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Acoustic Power Radiated Into the Quarter-Space by a Circular Membrane with an Asymmetric Excitation

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A circular membrane excited asymmetricaly is vibrating and radiating acoustic waves into the quarter-space limited by two rigid baffles arranged perpendicularly to one another. These processes are time harmonic. The classical Neumann boundary value problem has been solved using the complete eigenfunctions system together with the corresponding coupling matrix and including the acoustic attenuation effect.

Keywords: sound pressure, Green function, Neumann boundary value problem, modal analysis, acoustic impedance, complete eigenfunctions series.

1. Introduction

The acoustic interactions between sources and baffles are essential in the problems of sound radiation caused by some complex vibrating surface systems. They often require computing the acoustic power and the acoustic impedance as well. The complete eigenfunctions system and the coupling matrix can be used for describing the sound radiation of the mentioned vibrating systems. An adequate theoretical basis for such analysis was presented in [1–4].

A number of studies is devoted to the sound radiation emitted by various sources, some of them being the vibrating surface sources located within a flat rigid infinite baffle or within a rigid infinitely long cylinder. Noise control related to this sound radiation is also being studied [5–19]. The computations of the total, active and reactive, sound power radiated by a clamped circular plate have been presented in [5]. Having applied the modal analysis, one can consider the influence of the radiated sound pressure on the vibration velocity of the plate. The asymptotic formulas of the modal radiation resistance and reactance of an annular plate have been presented in [8]. An efficient method for visualization and analysis of the sound energy flow has been proposed in [20, 21]. However, so far only few papers have dealt with the sound radiation inside a quarter-space or inside a hemi-quarter-space. The analytical research in this field was initiated in [22] and successively extended in [23–30].

The acoustic impedance of the vibrating flat rigid circular piston radiating into the quarter-space has been presented in [23] and [24]. This paper proposes an extension of the previous results. The acoustic impedance has been computed for a circular membrane excited asymmetricaly including the acoustic attenuation. The spectral form of the Green function has been applied for this purpose. This function is the solution to the Helmholtz equation with the Neumann boundary conditions for the area of the two rigid baffles bounding the quarter-space. This has made it possible to join two different boundary value problems: one associated with the vibrations of the excited membrane and the second one associated with the sound radiation.

2. Sound pressure

A circular membrane of radius a vibrates asymmetrically and radiates the acoustic waves into the region of the quarter-space bounded by the two rigid baffles arranged perpendicularly to one another. The region can be defined by $-\infty < x < \infty$, $0 \le y < \infty$ and $0 \le z < \infty$. It is filled with the lossless gaseous medium of the rest density ϱ_0. The membrane is embedded into the half-plane $z = 0$ (cf., Fig. 1). This is the formulation of the classical Neumann boundary value problem.

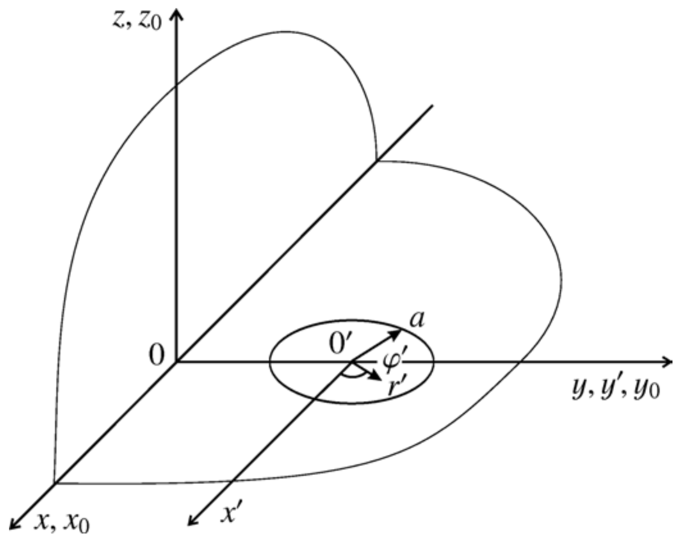

Fig. 1. The arrangement of the circular membrane located near the two-wall corner.

The sound pressure radiated by the membrane can be formulated for some time harmonic processes as $p(\mathbf{r},t) = p(\mathbf{r})e^{-i\omega t}$ where ω is the excitation circular frequency,

$$p(\mathbf{r}) = -ik_0\varrho_0 c \int\limits_{S_0} v(\mathbf{r}_0)\, G(\mathbf{r}\,|\,\mathbf{r}_0)\, \mathrm{d}S_0 \tag{1}$$

is the sound pressure amplitude, $k_0 = 2\pi/\lambda$ is the acoustic wavenumber, λ is the acoustic wavelength, and c is the sound velocity in the gaseous medium, $v(\mathbf{r}_0)$ is the vibration velocity of the acoustic particle adjoining directly to the membrane's surface, $\mathbf{r}$ is the leading vector of the field point and $\mathbf{r}_0$ is the leading vector of the membrane's point. It has been assumed that $v(\mathbf{r}_0) = v_N(\mathbf{r}_0)$ where $v_N(\mathbf{r}_0) \equiv v_N(r_0,\varphi_0) = -i\omega W(r_0,\varphi_0)$ is the normal component of the membrane's vibration velocity amplitude. This assumption enables joining the two boundary value problems mentioned above. One associated with the Helmholtz equation and the second associated with the excited vibrations of the membrane. The membrane's transverse deflection amplitude has been formulated in terms of the complete eigenfunction series [2]

$$W(r_0,\varphi_0) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\left\{c_{mn}^{(c)}W_{mn}^{(c)}(r_0,\varphi_0) + c_{mn}^{(s)}W_{mn}^{(s)}(r_0,\varphi_0)\right\}. \tag{2}$$

This kind of series requires completeness to assure the correct results. Therefore, the cosine and sine eigenfunctions of the degenerated mode (m,n) have been used and formulated as follows

$$\left\{\begin{matrix} W_{mn}^{(c)}(r_0,\varphi_0) \\ W_{mn}^{(s)}(r_0,\varphi_0) \end{matrix}\right\} = W_{mn}(r_0)\left\{\begin{matrix} \cos m\varphi_0 \\ \sin m\varphi_0 \end{matrix}\right\}, \qquad W_{mn}(r_0) = \sqrt{\epsilon_m}\,\frac{J_m(k_{mn}r_0)}{J_{m+1}(\beta_{mn})} \tag{3}$$

for $m = 0,1,2,\ldots$ where $\epsilon_m = 1$ for $m = 0$ and $\epsilon_m = 2$ for $m \geq 1$, $\beta_{mn} = k_{mn}a$ is the membrane's eigenvalue as well as the root of the frequency equation $J_m(\beta_{mn}) = 0$, and k_{mn} is the modal structural wavenumber of the mode (m,n). It is obvious that $W_{0n}^{(s)}(r_0,\varphi_0) = 0$ as well as $c_{0n}^{(s)} = 0$. The eigenfunctions have been normalized by

$$\frac{1}{\pi a^2}\int\limits_0^a\int\limits_0^{2\pi}\left\{\begin{matrix} W_{mn}^{(c)}(r_0,\varphi_0) \\ W_{mn}^{(s)}(r_0,\varphi_0) \end{matrix}\right\}\left\{\begin{matrix} W_{m'n'}^{(c)}(r_0,\varphi_0) \\ W_{m'n'}^{(s)}(r_0,\varphi_0) \end{matrix}\right\} r_0\,\mathrm{d}r_0\,\mathrm{d}\varphi_0 = \delta_{mm'}\delta_{nn'} \tag{4$_1$}$$

and satisfy the homogeneous equation of motion

$$\left(k_{mn}^{-2}\nabla^2 + 1\right)\left\{\begin{matrix} W_{mn}^{(c)}(r_0,\varphi_0) \\ W_{mn}^{(s)}(r_0,\varphi_0) \end{matrix}\right\} = 0, \tag{4$_2$}$$

where $\nabla^2 = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2}{\partial\varphi^2}$. The normal component of the membrane vibration velocity is

$$v_N(r_0,\varphi_0)=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\frac{\omega}{\omega_{mn}}\left\{c_{mn}^{(c)}v_{mn}^{(c)}(r_0,\varphi_0)+c_{mn}^{(s)}v_{mn}^{(s)}(r_0,\varphi_0)\right\}, \tag{5$_1$}$$

where

$$\left\{\begin{array}{c} v_{mn}^{(c)}(r_0,\varphi_0) \\ v_{mn}^{(s)}(r_0,\varphi_0)\end{array}\right\}=v_{mn}(r_0)\left\{\begin{array}{c}\cos m\varphi_0 \\ \sin m\varphi_0\end{array}\right\}, \qquad v_{mn}(r_0)=-i\omega_{mn}W_{mn}(r_0) \tag{5$_2$}$$

for $m=0,1,2,\dots$ and $v_{0n}^{(s)}(r_0,\varphi_0)=0$ where $\omega_{m,n}=c_M k_{m,n}$ is the eigenfrequency of the mode (m,n), $c_M=\sqrt{T/\sigma}$ is the velocity of the wave propagated over the membrane surface, σ is the membrane's mass per its surface unit and T is the uniform tension applied along its edge. The Green function for the Neumann boundary value problem under consideration for $z_0=0$ is [24]

$$\begin{aligned} G(\mathbf{r}\,|\,\mathbf{r}_0)&\equiv G(x,y,z\,|\,x_0,y_0,0)\\ &=\frac{i}{\pi^2}\int\limits_{\xi=-\infty}^{+\infty}\int\limits_{\eta=0}^{+\infty}e^{i\,[\xi\,(x-x_0)+\gamma z]}\cos\eta y\cos\eta y_0\frac{d\xi d\eta}{\gamma}, \end{aligned}\tag{6}$$

where $\gamma^2=k_0^2-\xi^2-\eta^2$. The Cartesian coordinates of the field point (x,y,z) and the source point $(x_0,y_0,0)$ have been converted into their local polar counterparts

$$\begin{aligned} x&=r\cos\varphi, & y&=l+r\sin\varphi,\\ x_0&=r_0\cos\varphi_0, & y_0&=l+r_0\sin\varphi_0, \end{aligned}\tag{7}$$

where l is the distance from the membrane center to the baffle $y=0$ (cf., Fig. 1).

Inserting Eqs. (6) and $(5)_1$ into Eq. (1) yields the sound pressure amplitude in the form of

$$p(r,\varphi,z)=\frac{k_0}{\pi}\,\varrho_0 c\,a^2\int\limits_{-\infty}^{\infty}\int\limits_{0}^{\infty}e^{i\,(\xi x+\gamma z)}\cos\eta y\,M(\xi,\eta)\,\frac{d\xi\,d\eta}{\gamma}, \tag{8$_1$}$$

where the characteristics function of the membrane's radiation is

$$M(\xi,\eta)=\frac{1}{\pi a^2}\int\limits_0^a\int\limits_0^{2\pi}v_N(r_0,\varphi_0)\,e^{-i\xi r_0\cos\varphi_0}\cos\eta\,(l+r_0\sin\varphi_0)\,r_0\,dr_0\,d\varphi_0. \tag{8$_2$}$$

These physical quantities can also be formulated in terms of their corresponding modal components as follows

$$\begin{aligned} p(r,\varphi,z)&=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\frac{\omega}{\omega_{mn}}\left\{c_{mn}^{(c)}p_{mn}^{(c)}(r,\varphi,z)+c_{mn}^{(s)}p_{mn}^{(s)}(r,\varphi,z)\right\},\\ M(\xi,\eta)&=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\frac{\omega}{\omega_{mn}}\left\{c_{mn}^{(c)}M_{mn}^{(c)}(\xi,\eta)+c_{mn}^{(s)}M_{mn}^{(s)}(\xi,\eta)\right\}, \end{aligned}\tag{9}$$

where

$$\begin{Bmatrix} p_{mn}^{(c)}(r,\varphi,z) \\ p_{mn}^{(s)}(r,\varphi,z) \end{Bmatrix} = \frac{k_0}{\pi}\,\varrho_0 c\, a^2 \int\limits_{-\infty}^{+\infty}\int\limits_{0}^{+\infty} e^{i\,(\xi x+\gamma z)} \cos\eta y \begin{Bmatrix} M_{mn}^{(c)}(\xi,\eta) \\ M_{mn}^{(s)}(\xi,\eta) \end{Bmatrix} \frac{\mathrm{d}\xi\,\mathrm{d}\eta}{\gamma},$$

$$\begin{Bmatrix} M_{mn}^{(c)}(\xi,\eta) \\ M_{mn}^{(s)}(\xi,\eta) \end{Bmatrix} = \frac{1}{\pi a^2}\int\limits_0^a\int\limits_0^{2\pi} \begin{Bmatrix} v_{mn}^{(c)}(r_0,\varphi_0) \\ v_{mn}^{(s)}(r_0,\varphi_0) \end{Bmatrix} e^{-i\xi r_0\cos\varphi_0} \cos\eta\,(l+r_0\sin\varphi_0)\, r_0\,\mathrm{d}r_0\,\mathrm{d}\varphi_0$$

$$= (-i)^{m+1}\omega_{mn}\sqrt{\epsilon_m}\,\Psi_{mn}(\tau) \begin{Bmatrix} \cos\eta l\cos m\alpha \\ -i\sin\eta l\sin m\alpha \end{Bmatrix}, \tag{10}$$

$$\Psi_{mn}(\tau) = \frac{2}{a^2}\int\limits_0^a \frac{J_m(k_{mn}r_0)}{J_{m+1}(\beta_{mn})} J_m(\tau r_0)\, r_0\,\mathrm{d}r_0 = \frac{2\beta_{mn}J_m(\tau a)}{\beta_{mn}^2-(\tau a)^2},$$

$$\xi = \tau\cos\alpha, \qquad \eta = \tau\sin\alpha$$

for $m = 0,1,2,\ldots$ and $M_{0n}^{(s)}(\xi,\eta) = 0$. Equations (8)–(10) constitute the complete set of data necessary to express the distribution of the sound pressure amplitude above the membrane's area. This quantity will be further used to obtain the total sound power radiated.

3. The acoustic power and the normalized acoustic impedance

The time-averaged acoustic power of the excited membrane can be expressed using the well-known equation $\Pi = \frac{1}{2}\int_S p v_N^* \,\mathrm{d}S$ where S is the surface enclosing the sound source. Applying the impedance approach implies that the computations are preformed for $z = 0$. Consequently, $p \equiv p(r,\varphi,0)$ (cf., Eq. $(8)_1$), $v_N^* = i\omega W^*$ (cf., Eq. (2)), and

$$\Pi = \frac{1}{2}\,k_0\,\varrho_0 c\,a^4 \int\limits_{-\infty}^{\infty}\int\limits_0^{\infty} M(\xi,\eta)\,M^*(\xi,\eta)\,\frac{\mathrm{d}\xi\mathrm{d}\eta}{\gamma}, \tag*{$(11)_1$}$$

where the conjugate value of the characteristic function is

$$M^*(\xi,\eta) = \frac{1}{\pi a^2}\int\limits_0^a\int\limits_0^{2\pi} v_N^*(r,\varphi)\,e^{i\xi r\cos\varphi}\cos\eta\,(l+r\sin\varphi)\,r\,\mathrm{d}r\,\mathrm{d}\varphi, \tag*{$(11)_2$}$$

which can also be expressed as

$$M^*(\xi,\eta)=\sum_{m'=0}^{\infty}\sum_{n'=1}^{\infty}\frac{\omega}{\omega_{m'n'}}\left\{c_{m'n'}^{(c)*}M_{m'n'}^{(c)*}(\xi,\eta)+c_{m'n'}^{(s)*}M_{m'n'}^{(s)*}(\xi,\eta)\right\} \tag{12$_1$}$$

in term of the following modal conjugate quantities

$$\begin{Bmatrix} M_{m'n'}^{(c)*}(\xi,\eta) \\ M_{m'n'}^{(s)*}(\xi,\eta) \end{Bmatrix}=\frac{1}{\pi a^2}\int_0^a\int_0^{2\pi}\begin{Bmatrix} v_{m'n'}^{(c)*}(r,\varphi) \\ v_{m'n'}^{(s)*}(r,\varphi) \end{Bmatrix} e^{i\xi r\cos\varphi}\cos\eta\,(l+r\sin\varphi)r\,\mathrm{d}r\,\mathrm{d}\varphi$$

$$=i^{m'+1}\omega_{m'n'}\sqrt{\epsilon_{m'}}\,\Psi_{m'n'}(\tau)\begin{Bmatrix} \cos\eta l\cos m'\alpha \\ -i\sin\eta l\sin m'\alpha \end{Bmatrix}, \tag{12$_2$}$$

$$\begin{Bmatrix} v_{m'n'}^{(c)*}(r,\varphi) \\ v_{m'n'}^{(s)*}(r,\varphi) \end{Bmatrix}=v_{m'n'}^*(r)\begin{Bmatrix} \cos m'\varphi \\ \sin m'\varphi \end{Bmatrix}, \qquad v_{m'n'}^*(r)=i\omega_{m'n'}W_{m'n'}(r) \tag{12$_3$}$$

for $m'=0,1,2,\ldots$, $v_{0n'}^*(r)=0$ and $M_{0n'}^{(s)*}(\xi,\eta)=0$. Further inserting Eqs. (9)$_2$ and (12)$_1$ into Eq. (11)$_1$ yields the sound power as the following fourfold series

$$\Pi=\sum_{mm'}\sum_{nn'}\frac{\omega^2}{\omega_{mn}\omega_{m'n'}}\Big\{c_{mn}^{(c)}c_{m'n'}^{(c)*}\Pi_{mn,\,m'n'}^{(c,c)}+\mathrm{sign}_{m'}c_{mn}^{(c)}c_{m'n'}^{(s)*}\Pi_{mn,\,m'n'}^{(c,s)}$$

$$+\mathrm{sign}_m c_{mn}^{(s)}c_{m'n'}^{(c)*}\Pi_{mn,\,m'n'}^{(s,c)}+\mathrm{sign}_{m,\,m'}c_{mn}^{(s)}c_{m'n'}^{(s)*}\Pi_{mn,\,m'n'}^{(s,s)}\Big\} \tag{13}$$

in terms of the mutual acoustic power of the two interacting modes (m,n) and (m',n') where sign_m is the signum function for an integer argument m, and consequently $\mathrm{sign}_0=0$, $\mathrm{sign}_m=1$ and $\mathrm{sign}_{-m}=-1$ for $m\geq 1$, and $\mathrm{sign}_{m,\,m'}=\mathrm{sign}_m\mathrm{sign}_{m'}$. Each of the modes can be cosine and sine which gives the following four different forms of the mutual acoustic power

$$\begin{Bmatrix} \Pi_{mn,\,m'n'}^{(c,c)} \\ \Pi_{mn,\,m'n'}^{(c,s)} \\ \Pi_{mn,\,m'n'}^{(s,c)} \\ \Pi_{mn,\,m'n'}^{(s,s)} \end{Bmatrix}=\frac{1}{2}k_0\,\varrho_0 c\,a^4\int_{-\infty}^{\infty}\int_0^{\infty}\begin{Bmatrix} M_{mn}^{(c)}(\xi,\eta) \\ M_{mn}^{(c)}(\xi,\eta) \\ M_{mn}^{(s)}(\xi,\eta) \\ M_{mn}^{(s)}(\xi,\eta) \end{Bmatrix}\begin{Bmatrix} M_{m'n'}^{(c)*}(\xi,\eta) \\ M_{m'n'}^{(s)*}(\xi,\eta) \\ M_{m'n'}^{(c)*}(\xi,\eta) \\ M_{m'n'}^{(s)*}(\xi,\eta) \end{Bmatrix}\frac{\mathrm{d}\xi\mathrm{d}\eta}{\gamma} \tag{14}$$

for $m,m'=0,1,2,\ldots$ and $n,n'=1,2,3,\ldots$ The signum function has been used in Eq. (13) to highlight the fact that $\sin m\varphi=0$ and $\sin m'\varphi=0$ within the integrands in Eq. (14).

The modal reference acoustic power must be the non-zero value for $m,m'=0,1,2,\ldots$ and $n,n'=1,2,3,\ldots$ Remembering that Eq. (4)$_1$ is non-zero only for $m=m'$ and $n=n'$, and that

$$\frac{1}{\pi a^2}\int_0^a\int_0^{2\pi}\left\{\begin{matrix} W_{mn}^{(c)}(r_0,\varphi_0) \\ W_{mn}^{(s)}(r_0,\varphi_0)\end{matrix}\right\}\left\{\begin{matrix} W_{m'n'}^{(s)}(r_0,\varphi_0) \\ W_{m'n'}^{(c)}(r_0,\varphi_0)\end{matrix}\right\} r_0\,\mathrm{d}r_0\,\mathrm{d}\varphi_0 = 0 \tag{15}$$

the modal reference acoustic power has been formulated as

$$\mathit{\Pi}_{mn,\,m'n'}^{(\mathrm{Ref.})} = \sqrt{\mathit{\Pi}_{mn}^{(\mathrm{Ref.})}\mathit{\Pi}_{m'n'}^{(\mathrm{Ref.})}} = \frac{S}{2}\,\varrho_0 c\,\omega_{mn}\omega_{m'n'}, \tag*{$(16)_1$}$$

where

$$\mathit{\Pi}_{mn}^{(\mathrm{Ref.})} = \frac{1}{2}\,\varrho_0 c\int_S \left|v_{mn}^{(c)}(r,\varphi)\right|^2\mathrm{d}S = \frac{S}{2}\,\varrho_0 c\,\omega_{mn}^2. \tag*{$(16)_2$}$$

Normalizing the modal mutual acoustic power from Eq. (14) by the modal reference acoustic power from Eq. $(16)_1$ gives the modal mutual acoustic impedance

$$\left\{\begin{matrix} \zeta_{mn,\,m'n'}^{(c,c)} \\ \zeta_{mn,\,m'n'}^{(c,s)} \\ \zeta_{mn,\,m'n'}^{(s,c)} \\ \zeta_{mn,\,m'n'}^{(s,s)} \end{matrix}\right\} = \frac{1}{\mathit{\Pi}_{mn,\,m'n'}^{(\mathrm{Ref.})}}\left\{\begin{matrix} \mathit{\Pi}_{mn,\,m'n'}^{(c,c)} \\ \mathit{\Pi}_{mn,\,m'n'}^{(c,s)} \\ \mathit{\Pi}_{mn,\,m'n'}^{(s,c)} \\ \mathit{\Pi}_{mn,\,m'n'}^{(s,s)} \end{matrix}\right\} = \frac{1}{4}\,k_0 a^2\,(-1)^m\,i^{m+m'}\sqrt{\epsilon_m\epsilon_{m'}}$$

$$\cdot\int_0^\infty \left\{\begin{matrix} \mathit{\Phi}_{mm'}^{(c,c)} \\ \mathit{\Phi}_{mm'}^{(c,s)} \\ \mathit{\Phi}_{mm'}^{(s,c)} \\ \mathit{\Phi}_{mm'}^{(s,s)} \end{matrix}\right\} \mathit{\Psi}_{mn}(\tau)\,\mathit{\Psi}_{m'n'}(\tau)\,\frac{\tau\mathrm{d}\tau}{\sqrt{k_0^2-\tau^2}}, \tag{17}$$

where the integration is performed in the complex plane of $\tau = \tau' + i\tau''$ along the real axis $0\tau'$, omitting the branch point $\tau' = k_0$, and

$$\begin{aligned}
\left\{\begin{matrix} \mathit{\Phi}_{mm'}^{(c,c)} \\ \mathit{\Phi}_{mm'}^{(s,s)} \end{matrix}\right\} &= \frac{4}{\pi}\int_0^\pi \left\{\begin{matrix} \cos^2\eta l \\ \sin^2\eta l \end{matrix}\right\}\left\{\begin{matrix} \cos m\alpha\cos m'\alpha \\ \sin m\alpha\sin m'\alpha \end{matrix}\right\}\mathrm{d}\alpha \\
&= \left\{\begin{matrix} 1 \\ \mathrm{sign}_{m,\,m'} \end{matrix}\right\}\left\{\frac{2\delta_{mm'}}{\epsilon_m} + J_{m+m'}(2\tau l) \pm J_{m-m'}(2\tau l)\right\}, \\
\left\{\begin{matrix} \mathit{\Phi}_{mm'}^{(c,s)} \\ \mathit{\Phi}_{mm'}^{(s,c)} \end{matrix}\right\} &= \frac{4i}{\pi}\int_0^\pi \cos\eta l\sin\eta l\left\{\begin{matrix} \cos m\alpha\sin m'\alpha \\ -\sin m\alpha\cos m'\alpha \end{matrix}\right\}\mathrm{d}\alpha \\
&= i\left\{\begin{matrix} \mathrm{sign}_{m'} \\ \mathrm{sign}_m \end{matrix}\right\}\left\{\pm\mathrm{sign}_{m+m'}J_{m+m'}(2\tau l) - \mathrm{sign}_{m-m'}J_{m-m'}(2\tau l)\right\}.
\end{aligned} \tag{18}$$

Applying Eqs. (13), (16) and (17) yields the acoustic power formulated in terms of the modal mutual impedance as follows

$$\Pi = \frac{S}{2}\,\varrho_0 c\,\omega^2 \sum_{mm'}\sum_{nn'}\Big\{c^{(c)}_{mn}c^{(c)*}_{m'n'}\zeta^{(c,c)}_{mn,\,m'n'} + \mathrm{sign}_{m'}c^{(c)}_{mn}c^{(s)*}_{m'n'}\zeta^{(c,s)}_{mn,\,m'n'}$$
$$+ \mathrm{sign}_{m}c^{(s)}_{mn}c^{(c)*}_{m'n'}\zeta^{(s,c)}_{mn,\,m'n'} + \mathrm{sign}_{m,\,m'}c^{(s)}_{mn}c^{(s)*}_{m'n'}\zeta^{(s,s)}_{mn,\,m'n'}\Big\}, \tag{19}$$

where $S = \pi a^2$.

Prior to using Eq. (19) for the numerical computations of the total sound power, the complex coefficients c_{mn} together with the modal mutual acoustic impedance from Eq. (17) must be determined. For this purpose the equation of motion of the excited membrane has to be solved.

The modal impedance can be approximated for $k_0 a < \beta_{mn}$ using the formulas presented in [30]. It is worth noticing that the integrands in Eq. (17) are non zero for $m \neq m'$ which result in the fourfold sum in Eq. (19). Moreover, the expression within the brackets in Eq. $(18)_1$ is asymmetric for the odd values of $m - m'$ since $J_{m'-m}(2\tau l) = (-1)^{m-m'} J_{m-m'}(2\tau l)$.

4. The equation of motion of the excited circular membrane

The basis of analysis is the model of membrane subjected to the uniform tension T along its edge of radius $r = a$. The membrane is made of the material of the surface density σ and is embedded into a flat rigid baffle. The non-homogeneous equation of motion of the membrane including the excitation $f(\mathbf{r},t)$ and the acoustic attenuation $p(\mathbf{r},t)$ is

$$T\,\nabla^2 W(\mathbf{r},t) - \sigma\,\frac{\partial^2}{\partial t^2}W(\mathbf{r},t) = -f(\mathbf{r},t) - p(\mathbf{r},t). \tag{20}$$

The sound pressure amplitude from Eq. $(9)_1$ appearing in Eq. (20) represents only the radiation into the region of the quarter-space above the rigid baffle $z \geq 0$. It has been assumed that the radiation from the bottom of the membrane $z < 0$ is suppressed. If it would not then the sound pressure should be taken twice, i.e. it should be written $-2p(\mathbf{r},t)$ in Eq. (20) instead of $-p(\mathbf{r},t)$. Assuming the harmonic time dependence $e^{-i\omega t}$ yields

$$\left(k_T^{-2}\nabla^2 + 1\right) W(r,\varphi) = -\frac{1}{\omega^2\sigma}\left\{f(r,\varphi) + p(r,\varphi,0)\right\}, \tag{21}$$

where $k_T = \omega/c_M$ is the structural wavenumber and ω is the excitation circular frequency. Inserting Eq. (2) and noting that Eq. $(4)_2$ implied

$$\nabla^2 \left\{\begin{array}{c} W^{(c)}_{mn}(r,\varphi) \\ W^{(s)}_{mn}(r,\varphi) \end{array}\right\} = -k^2_{mn}\left\{\begin{array}{c} W^{(c)}_{mn}(r,\varphi) \\ W^{(s)}_{mn}(r,\varphi) \end{array}\right\}. \tag{22}$$

Using the orthogonality of the trigonometric functions

$$\frac{\epsilon_m}{2\pi}\int\limits_0^{2\pi}\left\{\begin{matrix}\cos m\varphi\\ \sin m\varphi\end{matrix}\right\}\left\{\begin{matrix}\cos m'\varphi\\ \sin m'\varphi\end{matrix}\right\}\mathrm{d}\varphi=\delta_{mm'} \tag{23}$$

makes it possible to formulate Eq. (21) in the form of

$$\sum_{mn}\left(\frac{k_{mn}^2}{k_T^2}-1\right)\left\{c_{mn}^{(c)}W_{mn}^{(c)}(r,\varphi)+c_{mn}^{(s)}W_{mn}^{(s)}(r,\varphi)\right\}$$

$$=\frac{1}{\omega^2\sigma}\left\{f(r,\varphi)+p(r,\varphi,0)\right\}. \tag{24}$$

Multiplying it side by side by the cosine eigenfunction $W_{m'n'}^{(c)}(r,\varphi)$ and by the sine eigenfunction $W_{m'n'}^{(s)}(r,\varphi)$ and further applying Eq. $(4)_1$ gives one equation for $m=0$ and degenerates to the two complementary equations for $m\geq 1$

$$\left\{\begin{matrix}c_{mn}^{(c)}\\ c_{mn}^{(s)}\end{matrix}\right\}\left(\frac{k_{mn}^2}{k_T^2}-1\right)=\frac{1}{\omega^2\sigma}\left(\left\{\begin{matrix}f_{mn}^{(c)}\\ f_{mn}^{(s)}\end{matrix}\right\}+\left\{\begin{matrix}p_{mn}^{(c)}\\ p_{mn}^{(s)}\end{matrix}\right\}\right), \tag*{$(25)_1$}$$

where

$$\left\{\begin{matrix}f_{mn}^{(c)}\\ f_{mn}^{(s)}\end{matrix}\right\}=\frac{1}{S}\int\limits_S\left\{\begin{matrix}W_{mn}^{(c)}(r,\varphi)\\ W_{mn}^{(s)}(r,\varphi)\end{matrix}\right\}f(r,\varphi)\,\mathrm{d}S, \tag*{$(25)_2$}$$

$$\left\{\begin{matrix}p_{mn}^{(c)}\\ p_{mn}^{(s)}\end{matrix}\right\}=\frac{1}{S}\int\limits_S\left\{\begin{matrix}W_{mn}^{(c)}(r,\varphi)\\ W_{mn}^{(s)}(r,\varphi)\end{matrix}\right\}p(r,\varphi,0)\,\mathrm{d}S \tag*{$(25)_3$}$$

and $S=\pi a^2$, and $\mathrm{d}S=r\,\mathrm{d}r\,\mathrm{d}\varphi$. It is obvious here that when the acoustic attenuation is neglected, i.e. $p_{mn}^{(c)}=0$ and $p_{mn}^{(s)}=0$, then the coupling matrix can be determined immediately from Eq. $(25)_1$

$$\left\{\begin{matrix}c_{mn}^{(c)}\\ c_{mn}^{(s)}\end{matrix}\right\}=\frac{1}{\omega^2\sigma\left(k_T^{-2}k_{mn}^2-1\right)}\left\{\begin{matrix}f_{mn}^{(c)}\\ f_{mn}^{(s)}\end{matrix}\right\}. \tag{26}$$

However, this gives some improper solutions around the coincidence frequencies $k_T^2\approx k_{mn}^2$, and therefore the acoustic attenuation should to be included for these frequencies. Further inserting Eq. $(9)_1$ into Eq. $(25)_3$, and applying Eqs. (16) and (17) yields

$$\left\{\begin{matrix}p_{mn}^{(c)}\\ p_{mn}^{(s)}\end{matrix}\right\}=-i\omega\,\varrho_0 c\sum_{m'n'}\left(c_{m'n'}^{(c)}\left\{\begin{matrix}\zeta_{m'n',mn}^{(c,c)}\\ \zeta_{m'n',mn}^{(c,s)}\end{matrix}\right\}+c_{m'n'}^{(s)}\left\{\begin{matrix}\zeta_{m'n',mn}^{(s,c)}\\ \zeta_{m'n',mn}^{(s,s)}\end{matrix}\right\}\right) \tag{27}$$

which inserted into Eq. $(25)_1$ gives

$$\left\{\begin{matrix} c_{mn}^{(c)} \\ c_{mn}^{(s)} \end{matrix}\right\}\left(\frac{k_{mn}^2}{k_T^2}-1\right)+i\varepsilon_0\sum_{m'n'}\left(c_{m'n'}^{(c)}\left\{\begin{matrix} \zeta_{m'n',mn}^{(c,c)} \\ \zeta_{m'n',mn}^{(c,s)} \end{matrix}\right\}+c_{m'n'}^{(s)}\left\{\begin{matrix} \zeta_{m'n',mn}^{(s,c)} \\ \zeta_{m'n',mn}^{(s,s)} \end{matrix}\right\}\right)$$
$$=\frac{1}{\omega^2\sigma}\left\{\begin{matrix} f_{mn}^{(c)} \\ f_{mn}^{(s)} \end{matrix}\right\}, \tag{28}$$

where the dimensionless acoustic attenuation factor has been denoted as

$$\varepsilon_0=\frac{\varrho_0}{k_0\sigma}. \tag{29}$$

Given that $\varepsilon_0>0$, the double sum from Eq. (28) can be separated and inserted into Eq. (19) providing another expression for the total acoustic power radiated by the membrane formulated as the following double series

$$\Pi=\frac{i}{2}S\omega\sum_{mn}\left\{\sigma\omega^2\left[|c_{mn}^{(c)}|^2+|c_{mn}^{(s)}|^2\right]\left(\frac{k_{mn}^2}{k_T^2}-1\right)\right.$$
$$\left.-\left[c_{mn}^{(c)*}f_{mn}^{(c)}+c_{mn}^{(s)*}f_{mn}^{(s)}\right]\right\}, \tag{30}$$

where $|c_{mn}^{(c)}|^2=c_{mn}^{(c)}c_{mn}^{(c)*}$ and $|c_{mn}^{(s)}|^2=c_{mn}^{(s)}c_{mn}^{(s)*}$. It is important to note that this formulation cannot be used together with the coupling matrix computed from Eq. (26). Instead, it must be used with the coupling matrix computed by solving the algebraic equation system given in Eq. (28).

5. Numerical analysis

The following dimensionless parameters have been introduced to perform some numerical computations of the total sound power radiated by the membrane excited by the time harmonic surface force

$$\varepsilon_1=\varepsilon_0\,\frac{\omega}{\omega_{01}},\qquad \varepsilon_T=\omega_{01}\,\frac{a}{c}=\beta_{01}\,\frac{c_M}{c}, \tag{31}$$

where ω_{01} is the eigenfrequency of the membrane's mode $(0,1)$. The parameter ε_1 determines the influence of the acoustic attenuation on the membrane's vibration whereas the parameter ε_T determines its physical properties. The following four excitations have been used

$$
\begin{aligned}
f_1(r,\varphi) &= S\frac{f_0}{r}\,\delta(r-\overline{r}_0)\,\delta(\varphi-\overline{\varphi}_0),\\
f_2(r,\varphi) &= \left\{\begin{array}{c} f_0;\;\; 0 \le r \le b \\ 0;\;\; b < r \le a \end{array}\right\}(\cos M\varphi - \overline{\varphi}_0),\\
f_3(r,\varphi) &= f_0 W_{MN}(r)\,(\cos M\varphi - \overline{\varphi}_0),\\
f_4(r,\varphi) &= \left\{\begin{array}{c} f_0;\;\; 0 \le r \le b \\ 0;\;\; b < r \le a \end{array}\right.
\end{aligned}
\tag{32}
$$

for $0 \le b \le a$, where f_0 [N/m^2] is the excitation amplitude. In the case of the excitation given by Eq. $(32)_1$, the product Sf_0 can be interpreted as the excitation force exerted to the membrane at the point $(\overline{r}_0, \overline{\varphi}_0)$ for $\overline{r}_0 \in [0,a]$ and $\overline{\varphi}_0 \in [0, 2\pi]$. In the case of the excitations given by Eqs. $(32)_2$ and $(32)_3$, $M = 0, 1, 2, ...$ is the number of their nodal diameters. The variable $\overline{\varphi}_0$ is the excitation rotation angle with the rotation center located at the membrane's center. These three excitations are essentially asymmetric exept for $\overline{r}_0 = 0$ and $M = 0$ whereas the fourth excitation is axisymmetric for any value of b. The reference sound power has been defined as (cf., [7])

$$
\varPi_0 = \frac{S\varrho_0 c f_0^2}{\sigma^2\omega_{01}^2}. \tag{33}
$$

Inserting the excitation distribution from Eqs. (32) into Eq. $(25)_2$ and applying Eqs. $(4)_1$ and (15) yields

$$
\begin{aligned}
\left\{\begin{array}{c} f^{(c)}_{1,mn} \\ f^{(s)}_{1,mn} \end{array}\right\} &= f_0 \left\{\begin{array}{c} W^{(c)}_{mn}(\overline{r}_0,\overline{\varphi}_0) \\ W^{(s)}_{mn}(\overline{r}_0,\overline{\varphi}_0) \end{array}\right\},\\
\left\{\begin{array}{c} f^{(c)}_{2,mn} \\ f^{(s)}_{2,mn} \end{array}\right\} &= f_0\,\frac{\delta_{mM}}{\epsilon_m}\,\overline{F}_{mn}(b) \left\{\begin{array}{c} \cos\overline{\varphi}_0 \\ \sin\overline{\varphi}_0 \end{array}\right\},\\
\left\{\begin{array}{c} f^{(c)}_{3,mn} \\ f^{(s)}_{3,mn} \end{array}\right\} &= f_0\,\delta_{mM}\,\delta_{nN} \left\{\begin{array}{c} \cos\overline{\varphi}_0 \\ \sin\overline{\varphi}_0 \end{array}\right\},\\
\left\{\begin{array}{c} f^{(c)}_{4,mn} \\ f^{(s)}_{4,mn} \end{array}\right\} &= f_0\,\overline{F}_{mn}(b) \left\{\begin{array}{c} 1;\;\; m = 0 \\ 0;\;\; m \ge 0 \end{array}\right\},
\end{aligned}
\tag{34}
$$

where

$$\overline{F}_{mn}(b) = \frac{2}{a^2} \int_0^b W_{mn}(r)\, r\, \mathrm{d}r.$$

It is worth noticing that Eq. $(32)_4$ gives an axisymmetric excitation and therefore all the coefficients in Eq. $(34)_4$ are equal to zero except for the axisymmetric cosine ones, i.e. for $m = 0$.

Both Eqs. (19) and (30) can be used for numerical computations of the total sound power. From the practical viewpoint, the number of terms within the infinite sums must be finite. The appropriate value of this number is determined by the excitation frequency, i.e. at least all the modes for which the eigenfrequencies are smaller than the excitation frequency should be included. If higher accuracy is necessary, a greater number of modes should used. In this paper, the matrix of $(M = 3) \times (N = 3) = 9$ modes has been included (the normalized eigenfrequencies are given in Table 1). This set of modes enables calculating numerically the total sound power with the satisfactory accuracy for the qualitative analysis within the excitation frequency range $\omega/\omega_{01} \in (0, 4.83)$. As a result, we obtain the set of $R = (2M - 1) \times N = 15$ algebraic equations necessary to calculate the coupling matrix

$$\boldsymbol{c} \equiv [\boldsymbol{c}^{(\boldsymbol{c})} \,|\, \boldsymbol{c}^{(\boldsymbol{s})}] = \left[\begin{array}{cccc|cccc} c_{0,1}^{(c)} & c_{0,2}^{(c)} & c_{0,3}^{(c)} & \cdots & - & - & - & \cdots \\ c_{1,1}^{(c)} & c_{1,2}^{(c)} & c_{1,3}^{(c)} & \cdots & c_{1,1}^{(s)} & c_{1,2}^{(s)} & c_{1,3}^{(s)} & \cdots \\ c_{2,1}^{(c)} & c_{2,2}^{(c)} & c_{2,3}^{(c)} & \cdots & c_{2,1}^{(s)} & c_{2,2}^{(s)} & c_{2,3}^{(s)} & \ddots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots \end{array}\right] \tag{35}$$

consisting of $M \times N = 9$ cosine coefficients and $(M-1) \times N = 6$ sine coefficients. Subsequently, it is necessary to calculate numerically $R^2 = 225$ different values of the modal impedance. The triple dots in Eq. (35) indicate the fact that a greater number of modes can also be used.

Table 1. Normalized eigenfrequencies of the membrane $\omega_{m,\,n}/\omega_{0,1}$.

$m\backslash n$	1	2	3	$\cdots$
0	1.00	2.29	3.60	$\cdots$
1	1.58	2.92	4.23	$\cdots$
2	2.13	3.50	4.83	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

The modulus of the normalized total sound power grows as the parameter ε_T grows for the excitation frequencies different than the membrane's eigenfrequencies (cf., Figs. 2a, 3a, 4a, 5a). The phase cosine also grows as the parameter ε_T grows. The normalized total sound power of the membrane excited by the Dirac delta function has been presented in Fig. 2a. The nine local maxima can be noticed. They are associated with the nine eigenfrequencies of the membrane's modes included. In this case all the included modes are significantly excited and significantly contribute the sound power. The excitation given by Eq. $(32)_2$ has been described by the cosine function. In this case the modulus of the normalized sound power has only its three local maxima associated with the eigenfrequencies of the three modes of the number of the nodal diameters $m = 2$ (cf., Fig. 3a). In the general case this means that only the modes of the nodal diameters equal to the nodal diameters of the excitation, i.e. for $m = M$, are significantly excited. The normalized sound power of the membrane excited by the surface force given by Eq. $(32)_3$ has been presented in Fig. 4a. In this case only one mode is significantly excited. This mode satisfies the conditions $m = M$ and $n = N$ (this mode is $(2, 1)$ in Fig. 4a).

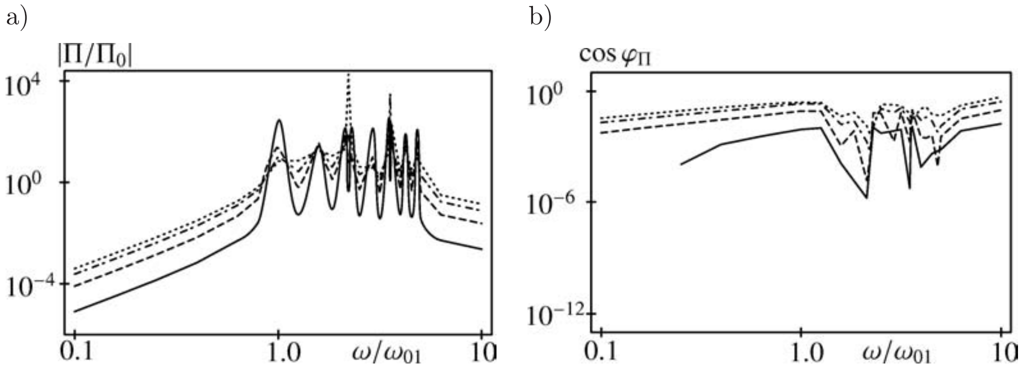

Fig. 2. The normalized total sound power radiated: a) modulus, b) phase cosine. Excitation given by Eq. $(32)_1$ for $\overline{r}_0 = 0.5a$, $\overline{\varphi}_0 = \pi/4$, $l/a = 3$ and $\varepsilon_1 = 0.6$. Lines: solid $\varepsilon_T = 0.01$, dashed $\varepsilon_T = 0.1$, dash-dotted $\varepsilon_T = 0.3$ and dotted $\varepsilon_T = 0.5$.

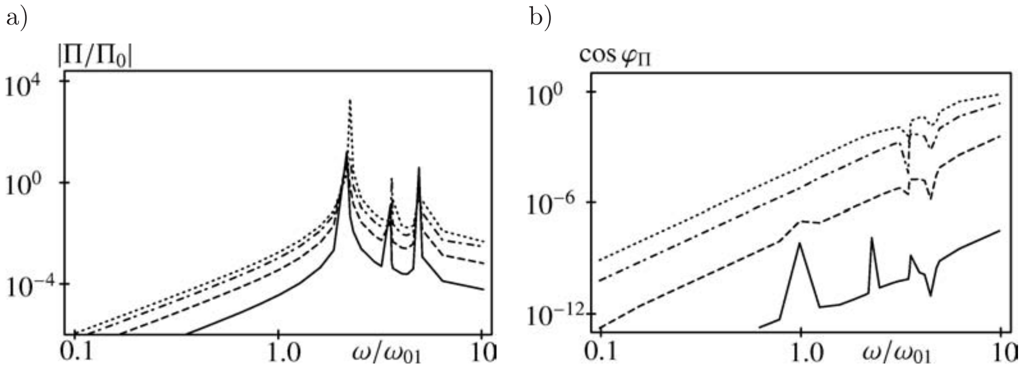

Fig. 3. The normalized total sound power radiated: a) modulus, b) phase cosine. Excitation given by Eq. $(32)_2$ for $M = 2$, $\overline{\varphi}_0 = \pi/4$, $l/a = 3$ and $\varepsilon_1 = 0.6$. Lines: solid $\varepsilon_T = 0.01$, dashed $\varepsilon_T = 0.1$, dash-dotted $\varepsilon_T = 0.3$ and dotted $\varepsilon_T = 0.5$.

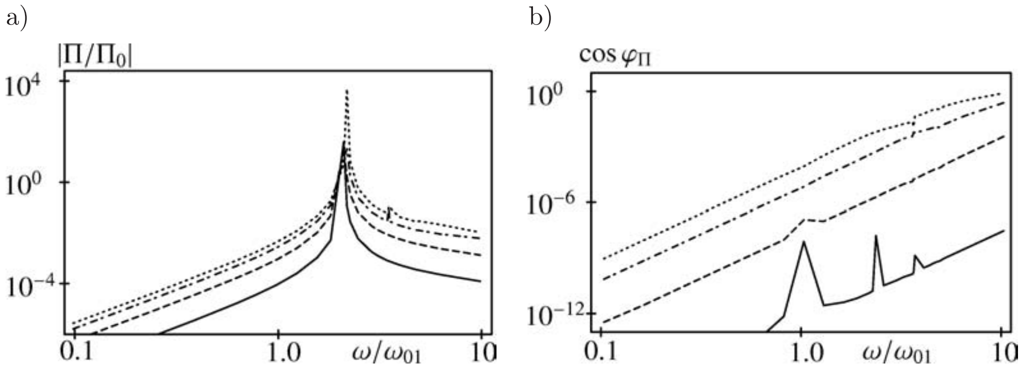

Fig. 4. The normalized total sound power radiated: a) modulus, b) phase cosine. Excitation given by Eq. $(32)_3$ for $\overline{\varphi}_0 = \pi/4$, $M = 2$, $N = 1$, $l/a = 3$ and $\varepsilon_1 = 0.6$. Lines: solid $\varepsilon_T = 0.01$, dashed $\varepsilon_T = 0.1$, dash-dotted $\varepsilon_T = 0.3$ and dotted $\varepsilon_T = 0.5$.

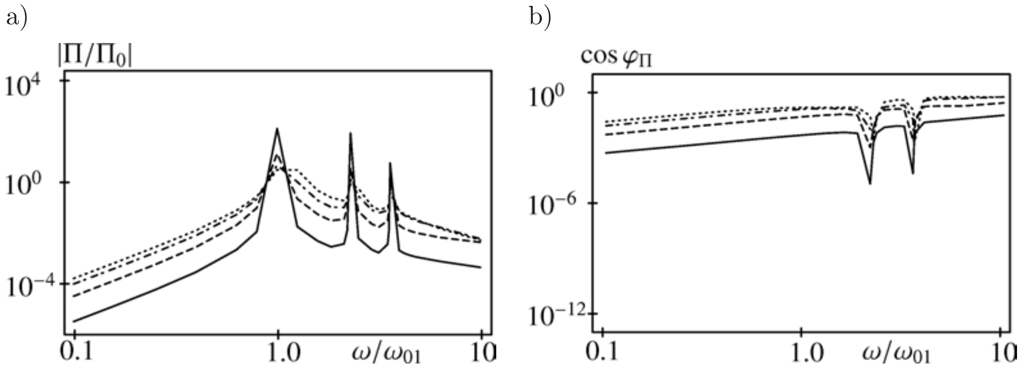

Fig. 5. The normalized total sound power radiated: a) modulus, b) phase cosine. Excitation given by Eq. $(32)_4$ for $l/a = 3$, $b = a$ and $\varepsilon_1 = 0.6$. Lines: solid $\varepsilon_T = 0.01$, dashed $\varepsilon_T = 0.1$, dash-dotted $\varepsilon_T = 0.3$ and dotted $\varepsilon_T = 0.5$.

In the case of the axisymmetric excitation given by Eq. $(32)_4$, only the axisymmetric modes are significantly excited for $m = 0$. This results in the three local maxima of the sound power for the eigenfrequencies of the membrane's axisymmetric modes $(0,1)$, $(0,2)$, $(0,3)$ (see Fig. 5a). The influence of some modes on the changes in the sound power phase cosine can be noticed in Figs. 2b, 3b, 4b and 5b for the excitation frequencies equal to the corresponding eigenfrequencies. This effect results from the influence of the acoustic attenuation on the membrane's vibrations and sound power radiated. Comparing the sound power radiated for the four different excitations leads to the conclusion that the sound power assumes greater values for the axisymmetric excitation given in Eq. $(32)_4$ and for the excitation described by the Dirac delta function given in Eq. $(32)_1$ where all the axisymmetric modes are significantly excited.

The modulus of the normalized total sound power radiated as the function of ε_1 has been presented in Fig. 6. The parameter ε_1 has been given in Eq. (31) and determines the acoustic attenuation. The most significant influence of this parameter on the sound power radiated can be noticed for the excitation frequencies equal to and close to the successive eigenfrequencies of the membrane when the effect of the resonance can be observed. Therefore all the curves have been plotted for $\omega = \omega_{mn}$. On the other hand, the modulus depends very weakly on the parameter ε_1 for the excitation frequencies much different from the successive eigenfrequencies and for these excitation frequencies the acoustics attenuation can neglected. This fact has been illustrated, e.g., by the flat solid line in Fig. 6b plotted for such the excitation frequency that the sound power resonance does not appear (cf., Fig. 3a).

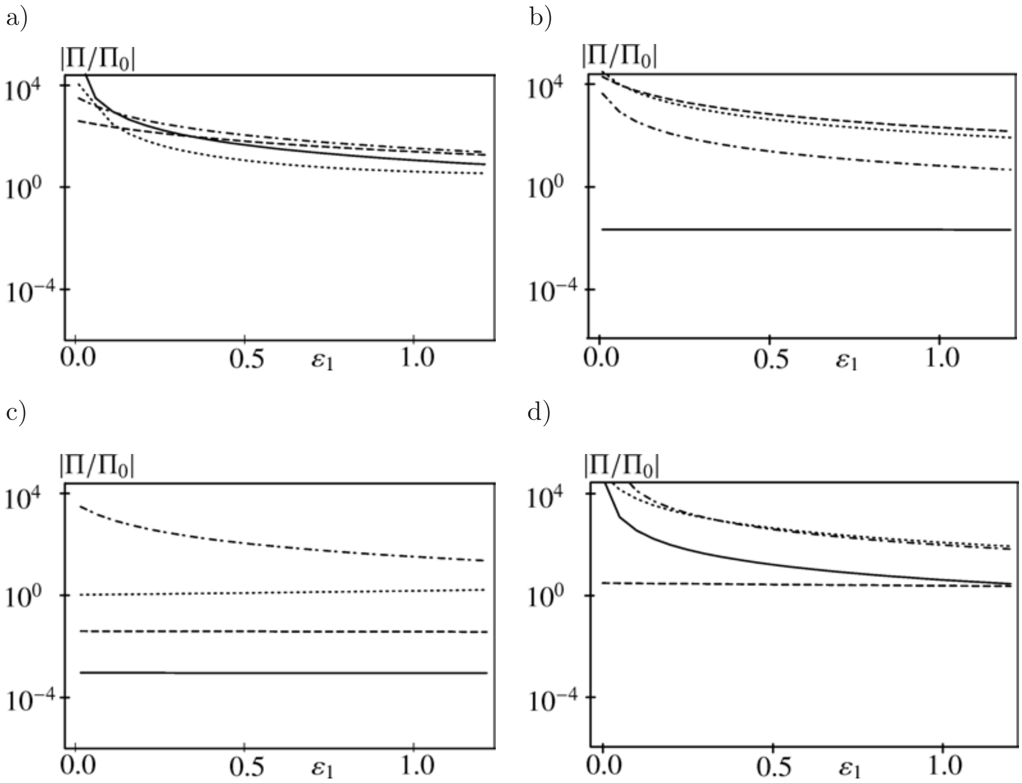

Fig. 6. The normalized total sound power radiated for $\varepsilon_T = 0.1$ and $l/a = 3$: a) excitation given by Eq. $(32)_1$ for $\overline{r}_0 = 0.5a$, $\overline{\varphi}_0 = \pi/4$ and lines: solid $\omega/\omega_{01} = 1.0$, dashed $\omega/\omega_{01} = 1.6$, dash-dotted $\omega/\omega_{01} = 2.13$ and dotted $\omega/\omega_{01} = 2.3$; b) excitation given by Eq. $(32)_2$ for $M = 2$, $\overline{\varphi}_0 = \pi/4$ and lines: solid $\omega/\omega_{01} = 1.0$, dashed $\omega/\omega_{01} = 1.6$, dash-dotted $\omega/\omega_{01} = 2.92$ and dotted $\omega/\omega_{01} = 4.83$; c) excitation given by Eq. $(32)_3$ for $\overline{\varphi}_0 = \pi/4$, $M = 2$, $N = 1$ and lines: solid $\omega/\omega_{01} = 1.0$, dashed $\omega/\omega_{01} = 1.6$, dash-dotted $\omega/\omega_{01} = 2.13$ and dotted $\omega/\omega_{01} = 2.3$; d) excitation given by Eq. $(32)_4$ for $b = a$ and lines: solid $\omega/\omega_{01} = 1.0$, dashed $\omega/\omega_{01} = 2.13$, dash-dotted $\omega/\omega_{01} = 2.3$ and dotted $\omega/\omega_{01} = 3.6$.

The relative difference modulus

$$\delta \Pi = \frac{|\Pi_2 - \Pi_1|}{|\Pi_1|}, \tag{36}$$

as the function of ω/ω_{01} has been presented in Fig. 7 where Π_2 is the total sound power radiated by the membrane located near the two-wall corner and Π_1 is the total sound power radiated by the same membrane embedded into a single flat baffle. This quantity is the measure of the relative change in the sound power radiated in the presence of the vertical baffle located in the distance l from the center of the vibrating membrane compared with the sound power radiated when the vertical baffle is not present. The phase difference cosine $\cos(\varphi_2 - \varphi_1)$, where φ_2 is the phase of Π_2 and φ_1 is the phase of Π_1, is nearly equal to the unity for all the considered excitation frequencies $\omega \in [0.1; 10]$. The changes in this quantity do not exceed the value of $2 \cdot 10^{-3}$ when the excitation frequency varies. These changes are so weak that they can be neglected and hence have not been plotted. The curves presented in Fig. 7 have been plotted for the Dirac delta function as the excitation $(32)_1$ and for the axisymmetric excitation $(32)_4$. The analysis of the curves makes it possible to conclude that the closer the vibrating membrane center is located to the vertical baffle the greater is the baffle's influence on the sound power radiated. The presence of the vertical baffle causes the greatest relative change in the sound power modulus for the low frequencies, i.e. for $\omega/\omega_{01} < 1$, and the smallest change for the excitation frequencies equal or close to the successive eigenfrequencies when the effect of resonance apears. The normalized total sound power radiated by the membrane embedded into a single flat baffle as the function of ω/ω_{01} has been presented in Fig. 8 for the reference of the curves presented in Fig. 7.

Four different formulations for the modal radiation impedance are necessary to assure their completeness together with the computing results of the total sound power radiated by the vibrating circular membrane into the region of

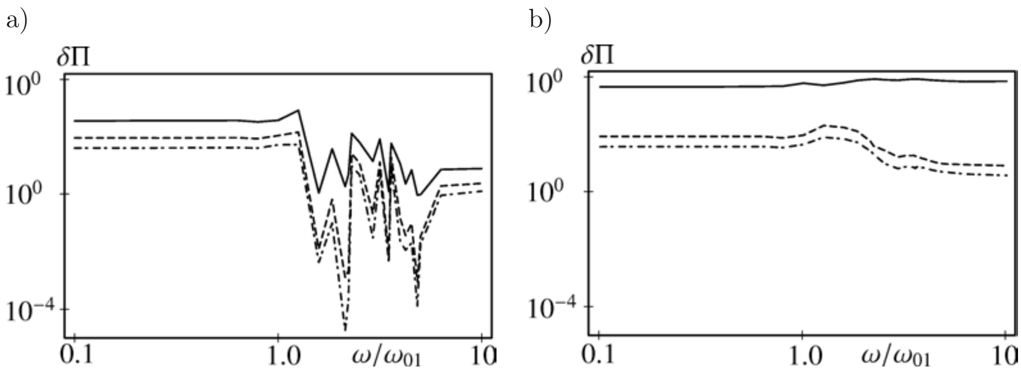

Fig. 7. The relative difference modulus $\delta\Pi$ of the total sound power radiated for $\varepsilon_1 = 0.6$ and $\varepsilon_T = 0.1$: a) excitation given by Eq. $(32)_1$ for $\overline{r}_0 = 0.5a$ and $\overline{\varphi}_0 = \pi/4$, b) excitation given by Eq. $(32)_4$ for $b = a$. Lines: solid $l/a = 1$, dashed $l/a = 2$ and dash-dotted $l/a = 3$.

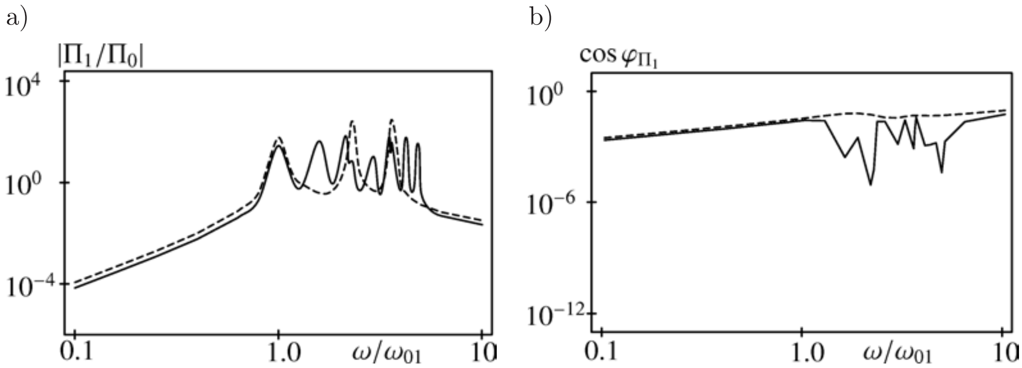

Fig. 8. The normalized total sound power radiated by the membrane embedded into a single flat baffle for $\varepsilon_T = 0.1$ and $\varepsilon_1 = 0.6$: a) modulus, b) phase cosine. Lines: solid – excitation given by Eq. $(32)_1$ for $\overline{r}_0 = 0.5a$ and $\overline{\varphi}_0 = \pi/4$; dashed – axisymmetric excitation given by Eq. $(32)_4$ for $b = a$.

the quarter-space. The analysis of Eqs. (18) containing the mentioned formulations leads to the conclusion that they can be asymmetric as the order of the two influencing modes is inverted, i.e. there exist such pairs of the modes (m, n) and (m', n') that $\xi_{mn,m'n'} \neq \xi_{m'n',mn}$ where $\xi_{mn,m'n'}$ represents one of the four mentioned impedance formulations. Moreover, $\xi^{(s,s)}_{0n,m'n'} = \xi^{(s,s)}_{mn,0n'} = 0$, $\xi^{(c,s)}_{mn,0n'} = \xi^{(s,c)}_{0n,m'n'} = 0$, $\xi^{(c,c)}_{mn,m'n'} = \xi^{(s,s)}_{mn,m'n'}$ for $m, m' \neq 0$ and the odd $m - m'$, $\xi^{(c,s)}_{mn,m'n'} = \xi^{(s,c)}_{mn,m'n'}$ for $m, m' \neq 0$ and $m - m'$ being even. The asymmetry of the modulus and the phase cosine of the modal radiation impedance results from the term containing $J_{m-m'}(2\tau l) = (-1)^{m'-m} J_{m'-m}(2\tau l)$ within Eqs. (18) (cf., [31]).

6. Concluding remarks

Applying the modal analysis and the Green function has made it possible to join two different boundary value problems: one concerning the vibrations of the excited circular membrane in the polar coordinates and the second one regarding the sound radiated by the membrane into the region of the quarter-space. As a result, all the modal and physical quantities have been presented for describing the acoustic interactions within the region of the quarter-space.

Solving the algebraic equations system (28) leads to determining the coefficients c_{mn} given that the modal mutual impedance $\zeta_{mn,m'n'}$ has been determined earlier. Assuming the surface excitation force makes it possible to determine the modal coefficients f_{mn} from Eq. (34)). The modal mutual impedance $\zeta_{mn,m'n'}$ has been determined using Eq. (17) performing integration over the variable τ within the limits $\tau \in (0, k_0)$ for the mutual resistance $\theta_{mn,m'n'}$ and within the limits $\tau \in (k_0, \infty)$ for the mutual reactance $\chi_{mn,m'n'}$. It has been noted that the reciprocity law is not satisfied for the mutual impedance for each pair of the two different modes (m, n) and (m', n').

The total sound power active and reactive both can be computed from Eqs. (19) and (30). It is worth noticing that Eq. (30) does not contain the modal radiation resistance from Eqs. (17) and (18). However, it can only be used with the coefficients c_{mn} computed from Eq. (28) where the modal radiation impedance have to be included.

Acknowledgments

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