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Self-Tuning Control with Regularized *RLS* Algorithm for Vibration Cancellation of a Circular Plate

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The self-tuning control assumes that the vibrating system is unknown and the controller procedure has the ability to identify the process and to update the necessary control law. Such an algorithm provides the relevant regulator parameters according to the obtained parametric object model. The algorithm can be described as a combination of the following two procedures: the online identification and the computation of the controller parameters. Nearly all of the identification procedures are related to the *Least Squares* (LS) estimate of a model output. Classified as an ill-posed problem, it implies that the obtained solution is potentially very sensitive to the data perturbations. In order to avoid such problems, the regularized version of the *RLS* method has been considered in this paper. By solving the linear system of equations with a non-singular Sylvester matrix, the formulas for the unknown coefficients of the considered PID-type controller structure have been obtained. The results of the tests and simulations for the circular plate vibration cancellation have been also included.

Keywords: self-tuning control, online identification, *RLS* algorithm, regularization, Sylvester matrix.

1. Introduction

The majority of the complex industrial systems are characterized by the parameters varying with the system operating point change. Conventional controllers, for which the parameters are computed, are efficient only when the system to be controlled is characterized by constant parameters. These coefficients are usually obtained with the well-known methods which have been applied for many years in automatic control [1]. In the situation where the controlled process parameters are unknown or time varying, as the errors between controller and actual process parameters will increase, a not-retuned constant-parameter-controller will lead to progressive degradation of system operation. The common alternative for improving the quality of control for such systems is the use of a self-tuning controller, which has become possible by the development of IT technology and adaptive algorithms [5–7]. Such technique can be considered as a combination of two procedures: (1) – an identification algorithm using measurements to form the system model; (2) – a control law computation for determination of the regulator parameters and the control to be applied to the system [5, 6].

The purpose of the parametric model is to approximate the behavior of the system as closely as possible, which can be achieved by ensuring the closest possible match between the predicted and observed outputs. The *Recursive Least Square* (*RLS*) method has been used for calculating the model parameters of a considered plate. It is well-known that this algorithm can be classified as an ill-posed problem because it involves the inversion of covariance matrix which can be ill-conditioned or even singular. It implies that the achieved solution is potentially very sensitive to perturbations of the measured data. The most common technique to overcome the described issue is to apply the regularization methods [7, 8].

The described control procedure was considered by the authors for a plate vibration cancellation [3, 4], where the proposed self-tuning regulator was designed with the use of results of the online identification and the classic, unregularized *RLS* algorithm, to update the control law. In this paper, the structure of the PID-type algorithm, which is the most frequently used in industry and relatively easy to implement, has been chosen. The relationship between the model parameters and regulator coefficients presented herein, has been obtained by solving the linear system of equations with the non-singular Sylvester matrix. For this discrete ill-posed problem, which is indeed difficult to be treated numerically, the Tikhonov regularization method [8] has been applied.

2. Identification experiment

Identification is a data-driven technique, which enables to construct an experimental model of the considered system based on the registered response of the structure as a result of the input signal chosen. For small displacements, the input/output model can be reasonably approximated by a linear finite-difference model. Techniques to infer a model from the measured signals typically contain three steps:

1. Design and conducting of identification experiment.
   Identification requires data that accurately represents the behavior of the system. The well-designed experiment ensures that the chosen variables are measured with sufficient accuracy and duration to capture the dynamics of the model.
2. Data analysis and preprocessing.
   The measured data should be analyzed and prepared for model calculation. This step includes basic data-cleaning operations, such as: removing outliers, offsets and linear trends, filtering, resampling etc.

3. Estimation and validation of models.
   Usually one can estimate several models and chose the simplest one (of the lowest possible order) that have a desired performance and best describes the dynamics of the system.

The identification experiment is the most complex and crucial step of the whole identification process because, not like the other steps, it requires direct interference to the process. The data acquisition process is usually conducted using PC computers with specially designed input/output interfaces and other measuring track devices like A/D, D/A converters, amplifiers and sensors (see Fig. 1).

Preparing of the experiment involves several choices, where the most important of them are:
(1) to pick out the bands of input/output signals,
(2) to choose the sampling frequency,
(3) to decide on the character of the input signal.

When sampling any continuous time system, one must chose a sampling rate which is neither too fast nor too slow. The minimum value of sampling frequency f_s is described by Shannon's theorem:

$$f_s \geq 2f_{\max}, \tag{1}$$

where $f_{\max}$ is the maximum frequency in output signal spectra. However, experience has shown [3, 4] that a sampling rate between ten to twelve times the highest frequency, results in the best performance. The top band of sampling frequency is also limited because of hardware restriction and numerical errors which can occur when the differences between two samples are too small. After trying several different sampling rates, 10 kHz was found to yield reasonable performance for the experimental setup constructed.

The other important parameter of identification experiment is the input signal. One of the most frequently chosen parameters is the *chirp* signal, which guarantees that all of the frequencies from the considered interval will occur. Assuming that the input and the output are recorded for N samples and denoting input at discrete time k as $u(kT)$, where T is the sampling time, the *chirp* signal can be described by

$$u(kT) = u_0 + A\,sat\left(\frac{k}{0.1N}\right) sat\left(\frac{N-k}{0.1N}\right)\sin(\omega_k kT). \tag{2}$$

In this expression $sat(\ )$ is a kind of weighting function that causes the input to start at zero, ramp up to amplitude A and ramp down to zero at the end. The constant u_0 is added to make the input have zero average value.

3. Experimental set-up

The main aim of the control system designed for the considered circular plate (see Fig. 1.) is to cancel its vibrations. An effective application of adaptive feedback control is possible if PC computer works under supervision of a real-time operating system, like *RTAI*. This extension of *Linux* changes priority of the running process and guarantees deterministic response time of *RTAI modules*. It also significantly increases the rate of executing the tasks and gives opportunity to use more advanced control algorithms which involve online identification and updating of the controller coefficients.

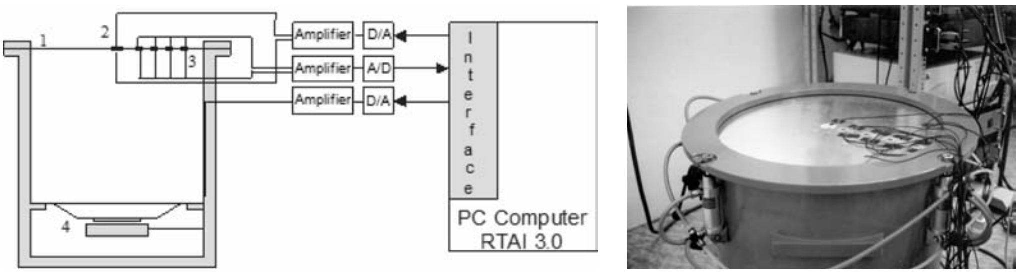


Fig. 1. Research position: 1 – circular plate, 2 – PZT elements, 3 – strain sensors, 4 – loudspeaker.

The data used in estimating system model parameters were obtained from several pairs of strain sensors. For vibration suppressing, the 2-layer piezo-disk elements, working in a pair, centrally mounted on the plate surface, have been chosen. To investigate the performance of the system, a *chirp* signal (10–1000 Hz) generated by computer software has been applied. With a sample rate of 10 kHz, the input and output data were gathered for system identification. It is desired to find the transfer function from the actuator to the error sensor.

4. Online identification

Assuming that the sampled signal values can be related through the linear difference equation given by Eq. (3):

$$\begin{aligned} y(k) + a_1 y(k-1) + ... + a_{nA} y(k-nA) \\ = b_1 u(k-d) + ... + b_{nB} u(k-d-nB+1) + e(k), \end{aligned} \tag{3}$$

where $y(k)$, $u(k)$ represent respectively the output and input at discrete time, $k = 1, 2, 3, \ldots$, nA – number of poles, nB – number of zeros plus 1, d is the number of samples before the input affects the system output, and $e(k)$ denotes white noise. The relationship shown above, known as autoregressive model with exogenous input (ARX), can be rewritten using the delay operator z^{-1} as Eq. (4):

$$y(k) = \frac{B(z^{-1})}{A(z^{-1})} u(k-d) + \frac{1}{A(z^{-1})} e(k), \tag{4}$$

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + ... + a_{nA} z^{-nA}, \tag{5}$$

$$B(z^{-1}) = b_1 + b_2 z^{-1} ... + b_{nB} z^{-nB+1}. \tag{6}$$

The model can be also expressed in terms of the parameters vector $\boldsymbol{\theta}$ and the registered vector $\boldsymbol{\varphi}$

$$y(k) = \boldsymbol{\theta}^{\mathrm{T}} \boldsymbol{\varphi}(k) + e(k), \qquad \forall k, \tag{7}$$

where

$$\boldsymbol{\varphi}(k) = [-y(k-1), ..., -y(k-nA),\, u(k-d), ..., u(k-d-nB+1)]^{\mathrm{T}}, \tag{8}$$

$$\boldsymbol{\theta} = [a_1, ..., a_{nA},\, b_1, ..., b_{nB}]^{\mathrm{T}}. \tag{9}$$

Vector $\boldsymbol{\theta}$ can be obtained by minimizing the classical *Least Squares* criterion expressed as:

$$V_N(\boldsymbol{\theta}) = \frac{1}{N} \sum_{k=1}^{N} \left[y(k) - \boldsymbol{\theta}^{\mathrm{T}} \varphi(k) \right]^2. \tag{10}$$

After derivation of Eq. (10) one can get the formula for calculation of vector $\boldsymbol{\theta}$:

$$\hat{\boldsymbol{\theta}} = \left[\sum_{k=1}^{N} \varphi(k) \varphi^T(k) \right]^{-1} \sum_{k=1}^{N} \varphi(k) y(k). \tag{11}$$

The Eq. (11) can be rewritten in a recursive fashion, as it is shown below:

$$\hat{\boldsymbol{\theta}}(k) = \hat{\boldsymbol{\theta}}(k-1) + \mathbf{L}(k) \varepsilon(k), \tag{12}$$

$$\mathbf{L}(k) = \frac{\mathbf{P}(k-1) \boldsymbol{\varphi}(k)}{1 + \boldsymbol{\varphi}^{\mathrm{T}}(k) \mathbf{P}(k-1) \boldsymbol{\varphi}(k)}, \tag{13}$$

$$\varepsilon(k) = y(k) - \hat{\boldsymbol{\theta}}^{\mathrm{T}}(k-1) \boldsymbol{\varphi}(k), \tag{14}$$

$$\mathbf{P}(k) = \mathbf{P}(k-1) - \frac{\mathbf{P}(k-1) \boldsymbol{\varphi}(k) \boldsymbol{\varphi}^{\mathrm{T}}(k) \mathbf{P}(k-1)}{1 + \boldsymbol{\varphi}^{\mathrm{T}}(k) \mathbf{P}(k-1) \boldsymbol{\varphi}(k)}, \tag{15}$$

where $\varepsilon(k)$ represents the prediction error at time k, and $\mathbf{P}$ – the covariance matrix. These formulas are known as the *Recursive Least Squares* (*RLS*) algorithm.

The problem of solving the Eq. (11), which involves inversion of covariance matrix, can be unstable numerically if this matrix is ill-conditioned. Classified as an ill-posed problem, it implies that the obtained solution is potentially very sensitive to perturbations of the data. The most commonly used technique to avoid that issue was proposed by Tikhonov [8] and is referred to as *regularization*.

To make provision for stability of the matrix P, it should satisfy the condition:

$$\lambda_{\min}\mathbf{I} \leq \mathbf{P}(k) \leq \lambda_{\max}\mathbf{I}, \tag{16}$$

where $\mathbf{I}$ is the identity matrix and λ denotes the parameter which controls the properties of the regularized solution. The authors have considered modification of classical *RLS* method proposed by Praly [7], which satisfies the above condition and involves calculation of the $\mathbf{P}$ matrix in two steps:

$$\mathbf{\Lambda}(k) = \mathbf{P}(k-1) - \frac{\mathbf{P}(k-1)\boldsymbol{\varphi}(k)\boldsymbol{\varphi}^{\mathrm{T}}(k)\mathbf{P}(k-1)}{1+\boldsymbol{\varphi}^{\mathrm{T}}(k)\mathbf{P}(k-1)\boldsymbol{\varphi}(k)}, \tag{17}$$

$$\mathbf{P}(k) = q\mathbf{\Lambda}(k) + r\mathbf{I}, \tag{18}$$

$$q = \frac{\gamma_{\max} - \gamma_{\min}}{\gamma_{\max}}, \tag{19}$$

$$r = \gamma_{\min}, \tag{20}$$

where $\gamma_{\min}$, $\gamma_{\max}$ denotes the lower and upper values of the regularization parameter, which is responsible for the sensitivity of the regularized solution to perturbations. The influence of described modification of classical *RLS* algorithm has been examined by comparing the results of online identification using the *RLS* method with and without regularization algorithms. In both cases, the *chirp* signal with frequency band 200–400 Hz and sample frequency set to 10 kHz was used. Figures below show the results of conducted tests for chosen order of *ARX* model, which was set to 2.

A first observation is that the regularized *RLS* algorithm (*RRLS*) shows a slightly, better convergence than the unregularized one for higher frequencies. It can be caused by the fact that the standard *RLS* algorithm loses ability to update model parameters if k is big enough (higher frequencies in the experiment), thus the obtained model is almost constant and coincidence between the real system output and model output might be worse. However, in the range of 10–200 Hz, the average divergences of the curves are slightly bigger. Despite this, the figures demonstrate that parametric model obtained using the considered *RRLS* algorithm reaches high accuracy and convergence with the measured output of the system, even if it is only the second-order model. The coincidence between the compared signals is very good in the whole range of considered frequencies (see Figs. b). Additionally, regularization, which is a standard technique for improving the condition of the covariance matrix $\mathbf{P}$ in the *RLS* estimate ($\mathbf{P}(k) \to 0$ if k is big enough), guarantees that such an ill-posed problem will turn into a well-posed one, while the additional computational cost is low.

In the next figures one can see how the parameters of the model were changing during the process. The values of the regularization parameters which were set as follows: $\gamma_{\min} = 0.01$, $\gamma_{\max} = 1000$, cause small modification of the $\mathbf{P}$ matrix (see Eqs. (17)–(20)) but this technique significantly increases stability of the *RLS* algorithm.

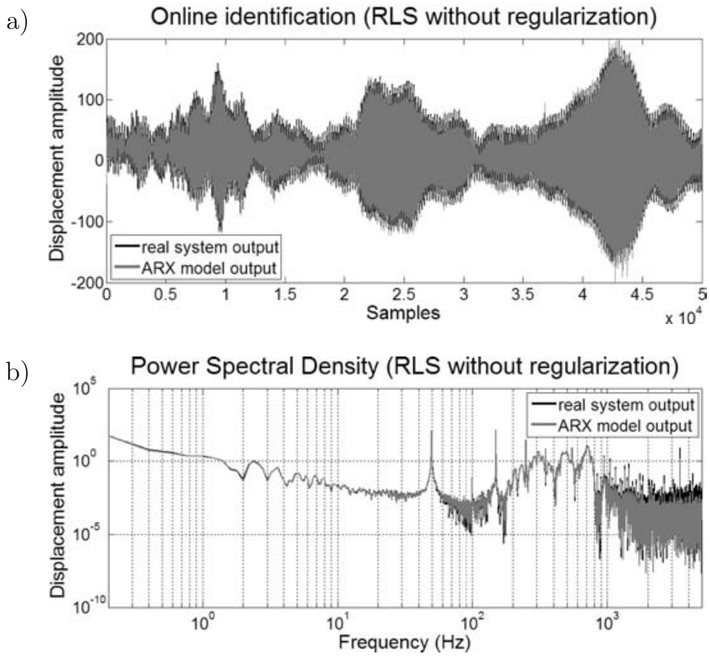


Fig. 2. Online identification using *RLS* without regularization algorithm: a) time responses; b) frequency responses.

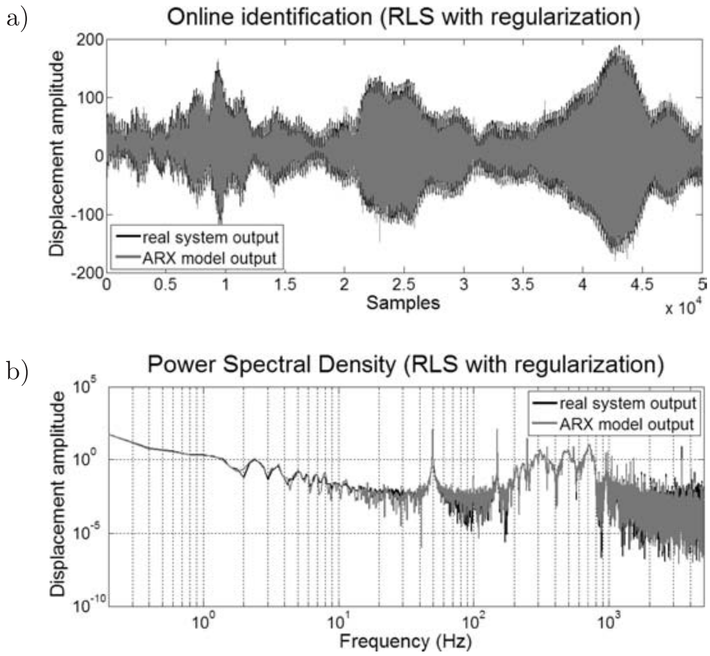


Fig. 3. Online identification using *RLS* with regularization algorithm ($\gamma_{\min} = 0.01$ and $\gamma_{\max} = 1000$): a) time responses; b) frequency responses.

Figures b show that both identification algorithms, the regularized and unregularized *RLS*, are able to tune the model parameters during the process of updating the plate model. It is easy to notice that in the case of unregularized *RLS* algorithm, the identified model parameters tend to reach almost stable values. It implies that parameter vector $\hat{\boldsymbol{\theta}}(i) \rightarrow \overline{\boldsymbol{\theta}} \cong \text{const}$, which might be not correct (an ill-posed problem), especially for online identification using a low-order model. Since the regularization of the matrix $\mathbf{P}$, based on a scaled identity matrix, provided a significant increase in algorithm stability, the obtained regularized solution make provision for better agreement between the real and estimated outputs.

a)

b)

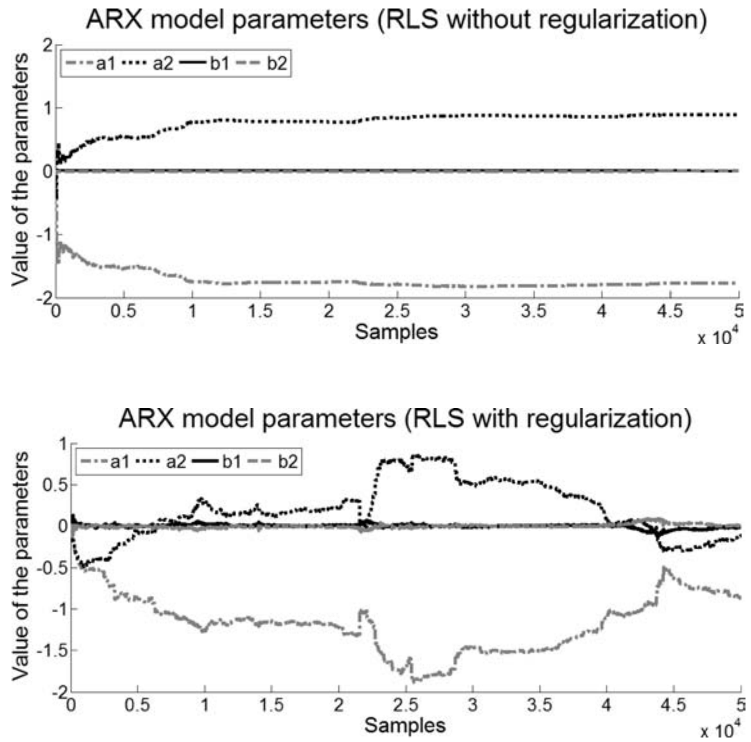


Fig. 4. *ARX* model parameters: a) *RLS* without regularization; b) *RLS* with regularization.

5. Self-tuning controller

Adaptive regulator uses identified model parameters to update its coefficients. To reduce the complexity of algorithm, the second-order *ARX* model of process has been chosen. It implies that control law can also be described as a second-order transfer function:

$$G_R = \frac{Q(z^{-1})}{P(z^{-1})} = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1 + p_1 z^{-1}}. \tag{21}$$

The regulator considered here is known as a PID-type controller and this is the most common type of controller used in industry because it provides good control results and it is also relatively easy to implement. The unknown parameters can be achieved using *pole placement* procedure. The controller based on assignment of poles in a closed-loop system presented in Fig. 5 is designed to stabilize the value of closed-loop poles (they should have previously determined values). Considering the stability requirement, good poles configuration can make it relatively easy to obtain a desired closed-loop system response (e.g., the maximum overshoot, damping, etc.).

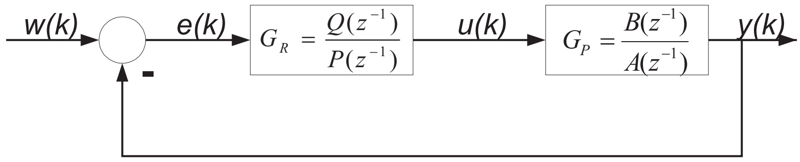


Fig. 5. Closed-loop system.

The closed-loop transfer function of the considered system, where G_P is the identified ARX model of the plant, can be expressed as Eq. (22):

$$G_W = \frac{Y(z^{-1})}{W(z^{-1})} = \frac{B(z^{-1})Q(z^{-1})}{A(z^{-1})P(z^{-1}) + B(z^{-1})Q(z^{-1})}. \tag{22}$$

The denominator of G_W should have chosen poles. It means that the denominator can be compared to the chosen polynomial D:

$$A(z^{-1})P(z^{-1}) + B(z^{-1})Q(z^{-1}) = D(z^{-1}), \tag{23}$$

$$D(z^{-1}) = \sum_{i=0}^{nd} d_i z^{-i} = d_0 + \sum_{i=1}^{nd} d_i z^{-i}. \tag{24}$$

Reorganizing Eq. (23) one can get the linear system of equation with non-singular Sylvester matrix:

$$\begin{bmatrix} 1 & b_1 & 0 & 0 \\ a_1 & b_2 & b_1 & 0 \\ a_2 & 0 & b_2 & b_1 \\ 0 & 0 & 0 & b_2 \end{bmatrix} \begin{bmatrix} p_1 \\ q_0 \\ q_1 \\ q2 \end{bmatrix} = \begin{bmatrix} d_1 - a_1 \\ d_2 - a_2 \\ d_3 \\ d_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \tag{25}$$

where numbers $d_0, \ldots, d_4$ are coefficients of chosen characteristic polynomial, as proposed in [2]:

$$D(z) = (z - \alpha)^2 [z - (\alpha + j\omega)][z - (\alpha - j\omega)]. \tag{26}$$

The chosen characteristic polynomial has a pair of complex conjugated poles $z_1 = \alpha + j\omega$ and $z_1 = \alpha - j\omega$ and double real pole $z_{3,4} = \alpha$, where α and ω have to satisfy the stability condition: $\alpha^2 + \omega^2 < 1$. Equations below show the relationship between the parameters of model and regulators obtained by solving Eq. (25):

$$p_1 = \frac{b_1^2(b_1x_4 - b_2x_3) + b_2^2(b_1x_2 - b_2x_1)}{b_2(a_1b_1b_2 - a_2b_1^2 - b_2^2)}, \tag{27}$$

$$q_0 = \frac{b_2^2(a_1x_1 - x_2) + b_1b_2(x_3 - a_2x_1) - b_1^2x_4}{b_2(a_1b_1b_2 - a_2b_1^2 - b_2^2)}, \tag{28}$$

$$q_1 = \frac{b_2^2(a_2x_1 - x_3) + b_1b_2(a_1x_3 - a_2x_2 + x_4) - a_1b_1^2x_4}{b_2(a_1b_1b_2 - a_2b_1^2 - b_2^2)}, \tag{29}$$

$$q_2 = \frac{x_4}{b_2}. \tag{30}$$

In the simulations, the proposed algorithm is applied in a closed-loop scenario for performing feedback vibration cancellation. The example results of performed simulations using MATLAB software are presented in figures below. In the first case, the excitation signal is sinusoidal with frequency of 210 Hz and the other is the *chirp* signal with frequency band 200–400 Hz.

In the case of sinusoidal disturbance, it can be seen (Fig. 6), that the plate displacement can be well suppressed when the controller is started.

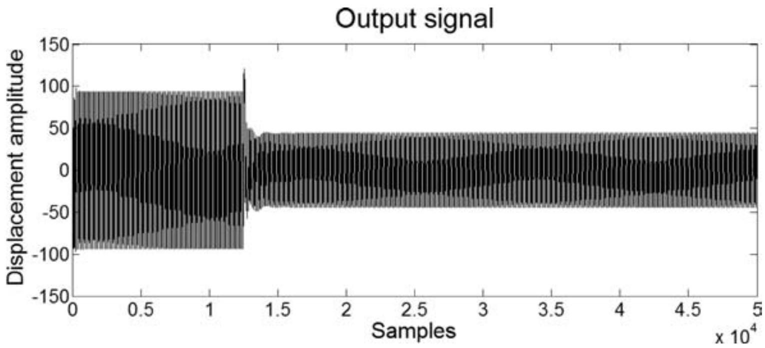
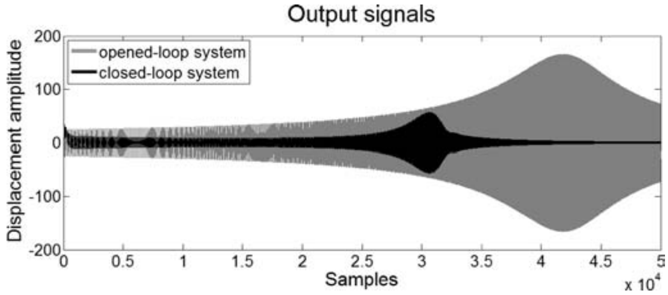


Fig. 6. Responses of open-loop system (0–1.25·10^4 samples) and closed-loop system (1.25–5·10^4 samples) for sinusoidal excitation of 210 Hz.

Figures show responses of open- and closed-loop system for sinusoidal and *chirp* signal excitations and for the following values of parameters: $\alpha = 0.992$, $\omega = 0.11$. The main observation is that, using of adaptation methods leads to substantial reduction of the plate vibrations for the periodic and non-periodic disturbance signals in the concerned frequency band and the considered control algorithm applied does not require high performance IT solutions.

a)



b)

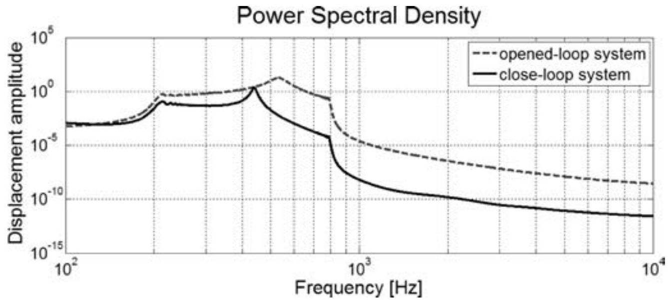


Fig. 7. Responses of open-loop system and closed-loop system for *chirp* signal (200–400 Hz); a) time responses; b) frequency responses.

6. Conclusions

The advances in microprocessor speed have made it possible to compute the control signal and apply it to the actuator within one sampling period, even if the control strategy is more time-consuming than typical various forms of PID controllers. However, in the case of adaptive methods applied to vibration cancellation of the structure, the techniques for the automatic, on-line adjustment of regulators designed to maintain a given level of system performance can't be widely implemented as yet.

The main aim of the paper was to design a self-tuning controller for circular plate vibration suppressing. Such regulator updates the control law during the process using an identified model. The goal for this study is to demonstrate an approach by which some of the benefits associated with the application of adaptive methods can be quantified. The authors chose second order of the *ARX* model and simple PID-type controller to reduce complexity of the design control algorithm. The high accuracy of such a solution was obtained by using high sampling rate (10 kHz) – more than 10 times higher than the considered frequency band of output signal. It is possible only by using real-time operating system like *Linux* and *RTAI* with the designed optimal code for control application. The conducted examinations show that modification of classical *RLS* algorithm used increases the accuracy of the model and reduces the influence of the measured data perturbation.

The considered adaptive PID-type controller causes substantial reduction of the plate vibrations for the input signals considered. Simulations show that there is no need for using such complicated control law if working terms of the considered plate are constant. In such situation, a more suitable solution is to use a static regulator whose parameters can be obtained basing on the model of the process identified in the off-line mode. The main advantage of the solutions proposed, is the fact that the adaptive controller has the ability to update the control law during the process, which makes it more suitable in case of any changes in terms of the working plate (e.g. temperature, boundary condition, surroundings etc.). From the simulation results it is clear that it may considerably increase the control performance. In addition, using a method for constructing a regularization matrix, a significant increase in robustness can be achieved.

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