

Motor Nonlinearities in Electrodynamic Loudspeakers: Modelling and Measurement

Benoit MERIT⁽¹⁾, Valérie LEMARQUAND⁽¹⁾, Guy LEMARQUAND⁽¹⁾,
Andrzej DOBRUCKI⁽²⁾

⁽¹⁾ *LAUM, CNRS, Université du Maine*
Av. O. Messiaen, 72085 Le Mans Cedex 9, France
e-mail: guy.lemarquand@univ-lemans.fr

⁽²⁾ *Wrocław University of Technology*
Institute of Telecommunications, Teleinformatics and Acoustics
Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland
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This paper studies the motor nonlinearities of a classical electrodynamic loudspeaker. Measurements show the dependence of the voice-coil inductance with its position and with the driving current as well as the force factor dependence on the coil position. They allow the tuning of the parameters of a model proposed for these dependences. Time and frequency analysis of the model help in the explanation of both the harmonic and intermodulation distortions observed in classical loudspeakers.

Keywords: electrodynamic loudspeaker, motor nonlinearity.

1. Introduction

The nonlinearities of electrodynamic loudspeakers lead to harmonic and intermodulation distortions in the sound production. Their prominent sources of nonlinearity are suspensions and the motor. In this paper, the motor nonlinearities have been studied.

Classical loudspeakers are traditionally represented by the *Thiele&Small* parameters [1, 2]. These parameters are used to simply model the functioning of a loudspeaker: the electrical domain is represented by a resistor R_e and an inductance L_e ; the mechanical domain is represented by the mechanical losses R_{ms} , the moving mass M_{ms} and the stiffness of the suspensions K_{ms} ; and the acoustical domain is represented by an acoustical impedance that depends on the loudspeaker conditions of use and on its dimensions. When only both mechanical and electrical domains are of interest, a loudspeaker can be represented by the diagram of Fig. 1.

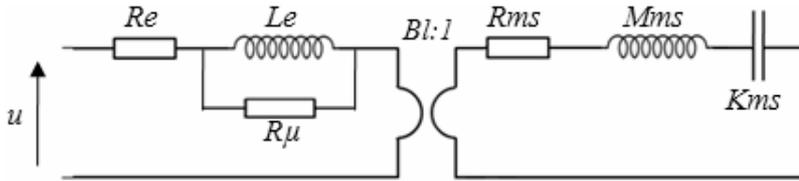


Fig. 1. Classical block diagram showing electrical and mechanical-mobility circuits of a loudspeaker and including the modelling of the eddy currents thanks to the resistor $R\mu$.

This diagram includes the modelling of eddy currents emergence via the resistor $R\mu$. This modelling is justified by the fact that eddy currents increase with the frequency of the input signal.

The coupling between the two domains is made thanks to the force factor Bl . In the mechanical domain, this parameter defines the motion force $F = Bl \cdot i$ that makes the voice-coil move when a current i runs through its wire. In the electrical domain, the force factor defines the back electromotive force $V_{\text{emf}} = Bl \cdot v$ that is created by the movement of the voice-coil with a velocity $v = dx/dt$ in the magnetic field, B , created by the magnetic circuit of the loudspeaker. When all the parameters are assumed to be linear, this coupling between the two domains leads to a system of two linear differential equations in which the current $i_L(t)$, is the current running through the voice-coil:

$$u(t) = \frac{Le(Re + R\mu)}{R\mu} \frac{di_L(t)}{dt} + Re \cdot i_L(t) + Bl \cdot v(t),$$

$$Mms \cdot \frac{d^2x(t)}{dt^2} + \left(Rms - \frac{Bl^2}{Re + R\mu} \right) \cdot \frac{dx(t)}{dt} + Kms \cdot x(t) = \frac{Bl}{Re + R\mu} \cdot (R\mu \cdot i_L(t) + u(t)). \quad (1)$$

In reality, it is known that a loudspeaker is a highly nonlinear system. The acoustic pressure it generates is not an accurate image of the driving voltage. Among the principal sources of nonlinear distortions, one finds the suspensions and the motor of the loudspeaker [3, 4]. The nonlinearities due to the suspensions are well-known, and loudspeaker manufacturers have developed materials and designs to minimize the drawbacks of the suspensions.

The nonlinearities causes due to a loudspeaker motor have also been identified [5–11]. In classical loudspeaker motors, the iron is a source of eddy currents, and makes the inductance dependent on the displacement of the voice-coil as well as on the driving current. And since the magnetic induction seen by the voice-coil varies with the axial position in the airgap, the force factor Bl is also dependent on the displacement of the voice-coil.

The effects of all these nonlinearities are well-known: they lead to harmonic and intermodulation distortions. Therefore, the aim of this paper is to highlight

the role of the motor alone in the generation of these distortions, thanks to a temporal resolution of the differential equations that govern the loudspeaker functioning.

2. Measurement of the motor nonlinearities

2.1. Evidence for nonlinearity

To help the study, each parameter of a loudspeaker motor is measured independently. Measurements are done on a typical loudspeaker whose moving part displacement can be ± 4 mm.

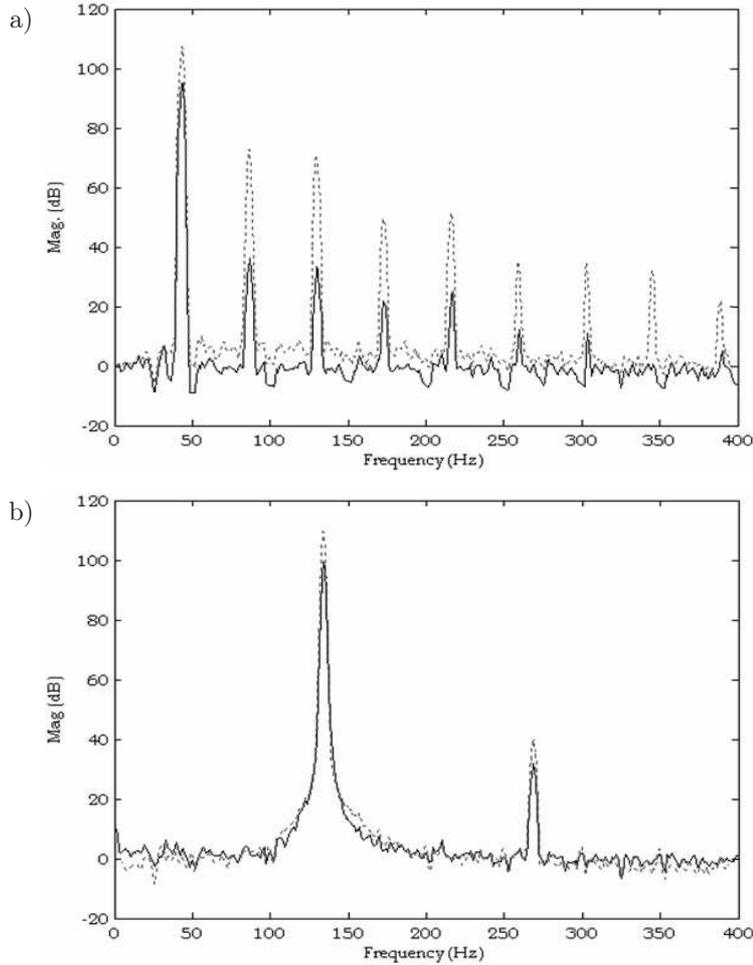


Fig. 2. Diaphragm velocity: input voltage 0.5 V (continuous line) and 2 V (dashed line) at a) 43 Hz and at b) 134 Hz. The reference is the measurement for the diaphragm at the rest position.

First, to point out the nonlinear nature of the loudspeaker, the velocity of the diaphragm when a simple sinusoidal signal drives the loudspeaker is measured thanks to a laser vibrometer (Fig. 2). At 43 Hz, the increase of the input voltage from 0.5 V to 2 V generates high amplitude harmonics.

At 134 Hz, the amplitude of the harmonics does not vary significantly from the 0.5 V to the 2 V input voltage. At this frequency, the displacements of the moving part are small compared with those at 43 Hz. Thus, we can conclude that the nonlinearities of the studied loudspeaker are related to the moving part displacement only.

In fact, the dependences of the stiffness, the inductance and the force factor on the displacement are at issue, but this paper plans to measure and model the contribution of the motor nonlinearities only.

2.2. Force factor nonlinearity

The most consequent source of nonlinearities in a loudspeaker motor is the dependence of the force factor Bl on the displacement x of the moving part [5]. To observe this dependence for the studied loudspeaker, the magnetic induction B in the airgap is first measured thanks to a Hall probe, and the force factor is deduced.

Figure 3 shows the force factor's non-symmetric dependence on the coil displacement. To take into account this dependence in the model, one has to approximate it thanks to a well-controllable mathematical function. The polynomial form is chosen to model the force factor variation. Its coefficient optimization must lead to a good conformity to the original curve and to its derivative. Figure 3 shows that a 4th order polynomial is necessary to reach an ideal conformity:

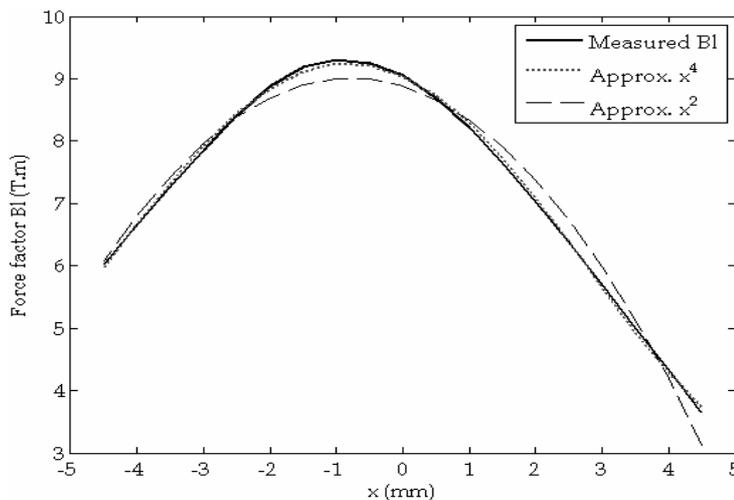


Fig. 3. Force factor Bl of the studied loudspeaker in the air gap vs. position x .

$$Bl(x) = Bl \cdot (1 + Bl_1 \cdot x + Bl_2 \cdot x^2 + Bl_3 \cdot x^3 + Bl_4 \cdot x^4), \quad (2)$$

with $Bl_1 = -54.01 \text{ m}^{-1}$, $Bl_2 = -31438.2 \text{ m}^{-2}$, $Bl_3 = 1.312 \cdot 10^6 \text{ m}^{-3}$ and $Bl_4 = 4.24 \cdot 10^8 \text{ m}^{-4}$.

2.3. Inductance nonlinearity

Another source of nonlinearities is the dependence of the coil inductance on the current intensity, i_L , and on the coil position in the air gap, x [12, 13]. To measure these dependences, a tailor-made equipment allows us to completely block the loudspeaker moving part –so, the voice-coil– in a required axial position x . For each position, for various current intensities from 20 mA to 180 mA, and for various frequencies from 30 Hz to 1000 Hz, the impedance is measured thanks to a Wayne–Kerr bridge. We model it as a resistor R_e in series with an inductance L_e in parallel with a resistor R_μ , which represents the eddy currents (Fig. 1).

Figure 4 presents the dependence of the inductance on the current intensity, for various positions of the voice-coil. The general trend shows that this inductance is not clearly dependent on the current intensity. Even if this observation agrees with previous observations on the velocity measurements, it would have been interesting to be able to supply the loudspeaker with higher current intensities, the bridge being limited to 200 mA.

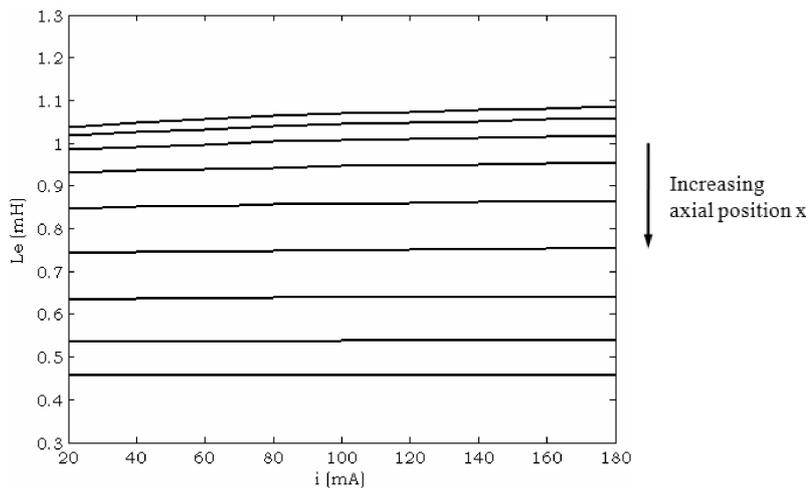


Fig. 4. Inductance L_e of the loudspeaker vs. driving current intensity i , for various positions of the voice-coil from $x = -0.004 \text{ mm}$ to $x = 0.004 \text{ mm}$.

Figure 5 shows the variation of the inductance with the axial position of the coil, for various current intensities. The grouped curves confirm that the inductance is not dependent on the current intensity. Nevertheless, the inductance

greatly depends on the coil axial position. The inductance mean value is approximated by a 4th order polynomial over x , which models the values as well as the derivative over x in the domain $x[m] \in [-0.0045; 0.0045]$:

$$Le(x, i_L) = Le \cdot L_x(x) = Le \cdot (1 + L_{x1} \cdot x + L_{x2} \cdot x^2 + L_{x3} \cdot x^3 + L_{x4} \cdot x^4), \quad (3)$$

with $L_{x1} = -112.65 \text{ m}^{-1}$, $L_{x2} = -11095.3 \text{ m}^{-2}$, $L_{x3} = 1.501 \cdot 10^6 \text{ m}^{-3}$ and $L_{x4} = 2.607 \cdot 10^8 \text{ m}^{-4}$.

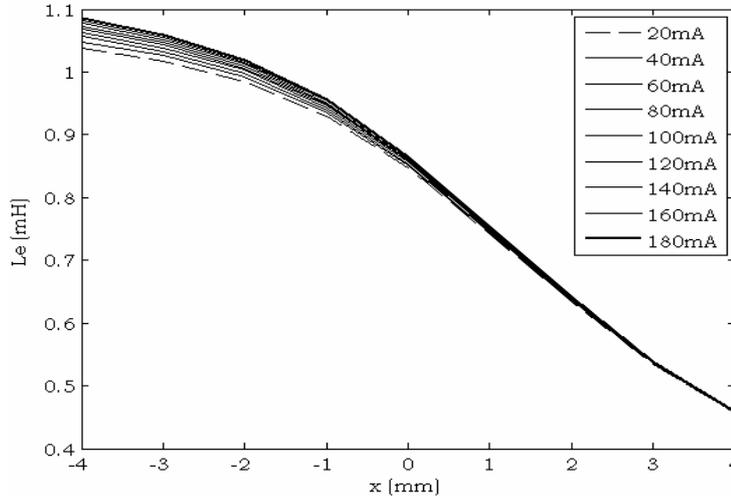


Fig. 5. Measured inductance Le of the loudspeaker vs. position in the airgap x .

In addition, we assume that $R\mu$ depends only on the axial position x , as Le , and show its values in Fig. 6. This is consistent with Fig. 5: a variation in the

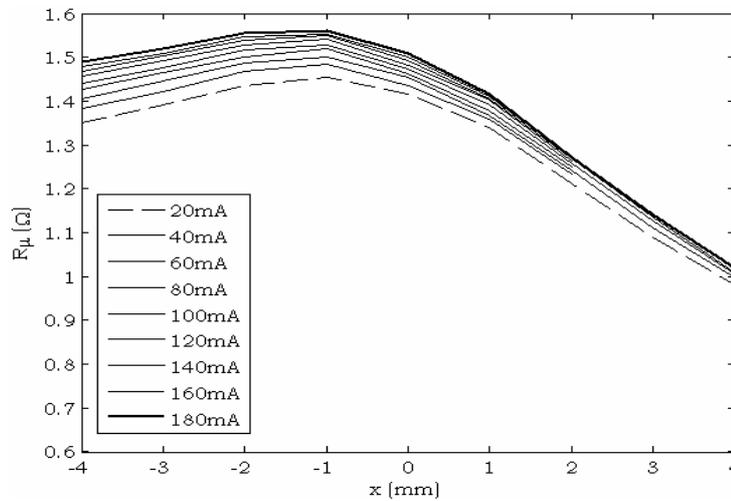


Fig. 6. Measured resistor $R\mu$ of the loudspeaker vs. position in the airgap x .

magnetic flux means an increase in the source of eddy currents, so $R\mu$ is related to the slope of Le . Thus, the variation of $R\mu$ with displacement x , is modelled by (4):

$$R\mu(x) = R\mu \cdot (1 + R\mu_1 \cdot x + R\mu_2 \cdot x^2 + R\mu_3 \cdot x^3 + R\mu_4 \cdot x^4), \quad (4)$$

with $R\mu_1 = -45.15 \text{ m}^{-1}$, $R\mu_2 = -17081.4 \text{ m}^{-2}$, $R\mu_3 = 536430 \text{ m}^{-3}$ and $R\mu_4 = 3.855 \cdot 10^8 \text{ m}^{-4}$.

3. Modelling of the loudspeaker with the motor nonlinearities

3.1. Differential equations

When the inductance Le , the force factor Bl and the resistor $R\mu$ are not assumed to be constant anymore but dependent on the displacement, the model of Fig. 1 does not lead to (1) anymore.

Especially, contrary to $Bl(x)$ and $R\mu(x)$, the nonlinear model of the inductance cannot simply substitute the linear parameter Le in the equation of equilibrium of the voltages in the electrical side of the equivalent circuit. The voltage on the inductance $u_L(t)$ is not expressed by the inductance times the derivative of the current over time anymore, but it is the derivative of the magnetic flux $\Phi(t)$:

$$u_L(t) = \frac{d\Phi(t)}{dt} = \frac{d [Le(x, i_L) \cdot i_L(t)]}{dt}. \quad (5)$$

Considering that $Le(x, i_L)$ varies independently with x and i_L , which are the two time-dependent unknowns of the problem, yields:

$$u(t) = \left(1 + \frac{Re}{R\mu}\right) \cdot \left[Le \frac{di_L(t)}{dt} + i_L(t) \cdot \left(\frac{di_L(t)}{dt} \frac{dLe}{di_L} + \frac{dx(t)}{dt} \frac{dLe}{dx} \right) \right] + Re \cdot i_L(t) + Bl \cdot \frac{dx(t)}{dt}, \quad (6)$$

$$Mms \cdot \frac{d^2x(t)}{dt^2} + \left(Rms + \frac{Bl^2}{Re + R\mu} \right) \cdot \frac{dx(t)}{dt} + Kms \cdot x(t) = \frac{Bl}{Re + R\mu} \cdot (R\mu \cdot i_L(t) + u(t)) + \frac{1}{2} \frac{dLe}{dx} i_L(t)^2.$$

Note that the second differential equation of (6) takes into account the reluctance force created by the dependence of the inductance on the displacement. It is proportional to the square of the current $i_L(t)$ in the voice-coil [14]. By switching the nonlinear parameters Le , Bl and $R\mu$, the resolution of (6) leads to the variables $i_L(t)$ and $x(t)$, used to calculate the acoustic pressure generated by the modelled loudspeaker.

3.2. Harmonic distortions

For the calculations, the loudspeaker input $u(t)$ is chosen to generate the maximal 4 mm displacement to emphasize the related nonlinearities. Figure 7 compares the three pressure responses calculated by including successively each of the three displacement-varying parameters Le , $R\mu$ and Bl previously measured and modelled. The input voltage is a 30 Hz single-tone signal. The variations of Le and $R\mu$ with the displacement do not create consequent distortions. Their corresponding temporal responses are indistinguishable from each other and the amplitudes of the created harmonics are at least 40 dB lower than the amplitude of the fundamental tone.

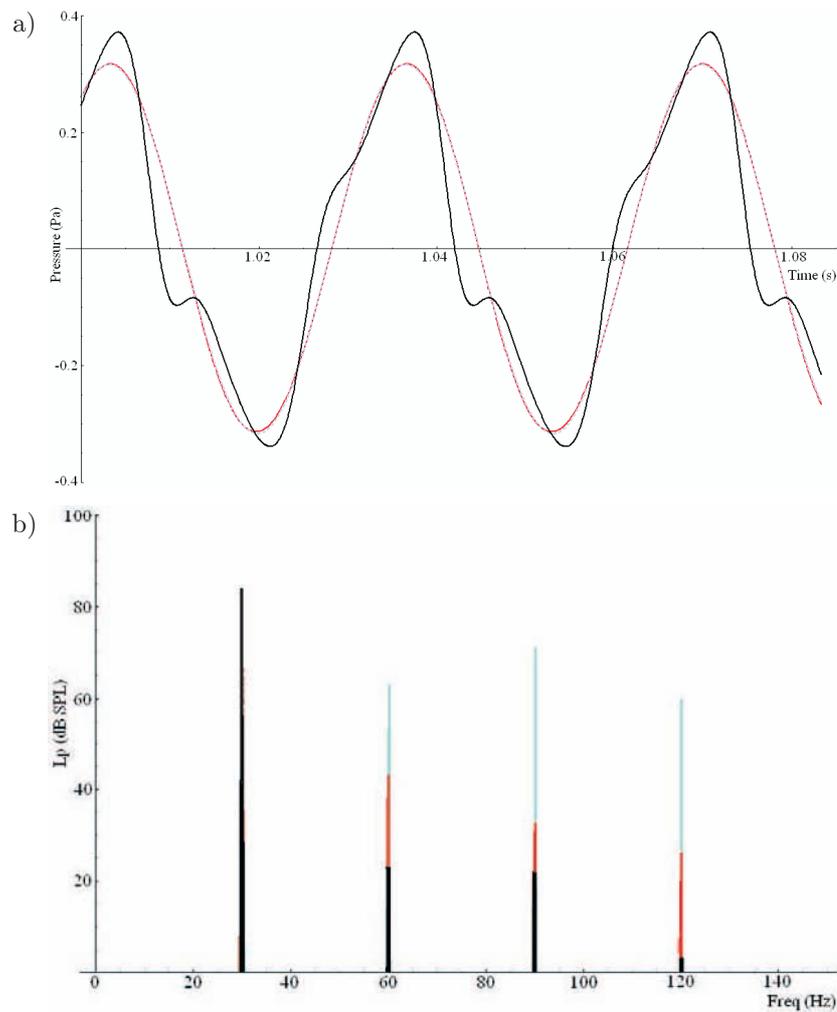


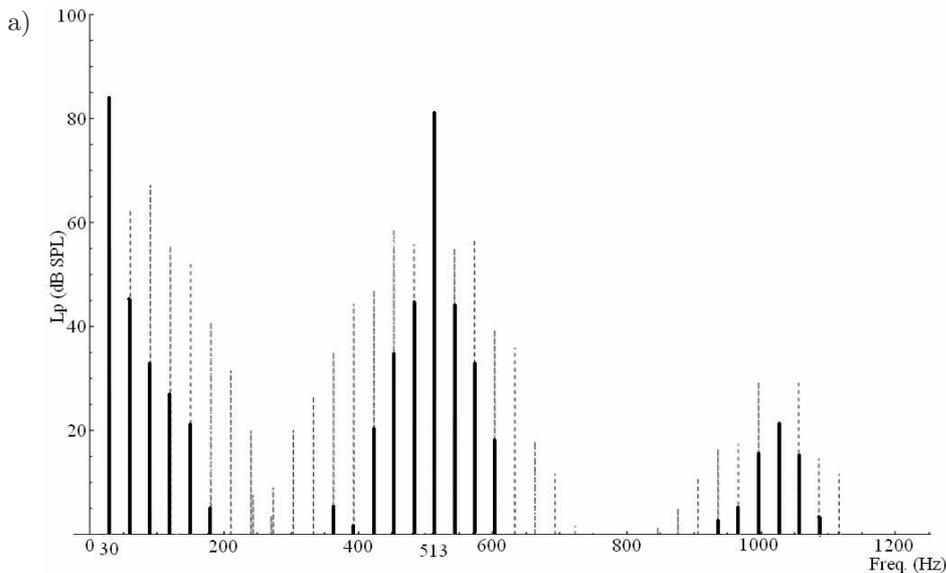
Fig. 7. Temporal pressure responses (a) and sound pressure spectra (b) of the loudspeaker with $Bl(x)$ -nonlinearity (black line), with $Le(x)$ -nonlinearity (red line) and with $R\mu(x)$ -nonlinearity (blue line).

The model used to calculate these contributions shows that dependence of Le and $R\mu$ on the displacement does not induce clear harmonic distortions. On the other hand, Fig. 7 shows that the non-uniform $Bl(x)$ leads to consequent harmonic distortions. The temporal response is highly distorted and high-amplitude harmonics are produced in the frequency response. Especially the third harmonic at 90 Hz is only 12 dB lower than the fundamental one.

3.3. Intermodulation distortions

In this section, the loudspeaker is virtually excited by a two-tone sinusoidal signal with frequencies 30 Hz and 513 Hz. As in the previous section, influences of displacement-varying Le and $R\mu$ are indistinguishable. Figure 8a shows that their dependence creates intermodulation distortion (thick line). But the level of the first intermodulation components is at least 36 dB lower than the fundamental at 513 Hz. These observations are true for various couples of frequencies. Assuming that the model well represents the loudspeaker, we conclude that the dependence of Le and $R\mu$ on the displacement is not critical in the generation of the intermodulation distortions.

Looking at the dotted-line of Fig. 8a, one can see that the dependence of the force factor Bl on the displacement leads to quite consequent intermodulation distortions. The level of the first intermodulation component is 22 dB lower than the fundamental at 513 Hz. This phenomenon is amplified at low frequencies and when the frequencies of the two-tone signal are close to each other (Fig. 8b). Adding the production of harmonic distortion, one can understand that the non-uniform $Bl(x)$ is a great source of intermodulation distortions.



[Fig. 8a.]

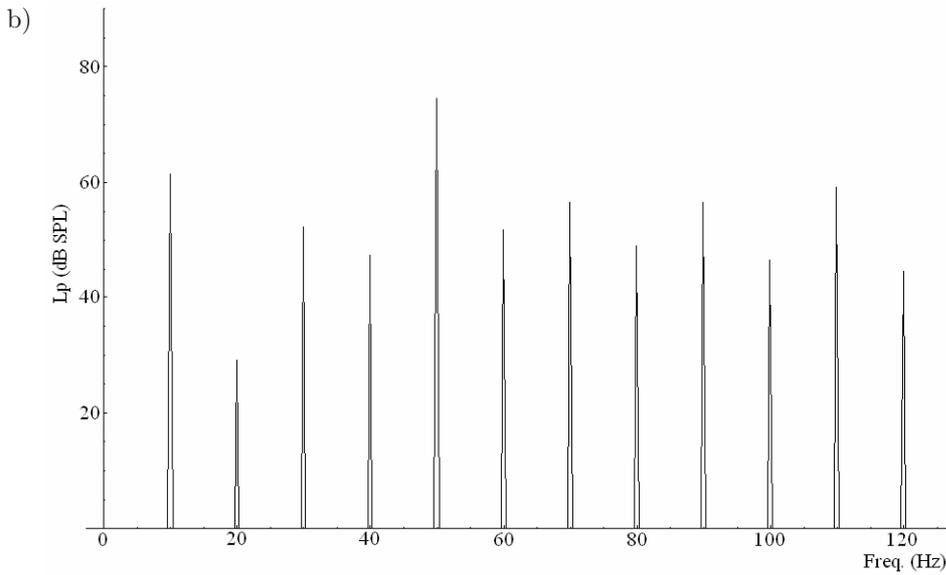


Fig. 8. Loudspeaker sound pressure spectrum in response a) – to a two-tone excitation signal (30 Hz and 513 Hz), with $Bl(x)$ -nonlinearity (dotted line) and with $Le(x)$ -nonlinearity (thick line) and b) – to a two-tone excitation signal (10 Hz and 50 Hz) with $Bl(x)$ -nonlinearity.

4. Discussion and conclusion

This paper highlights the audible distortions created by the motor of an electrodynamic loudspeaker of classical construction. For the studied loudspeaker, the most consequent source of nonlinearities seems to come from the displacement-varying force factor. The weaker, but still significant, effect is caused by the nonlinearity of voice-coil inductance. The nonlinearity of the voice-coil is usually neglected in the consideration of the nonlinear distortion in loudspeakers. The paper shows that this nonlinearity should be taken into account. Particularly, it produces bothersome intermodulations. However, these observations and conclusions are valid for the actual studied loudspeaker. More general conclusions can be obtained from further measurements and modelling of other loudspeakers. These conclusions could help loudspeaker designers in conceiving their loudspeaker motors, with the aim of reducing their nonlinearities, enough to be sure that no sound distortion would be audible. Another way to get rid of the distortions generated by a loudspeaker motor is to use a completely different construction of the motor, which consists in the total removal of the soft-iron magnetic circuit [15–17]. In this construction, eddy currents disappear and the voice-coil inductance is low and constant. A good design of such a motor can also lead to a constant force factor.

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