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Feedforward vs. Feedback Fixed-Parameter H_2 Control of Non-Stationary Noise

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Stationary random noise can be modelled as a wide-sense stationary white noise filtered by a minimum phase filter. Such filter can be used to design an optimal control filter minimising variance of the signal being the effect of the noise and the secondary sound interference. However, in many environments the noise is subject to change. For instance, some of the noisy devices are switched on and off, speed of some rotors or fans changes, etc. As a result contribution of different frequency components may significantly vary in time. Solving the optimisation problem to update control filter is rather avoided in on-line systems. In adaptive approach there are problems with convergence or some unpleasant transient acoustic effects. In this paper, the fixed-parameter approach to control is appreciated. Dominating frequency components/bands can usually be distinguished for the acoustic environment. Then, the idea of generalised disturbance defined by a frequency window of different type can be applied. If a reference signal, correlated with the disturbance to be reduced is available in advance, a feedforward structure can be applied, and otherwise, a feedback structure is used. Spectral and inner-outer factorisations are employed in order to cope with non-minimum phase character of the acousto-electric plant. Efficiency of the proposed approach for both control structures is verified based on the data obtained from an active personal headset. The generalised disturbance based control systems are confronted with the classical Wiener control systems designed for the given disturbance.

Keywords: active noise control, acousto-electric path, fixed-parameter control, non-stationary noise, feedforward control, feedback control, optimal control.

1. Introduction

Active noise control aims at reducing unwanted primary acoustic noise using secondary sound generated by a secondary source (usually a loudspeaker) driven by a control system [2–4]. If the acousto-electric plant differs marginally when compared to its model and the noise is stationary, it is justified to design a fixed-parameter control system. Such a system is generally free of transient acoustic effects unpleasant to the user, what is an important drawback of an adaptive system, which might even diverge under some circumstances [2, 10, 12, 13]. On the other hand, the fixed-parameter system does not require sophisticated calculations and is thus simpler in implementation and cheaper than an adaptive system.

The fixed-parameter control system to be effective, is often designed as a solution to an optimisation problem. In the H_2 approach the basic cost function can be defined as

$$L = E\left\{y^2(i)\right\}, \tag{1}$$

where $y(i)$ is the control system output, being the effect of acoustic waves interference, monitored by a microphone or estimated (in the problem of noise control at a location different than that where the physical microphone is placed). Minimisation of such a cost function corresponds to minimisation of the mean-square acoustic pressure. The cost function can be extended by including other terms due to control signal variance, control filter gain or plant uncertainty.

For optimisation, knowledge about the primary noise is often required. Dependent on the domain, the sampled primary noise itself (being the output disturbance for the control system), its model or power spectrum density can be used. The control system designed in such a way may be possibly best tuned to reduce the given stationary noise.

A very common application of active noise control is an active headset. It is a personal device directly protecting the human's hearing system by reducing the unwanted sound directly at the eardrum [5]. Since the headset is usually tightly sealed to the ear-shell the plant response changes little. However, the person wearing the headset may move in a noisy area, e.g. a factory, and pass close to different working mechanisms generating different type of noise. The devices may be turned on and off, and may generate non-stationary sound, although of a limited frequency band. A control system designed for a particular disturbance may then fail. For such a case a control system can be designed in order to minimise the integral of the sensitivity function over an assumed frequency range [11] or the H_∞ theory can be applied. In this paper another approach is used.

It takes advantage of the Wiener filter methodology [2]. First, the control filters are designed for a given disturbance. Then, the idea of a generalised disturbance is put forward and the Wiener filter design process is performed again for such disturbance. Feedforward and feedback architectures are used and compared [9]. Experiments are performed to validate proposed systems for different cases referring to reality.

2. Control of stationary noise

2.1. Feedforward control

The general feedforward structure is presented in Fig. 1, where $y(i)$ is the system output, $d(i)$ is the output disturbance to be reduced, $x(i)$ is a reference signal correlated with that disturbance, P is the primary path related to noise propagation between the points of $x(i)$ measurement and the noise reduction point, S is the secondary path related to secondary sound propagation between the secondary source and the noise reduction point, W is the control filter. Paths P and S include standard necessary electronics. $Y(z)$, $D(z)$, and $X(z)$ will represent Z transforms of respective signals.

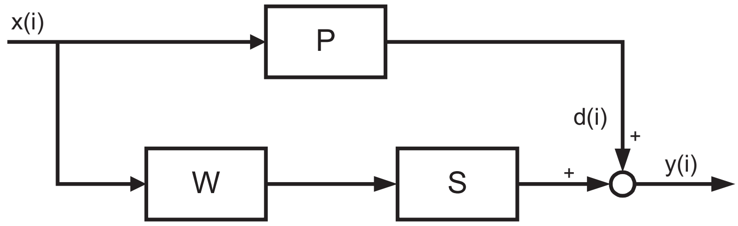

Fig. 1. Feedforward structure.

It follows from Fig. 1 that the system output can be described as

$$Y(z) = [P(z) + W(z)S(z)]X(z). \tag{2}$$

Stochastic and wide-sense stationary disturbance can be modelled by a zero-mean wide-sense stationary white noise (of Z transform $E(z)$) filtered by a minimum phase disturbance-shaping filter:

$$X(z) = F(z)E(z). \tag{3}$$

Contribution of the output disturbance to the system output could be completely removed by the control filter of the following transfer function in [2, 8, 10]

$$W_{\mathrm{opt}}(z) = -\frac{P(z)}{S(z)}. \tag{4}$$

Because transfer functions of the paths P and S are generally unknown, their models, noted with hats, should be used in practice. However, the control filter calculated in this way would be unstable because the secondary path model, $\widehat{S}(z)$, is generally non-minimum phase. It could also be non-causal, dependant on the relative delay introduced by models $\widehat{S}(z)$ and $\widehat{P}(z)$. One of the possible solutions is to minimise the mean square value of the output signal and factorise the plant model into the inner, $\widehat{S}^{(i)}(z)$, and outer, $\widehat{S}^{(o)}(z)$, parts:

$$\widehat{S}(z) = \widehat{S}^{(i)}(z)\widehat{S}^{(o)}(z). \tag{5}$$

Then, the sub-optimal causal control filter can be easily found using the Wiener approach in the discrete frequency domain as [2]:

$$W_{\mathrm{opt}+}(n) = -\frac{1}{F(n)\widehat{S}^{(o)}(n)} \left\{ \frac{\widehat{P}(n)F(n)}{\widehat{S}^{(i)}(n)} \right\}_+ , \tag{6}$$

where n is the frequency bin number and the other quantities being functions of n are frequency responses of their respective transfer functions. The causal part $\{\,\}_+$ in (6) can be obtained according to

$$\left\{ \frac{\widehat{P}(n)F(n)}{\widehat{S}^{(i)}(n)} \right\}_+ = FFT\left[\ell(n)\cdot IFFT\left(\frac{\widehat{P}(n)F(n)}{\widehat{S}^{(i)}(n)}\right)\right], \tag{7}$$

where FFT is the Fast Fourier Transform, $IFFT$ is its inverse, and $\ell(n)$ is:

$$\ell(n) = \begin{cases} 1 & n \geq 0, \\ 0 & n < 0. \end{cases} \tag{8}$$

$F(n)$ can be found from the disturbance Power Spectral Density, S_{xx}, using the cepstral method:

$$F(n) = \exp\left\{FFT[\ell_0(n)\cdot IFFT(\ln(S_{xx}(n)))]\right\}, \tag{9}$$

where $\ell_0(n)$ is defined as

$$\ell_0(n) = \begin{cases} 1 & n > 0, \\ 1/2 & n = 0, \\ 0 & n < 0. \end{cases} \tag{10}$$

Similar approach can be applied to obtain the frequency response of the outer part of the plant model:

$$\widehat{S}^{(o)}(n) = \exp\left\{FFT\left[2\ell_0(n)\cdot IFFT\left(\ln\left(\left|\widehat{S}(n)\right|\right)\right)\right]\right\}. \tag{11}$$

The inner part is obtained from (5).

The Wiener filter can also be designed in the time domain using the correlation-based approach, which does not require calculating the spectral and inner-outer factorisations. Both the plant model and the control filter are then assumed to have finite impulse responses,

$$\mathbf{w}_{\mathrm{opt}+}(n) = -E\left\{\widehat{\mathbf{r}}(i)\widehat{\mathbf{r}}^{\mathrm{T}}(i)\right\}^{-1} E\left\{\widehat{\mathbf{r}}(i)d(i)\right\}, \tag{12}$$

and $\widehat{r}(i)$ is defined as disturbance filtered by the plant model

$$\widehat{r}(i) = \widehat{\mathbf{s}}^{\mathrm{T}}x(i), \tag{13}$$

where $\mathbf{w}_{\mathrm{opt}+}(i)$ and $\widehat{\mathbf{s}}$ are vectors of impulse responses of respective transfer functions of FIR type, and $\widehat{\mathbf{r}}(i)$ is the vector of regressors of $\widehat{r}(i)$. In practice, the auto- and cross correlations in (12) are often computed from auto- and cross- spectral densities.

2.2. Feedback control

If a reference signal, correlated with the output disturbance, is unavailable, e.g. in case of mobile applications, feedback control is an interim solution. Among many possible feedback structures, Internal Model Control (IMC) is usually used for active control as presented in Fig. 2.

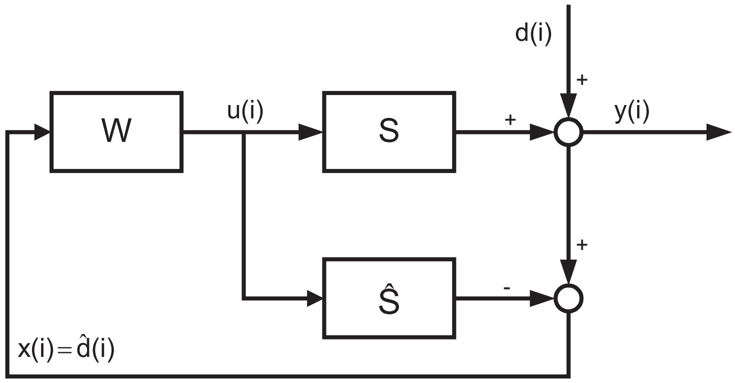

Fig. 2. IMC structure.

It follows from Fig. 2 that the system output can be described as

$$Y(z) = \frac{1 + W(z)\widehat{S}(z)}{1 + W(z)\left(\widehat{S}(z) - S(z)\right)} D(z). \tag{14}$$

If the plant model is accurate ($S \approx \widehat{S}$), the system can be analysed as a feedforward one. Control filter W, which minimises the mean-square value of the output signal can now be designed using the standard Wiener technique [2, 7, 10].

To avoid duplicate equations describing analogous control design process, changes to equations from Subsec. 2.1 are presented only. All of the occurrences of the signal $x(i)$ and path P (in both the time and frequency domains) as well as the power spectral density S_{xx} in the Eqs. (3)–(13) [excluding (12)] should be substituted by signal $d(i), 1$, and power spectral density S_{dd}, respectively.

3. Control of non-stationary noise

3.1. Fixed-parameter approach

Fixed-parameter control filters designed based on the disturbance model, as described in the previous sections, may be far from the optimal solution if the noise is non-stationary, and they may fail. Solving the optimisation problem to update control filter is time consuming and is rarely used in on-line systems. An adaptive approach may experience problems with convergence or some unpleasant transient acoustic effects may be generated. Therefore, it is avoided in industrial applications, where such effects might annoy users and cause a danger.

For many applications, although the noise may be non-stationary, its spectrum is limited to a narrow band, and only within that band contribution of different frequencies changes. Under such assumption, the same design procedure, presented in Sec. 2, can be used if the idea of the generalised disturbance is incorporated.

3.2. The idea of the generalised disturbance

The generalised disturbance signal can be defined in the frequency domain. First, it is necessary to identify dominating frequency components of the disturbance, what can easily be done, e.g. based on its spectral analysis. The PSD can be of a window-type shape for frequencies of interest (Fig. 3), what results in the window-type magnitude of frequency response of the disturbance shaping filter. Different windows can be used, dependent to what extend frequency components around the dominating ones should be taken into consideration. The PSD does not bear information about the phase of the disturbance. However, with the control system description presented in Sec. 2, the phase of the disturbance does not influence the design (note that spectral factorisation could be performed over the disturbance-shaping filter, what modifies the phase characteristic substantially). However, in case of the feedforward structure for the control filter to be successful, the phase relation between the reference signal and the disturbance to be reduced is crucial. Therefore, equations for the Wiener filter in the previous section were written based on the reference signal only, assumed to be acquired in advance,

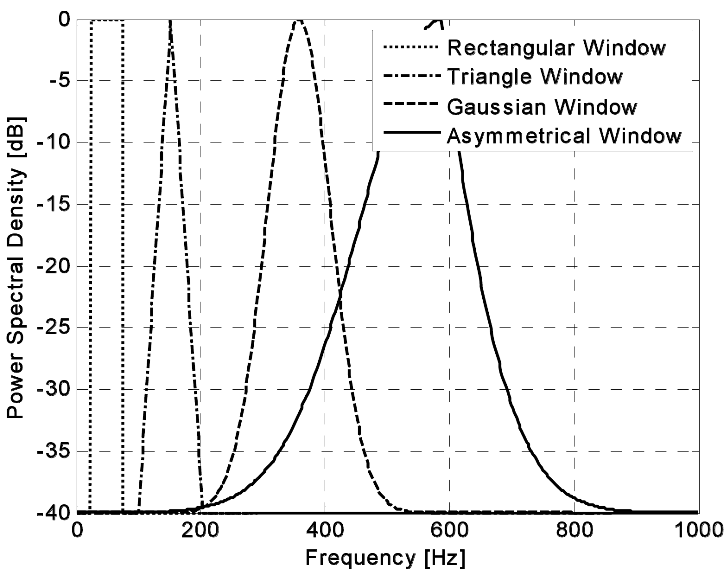

Fig. 3. Window-type magnitude of the generalised disturbance PSD.

and the output disturbance is estimated by filtering the reference signal by the primary path model.

Having the response of the disturbance shaping filter $F(n)$ calculated from the disturbance PSD as defined in (9), it can be directly applied to (6) and (7) to obtain the control filter. For practical applications, it is recommended to design off-line a set of control filters for generalised disturbances defined by window type, width and centre. Then, after simple identification of the dominating frequency components of the disturbance, the appropriate filter can be selected from a look-up table.

For success of the proposed approach it is important to use high sampling frequency. For the feedforward structure the reference signal should provide information about the output disturbance in advance. Assuming that for a small headset the distance between the reference microphone mounted outside the headset, and the eardrum is about 3 cm, the sampling frequency higher than 11 kHz should be used. In turn, a feedback system can perform effectively for the frequency band limited as [6]

$$\Delta f < \frac{1}{2\pi\tau_{\text{plant}}}. \tag{15}$$

Concluding, the lower is the ratio of the frequency band under control and the sampling frequency, the better reduction results can be obtained for a given group delay τ_{plant} introduced by the secondary path. Due to specific properties of the acoustic signal and the hearing mechanism, sufficiently high sampling frequency allows for not using analogue anti-aliasing and reconstruction filters, thereby reducing τ_{plant} and thus significantly enhancing the system performance.

4. Experiments

For experiments the active headset has been tightly sealed to the ear canal opening in the artificial head (Fig. 4). The reference microphone presented in Fig. 4 is used only when the feedforward structure is considered. Noise propagates between the points of reference microphone ($x(i)$ measurement) and the point, where noise reduction is required – position of the error microphone. The error microphone is in this case only used for identifying path models and for monitoring the system output during control. Position of the reference microphone is not crucial, provided the causality constraint is satisfied in the feedforward system for the chosen sampling frequency, since it strongly influences P (time delay of P should be larger than that of S).

For the feedback control, a reference signal correlated with the output disturbance is unavailable. In this case the error microphone is a single source of measurement.

The sampling frequency has been chosen as 22050 Hz. Frequency response of the plant model for low frequencies is presented in Fig. 5.

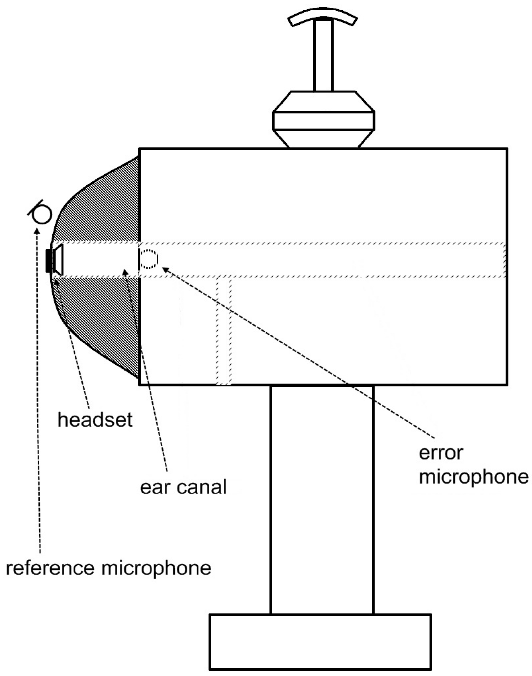

Fig. 4. Artificial head with active headset.

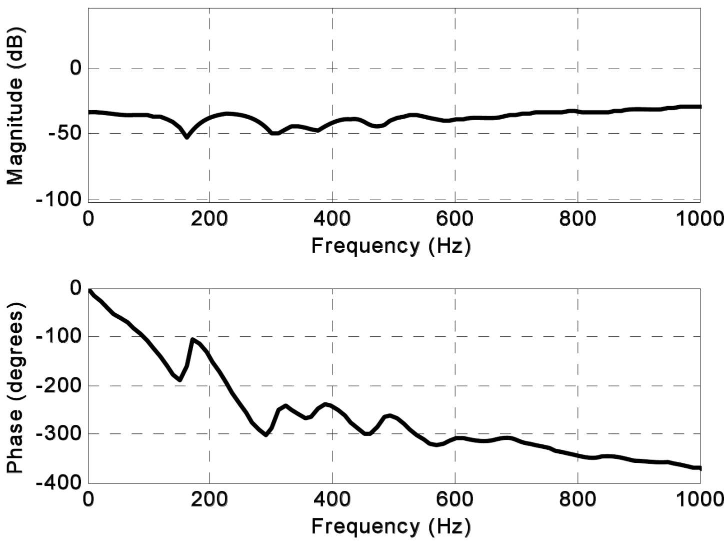

Fig. 5. Frequency response of the plant model.

Real-world sound recorded in the power plant in Rybnik, Poland, has been used as the noise to be reduced. Its PSD is presented in Fig. 6 by the dotted line. First the Wiener filter has been designed based on the full knowledge about the disturbance. Then, the proposed approach with the generalised disturbance of a rectangular window PSD has been used (covering the frequencies of 250–500 Hz, representing dominating components of the primary noise). Both the feedforward and feedback structures have been considered. Obtained results are presented in Fig. 6.

It is observed from the plots that the optimally designed feedforward and feedback systems satisfactorily reduce the noise in the frequency band of inter-

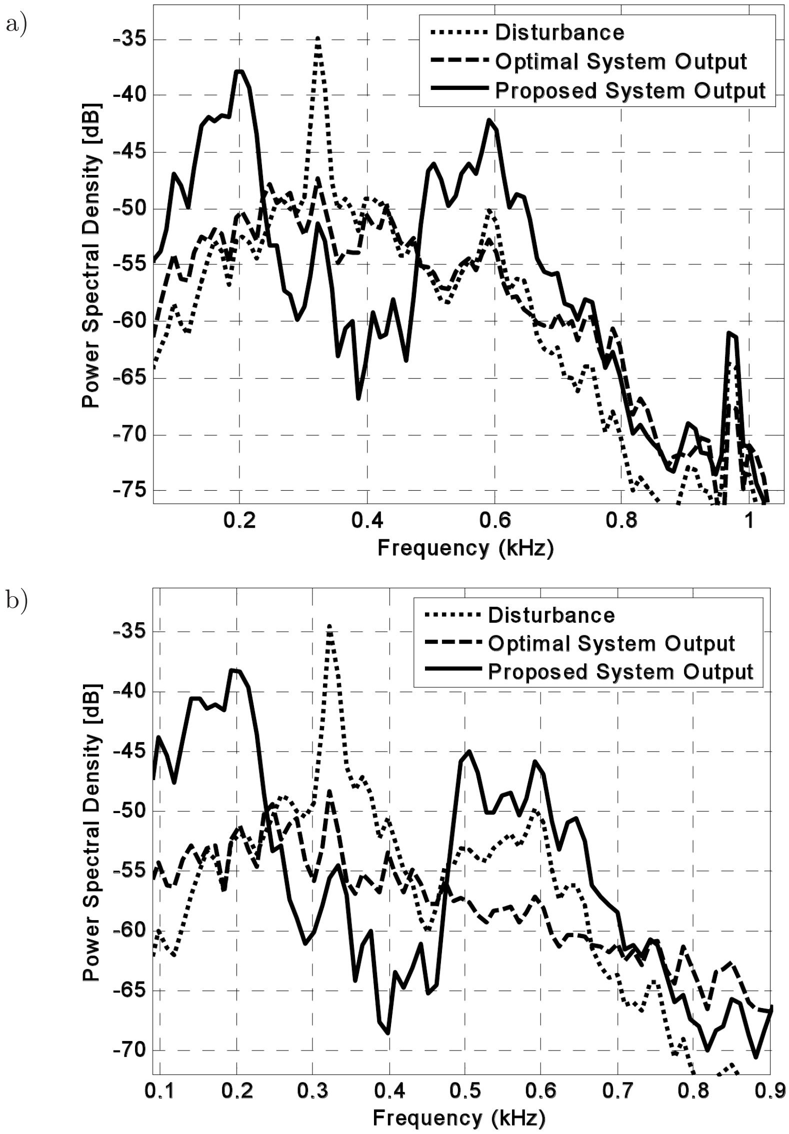

Fig. 6. Control results for the optimal and proposed systems (rectangular window disturbance PSD) a) IMC structure; b) feedforward structure.

est. However, the systems designed for the generalised disturbance reveal better performance for the selected band, because they are tuned to make the full effort there. Frequency components outside that band are reinforced. Such drawback can be successfully mitigated by generating another generalised disturbance (Figs. 7a, 8a, 9 and 10). Best results are obtained by using a window of gradually decaying PSD outside that band (Figs. 7a, 8a and 10). Sound reinforcement be-

a)

b)

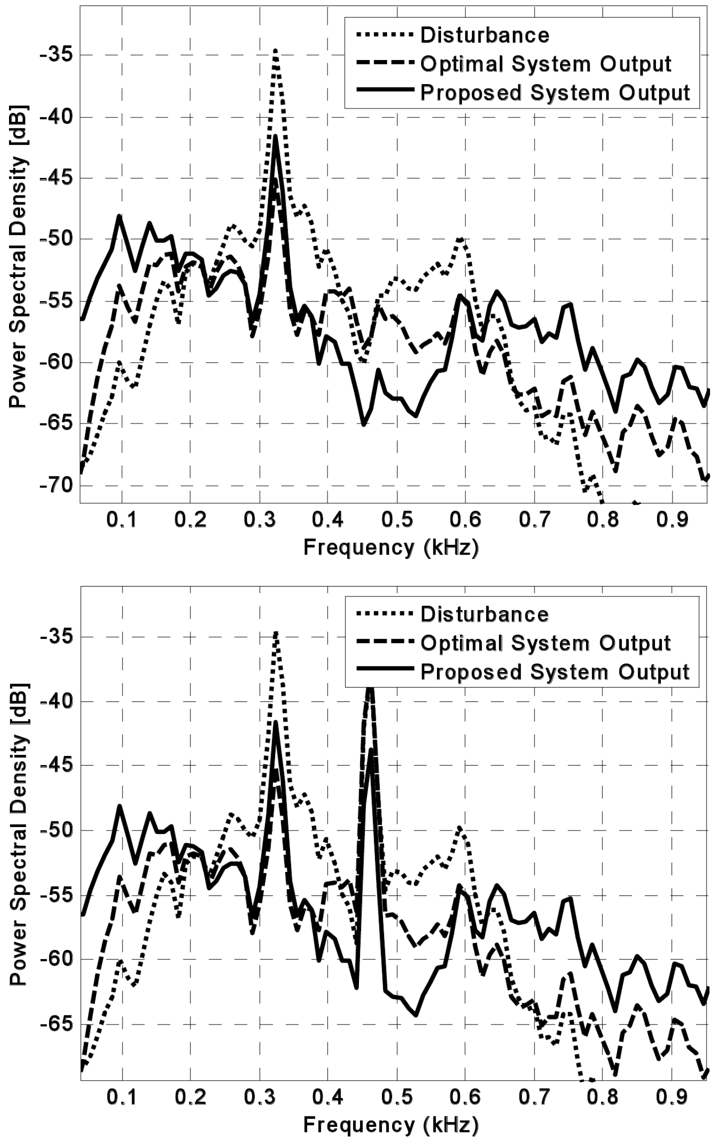

Fig. 7. Control results for the optimal and proposed systems in the feedforward structure (Gaussian window disturbance PSD) operating: a) for the disturbance used in design process; b) for the disturbance with additional components at 440–470 Hz.

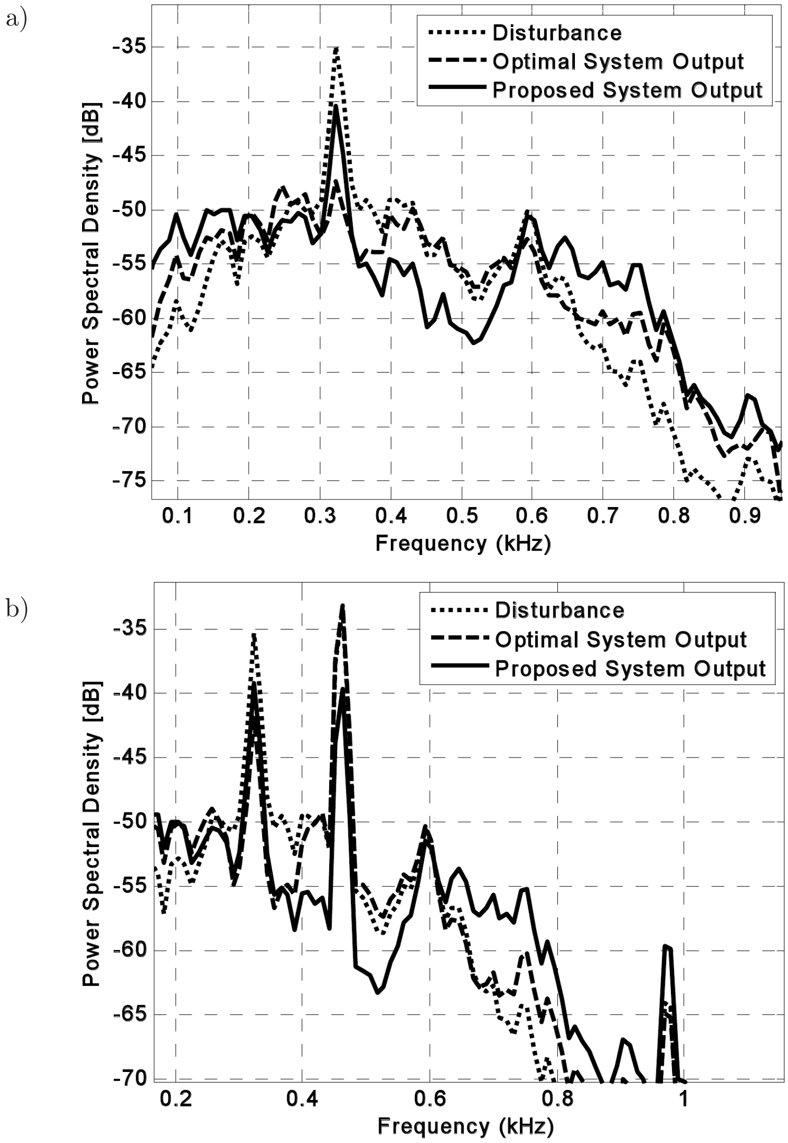

Fig. 8. Control results for the optimal and proposed systems in the IMC structure (Gaussian window disturbance PSD) operating: a) for the disturbance used in design process; b) for the disturbance with additional components at 440–470 Hz.

yond the band of interest is much smaller than in case of the rectangular window, but noise reduction in that band is smaller as well (Figs. 7a and 8a).

Another experiment was performed to examine how the two control systems behave if the control filter is designed for one disturbance and it operates for a different disturbance, which was generated by adding another component of frequencies 440–470 Hz (Figs. 7b and 8b). This experiment simulates switching a)

b)

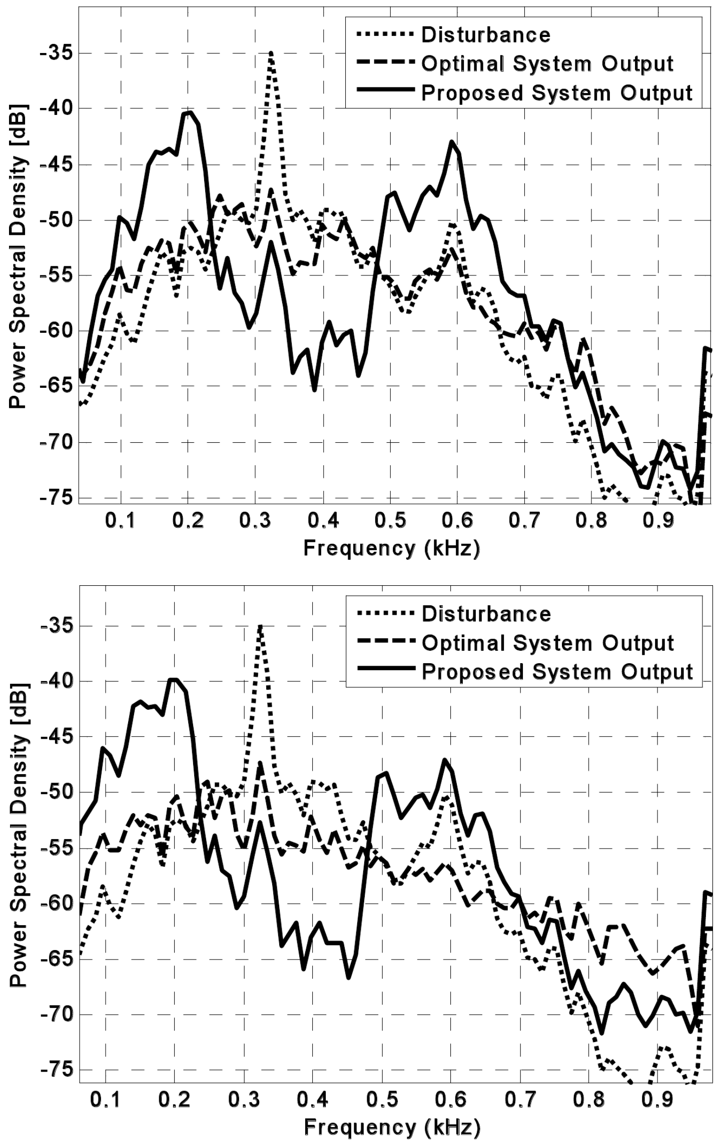

Fig. 9. Control results for the optimal and proposed systems (triangular window disturbance PSD) a) IMC structure; b) feedforward structure.

on or off some of the noise sources, or another type of noise nonstationarity. As expected, the system optimal for the particular noise is unable to reduce the added frequency components (440–470 Hz), because they were not considered during control filter design. They are, however, well reduced by the proposed system.

For all experiments reported above, the feedforward system yields better performance than the feedback system. Thus, provided the environmental and techa)

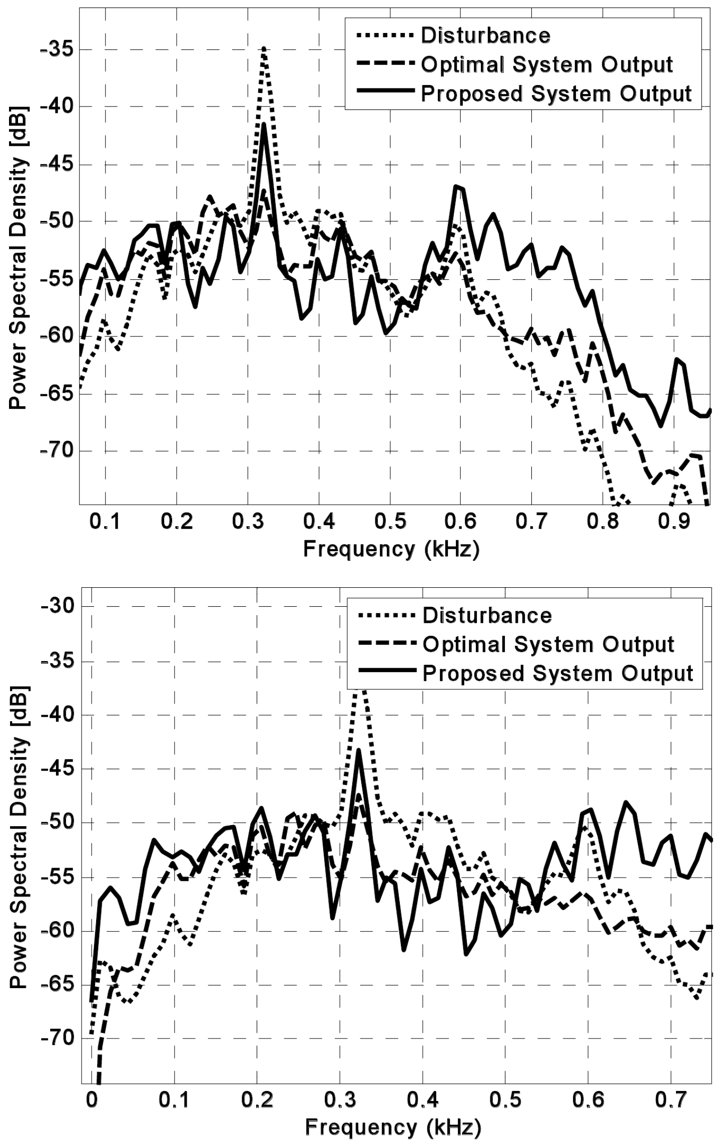

Fig. 10. Control results for the optimal and proposed systems (asymmetrical Gaussian window disturbance PSD) a) IMC structure; b) feedforward structure.

nical conditions allow for installing a reference microphone giving in advance a signal correlated with the output disturbance, the feedforward structure is worth considering in a diffuse sound field. However, for it to operate efficiently, the path models should be of sufficient accuracy, i.e. the headset should be sealed similarly during modelling and control. Otherwise, the feedback system might give better results, since it has the in-built potential to reduce effects of the modelling error. The error microphone, which is quite easy to be installed in case of classical headsets, is really difficult to be placed if small active earplugs are considered, which are appreciated for some applications due to ergonomic reasons.

5. Conclusions

In this paper fixed-parameter active control of sound in the feedforward and feedback (Internal Model Control) structures has been considered. Referring to practical applications where the noise to be reduced is non-stationary and the plant changes little, the idea of the generalised disturbance has been proposed for the both structures. The Wiener filter design procedure has been adequately adopted. Such approach has been proven to yield better performance than the optimal Wiener system designed for the given disturbance, which is then subject to change. The proposed systems exhibit much better behaviour for the assumed frequency band. The performance within and beyond that band can be controlled by applying different types of the frequency window defining the generalised disturbance. The overall noise reduction level is larger for the feedforward structure. Such structure requires involvement of the reference microphone being installed on the outside of the headset. However, it does not need the error microphone close to the eardrum, what could be a source of problems if the size of the headsets is reduced as required for some industrial applications. Nevertheless, if there is a risk of path variations, the feedback structure can be more beneficial.

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