of a higher rank ($r \geq 3$) shall be applied.

As for the first approximation rank and five indices that are not so strongly correlated to each other as in the simplified model (formula (4)), the approximation error is quite large. Therefore, if the approximation rank $r = 1$ is used to obtain the weights for five partial indices, the results are not satisfactory enough.

Table 2. Comparison of assessment of acoustic quality of the churches by the global indices.

No	Sacral objects	Partial indices					W_{AQS1}	W_{AQS2}	W_{AQS3}	W_{AQS4}
							(3 indicies)		(5 indicies)	
		W_r	W_m	W_{is}	W_{ed}	W_{ul}	traditional weights	weights from SVD	traditional weights	weights from SVD
1	The St. Sebastian's Church in Strzelce Wielkie	0.89	0.67	0.49	1.00	0.2	**0.75**	**0.75**	**0.71**	**0.74**
2	The Holiest Sacred Heart's Church in Kraków	0.98	0.6	0.34	0.39	0.53	**0.75**	**0.74**	**0.67**	**0.63**
3	The St. Clement's Church in Wieliczka	1	0.49	0.34	0.58	0.21	**0.75**	**0.72**	**0.65**	**0.61**
4	The Jesus Heart's Church in Kraków	0.91	0.42	0.23	0.57	0.43	**0.65**	**0.63**	**0.61**	**0.59**
5	The Reformati Fathers Church in Wieliczka	0.86	0.48	0.33	0.37	0.26	**0.66**	**0.64**	**0.57**	**0.52**
6	The St. Paul Apostle's Church in Bochnia	0.11	0.21	0.21	1.00	0.78	**0.15**	**0.16**	**0.34**	**0.46**

6. Conclusions

The possibility of application of the singular value decomposition for the analysis of sacral objects upon two calculation models has been presented in the paper. The calculation model of indices and sacral objects consisting of five partial indices and the simplified model containing three indices have been constructed. By means of the SVD decomposition it is possible to obtain particular numbers that can be deemed as the weights of partial indices during assessment by the global index. The performed analyses proved that such method of weight calculation is useful if all indices are well correlated to each other, so it relates to the simplified model containing three partial indices. Then, the applied first approximation rank is sufficient for approximating the structure of relations of the observation matrix of indices and objects. If weakly correlated indices occur in the model, i.e. five-index model, then after the SVD decomposition is applied and the weights are obtained, the results are not as good as for the three-index model.

Acknowledgment

References

[1] CEMPEL Cz., *Reduction of Data Set in Machines Diagnostics*, Zagadnienia Eksploatacji Maszyn, **4**, 44, 571–585 (1980).

[2] ENGEL Z., ENGEL J., *Application of Singular Value Decomposition in Analysis of Vibroacoustic processes*, Proceedings of XXXIV Ogólnopolskie Sympozjum "Diagnostyka Maszyn", pp. 17–24, Węgierska Górka 2007.

[3] ENGEL Z., KOSAŁA K., *Reverberation Indices in Acoustic Assessments of Sacral Structures*, Archives of Acoustics, **29**, 1, 45–59 (2004).

[4] ENGEL Z., KOSALA K., *Index Method of the Acoustic Quality Assessment of Sacral Buildings*, Archives of Acoustics, **32**, 3, 3–22 (2007).

[5] ENGEL Z., ENGEL J., KOSAŁA K., SADOWSKI J., *The Bases of Acoustics of Sacral Objects*, ITE, Kraków–Radom 2007.

[6] KOSAŁA K., *The Possibilities of Application of Singular Value Decomposition in Acoustical Properties Analysis of Sacral Objects*, Proceedings of Wibrotech 2007, Warszawa–Jachranka, 29–30.11.2007, Warszawa, PW, 159–168 (2007).

[7] KOSAŁA K., *Global Index of the Acoustic Quality of Sacral Buildings at Incomplete Information*, Archives of Acoustics, **33**, 2, 165–183 (2008).