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UNCERTAINTY IN VIBRATION ENERGY FLOW ANALYSIS

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Introducing uncertainty considerations into simulation will facilitate exactitude assessment and help to boost the designer's confidence in simulation. The sources of uncertainty marked as the sensitivity parameters have an impact on the vibrational energy flow simulation and must be factored into the solution process. One of these parameters, the excitation frequency, has been identified and analyzed. The techniques for quantifying the effects of uncertainties are presented. The transmission of vibrational energy flow in plates is analyzed by the structural intensity method. A typical case for vehicle structures has been selected as an example: a simply supported plate under tangential force excitation. The relations between the structural intensity distribution and structural mode shapes and the effects of excitation frequency on the changes of energy flow in plate are discussed. Finally, the potential application of the structural intensity technique towards for the design of vehicle structures is discussed.

Key words: vibration energy, structural intensity, uncertainty of energy distribution.

1. Introduction

Vehicles are always subjected to external dynamic loading with various excitation frequencies, from slowly varying wave loads to relatively high frequency engines and interaction of road and suspension induced forces. Generally speaking, when the frequencies of the external forces are close to one of the natural frequencies of the structural components, the permissible vibration levels may be exceeded, which may result in fatigue failure of the structure, destruction of electronic and mechanical equipment or very high noise level. Since plates are most commonly used built-up structural elements in vehicles, the damage of the plates will result in the collapse of overall system structures. An increasingly popular approach to undertake a vehicle ultimate strength analysis is to consider the failure of the individual plates and combine these to determine the failure load of the entire element cross-section.

Structural intensity is the power flow due to structural vibration per unit cross-sectional area in elastic medium and it is analogous to acoustic intensity in a fluid medium. The interest in the investigation of structural intensity arises from practical reasons, because structural intensity field indicates the magnitude and direction of vibrational energy flow at any point of a structure, and energy flow distribution offers information on energy transmission paths and positions of sources and sinks of mechanical energy. Dissipative elements, mechanical modification and active vibration control can be used for an alteration of the energy flow paths within the structure and for an alternation of the amount of mechanical energy injected into the structure. Of primarily practical concerns are complex built-up structures, which can be successfully treated only by measurements or by numerical computation when a prediction of structural behavior in various operating conditions is needed. For these reasons, the investigation of the energy flow paths in stiffened plates is very important to the response and damage detection for overall vehicle structures.

The present paper is concerned with energy transmission in plate-like structures, which undergo different frequency point force excitation. Using the structural intensity techniques, a detailed description of the transmission pattern of vibrational energy flow from the source of excitation to the sink through structures as simply supported plates can be obtained. The computational method for structural intensity is illustrated and formulas of structural intensity for beams and plates and their relation of force and strain are given in previous works [1, 3, 5, 7]. The presented in these works formulas were used in the post-processing algorithm for structural intensity calculations. The calculations were done with use of the FEM for obtaining harmonic response solution. A number of numerical examples which are linked to vehicle structures are presented. The relation between structural intensity and structural mode shapes as well as the changes of energy flow in a plate for the excitation frequency change are discussed. Finally, the potential application of the structural intensity technique in thin-walled structures engineering is presented.

2. Methods of uncertainty assessment

The most widely used traditional methods for assessing uncertainty are adopted from sensitivity analysis [11, 12]. The sensitivity analysis is used to assess the relation between variations in input parameters to variations in output (predicted) parameters. The parameters which have the greatest influence are termed as sensitive parameters in the model. For an uncertainty analysis a distinction must be introduced between a sensitive parameter and an important parameter. It may happen that a sensitive parameter is known to within a close tolerance, in which case uncertainty in this parameter will not lead to significant uncertainty in the predictions. It is therefore necessary to consider the realistic levels of uncertainty in the input parameters in detail.

The sensitivity analysis techniques can be used to address the following issues [12]:

- Model realism – how well and to what resolution the model represents reality,
- Input parameters – what values should be used in the absence of assumed data,
- Stochastic processes – to what extent the assumptions regarding operational factors affect the predictions,

- Simulation program capabilities – what uncertainties are associated with a particular choice of algorithms for the various vibration energy transfer processes,
- Design variations – what will be the effect of changing one aspect of the design.

These issues are related to the categories of uncertainty sources. The use of an assessment technique can range from a simple analysis, e.g. variation of a few parameters which are deemed to be important for the predictions through to a comprehensive analysis. Due to the difficulties of implementation and managing the analysis of the results, comprehensive analyses have remained in the research domain. Furthermore, as no integrated techniques are available, sensitivity studies are generally used in an ad hoc manner, with specific scripts being written to perform individual studies. There is a need for better simulation support to allow the user to assess uncertainty and to present predictions and their associated uncertainties as a matter of routine. To achieve better simulation support several issues have to be addressed. The sources of uncertainty affecting a model have to be identified and quantified. Then suitable techniques have to be identified before the structure assessment. These techniques can be implemented within a simulation package.

Statistical uncertainty analysis techniques can be categorized as structured and non-structured methods. Structured methods are adopted from experimental techniques, where by a series of experiments would be designed to analyze the response of pre-determined models. Non-structured methods are stochastic in nature. In the former category is Differential Sensitivity Analysis (DSA), which is the most popular method for application in building thermal simulation. In the latter category Monte Carlo Analysis (MCA) has been used.

Deterministic solution techniques are possible to carry the uncertainty information through the calculation procedure. These techniques rely on altering the underlying arithmetical functions as all operations are carried out on general numbers.

3. Structural intensity in thin-walled elements

For a steady state of vibration the surface structural intensity can be evaluated as the complex quantity [2]:

$$\widetilde{S}_{\sigma kl\, vl}(\omega) = I_k(\omega) + jJ_k(\omega), \tag{1}$$

where $\omega = 2\pi f$ – angular frequency, f – frequency of vibrations, $\widetilde{S}_{\sigma kl\, v_l}(\omega)$ is the cross spectrum function of complex components of stress and particle velocity.

In practical cases only the real part of structural intensity which is responsible for the energy transfer is analyzed. The imaginary part is connected with the standing waves and represents the energy conservation in the system. Instantaneous value of the real part of structural intensity $i_k(t)$ is time dependent vector quantity equal to the change of energy density in the infinitively small volume [6]. Its *k*-th component is given by the equation:

$$i_k(t) = -\sigma_{kl}(t)v_l(t), \qquad l = 1, 2, 3, \tag{2}$$

where $v_l(t)$ is the *l*-th component of velocity vector and $\sigma_{kl}(t)$ is the *kl*-th component of stress tensor.

Averaged in time value of real part of structural intensity (2) represents the net energy flow in mechanical structure [6]

$$I_k = \langle i_k(t) \rangle \tag{3}$$

in the direction of *k*-th coordinate of rectangular frame of reference corresponding to the analyzed constructional element.

In the numerical calculations by means of the finite element method the structural intensity is related to the neutral undeformed middle plane of the plate. There are assumed small deformations which enable the superposition of independent displacements for flat finite element of the plate or shell type. The two components of structural intensity are derived [9]:

$$\begin{aligned} I_x &= -\frac{\omega}{2}\mathrm{Im}\left[\widetilde{N}_x\, \widetilde{u}_0^* + \widetilde{N}_{xy}\, \widetilde{v}_0^* + \widetilde{Q}_x\, \widetilde{w}_0^* + \widetilde{M}_x \theta_y^* - \widetilde{M}_{xy} \theta_x^* \right], \\ I_y &= -\frac{\omega}{2}\mathrm{Im}\left[\widetilde{N}_y\, \widetilde{v}_0^* + \widetilde{N}_{yx}\, \widetilde{u}_0^* + \widetilde{Q}_y\, \widetilde{w}_0^* - \widetilde{M}_y \theta_x^* + \widetilde{M}_{yx} \theta_y^* \right], \end{aligned} \tag{4}$$

where $\widetilde{N}_x,\ \widetilde{N}_y$ – tension forces in plate, $\widetilde{N}_{xy} = \widetilde{N}_{yx}$ – internal forces in plate, $\widetilde{Q}_x$,$\widetilde{Q}_y$ – shear forces, $\widetilde{M}_x$, $\widetilde{M}_y$, – bending moments $\widetilde{M}_{xy} = \widetilde{M}_{yx}$ – twisting moments.

The procedure of structural intensity calculation is based on a complex response of the structure with the modal representation of the structure without the dissipation [8]. The damping is considered in two forms. The structural internal damping of the structure is taken into consideration traditionally as modal damping. The known damping, placed in a known location is treated as external loading.

4. Analysed model of plate

For the purpose of sensitivity analysis of structural intensity with excitation frequency as the parameter a numerical experiment was performed. The model chosen for the calculations was a homogeneous rectangular flat plate, simply supported at the edges. The plate had mechanical properties of construction steel and dimensions of 1.5 m in width and 2.5 m in length with thickness of 10^{-2} m. The FEM model was arranged using the NASTRAN software. The plate was divided into 3840 elements. The model consisted of the same square shell element of the QUAD4 type. The excitation force and damping force were introduced to the model at the special point locations. The harmonic excitation force was attached to the plate in the place indicated on figures by a star. The amplitude of the excitation was set to 10^3 N. The frequency of excitation was changed in the range from 25 to 1000 Hz. The damping force proportional to the velocity of vibration was attached to the plate in the place indicated on figures by a triangle. The magnitude of the damping force was set to 10^3 N. The direction of its action was chosen perpendicularly to the plane of the plate. Locations of damping and excitation forces were the same for all models. The only feasible motion of the plate was the rotation around the edges of the plate. Such model was in accordance with the most technical cases of plate-like element mounting in practice.

5. Results of calculations

The main target of the analysis of the rectangular plate was testing the distribution of structural intensity vectors for different frequency of force excitation. In the analysis an elaborated program specially developed for the calculations of complex modal model was applied. The program allowed the consideration of additionally localised damping in the system.

The results obtained from the calculations are presented in a graphical form on Fig. 1 to 8. The structural intensity vectors distribution over the plate is shown. Distributions of intensity vectors in places near to the places of excitation and damping are similar for all cases of load frequency. The positions of excitation and damping forces attachments are clearly shown. The distribution of vectors has distinctly shown the direction of energy flow from the excitation to the damper. However, in all cases the distribution of intensity vectors in the middle part of the plates has shown different shapes. During the excitation frequency change at a uniform rate from 25 Hz to 100 Hz with the increment of 1 Hz a uniform change of structural intensity vectors distribution was observed in most cases.

Table 1. Frequencies of the first 10 mode shapes, Hz.

Mode no	1	2	3	4	5	6	7	8	9	10
Frequency, Hz	14.82	26.58	46.19	47.55	59.23	73.65	78.72	102.08	106.04	108.94

For some frequencies there were observed abrupt changes of vectors distribution and, what is more significant, of the maximum value of vectors magnitude. The change in value of vectors magnitude was of 5 orders varying from 0.3 till 8900 W/m^2.

The vortices of vectors field in the plates region were observed far from excitation and damping. This effect means the circulation and conservation of vibration energy in the mechanical system take place.

Analyzing the results of calculations obtained for the plates presented here it can be noticed that the distribution of structural intensity vectors is not similar to the distribution of displacements for each mode shape. This conclusion results from the fact that the mode shapes are connected to the standing waves formed in the plate. In the system with small internal damping the energy flow is not observed or at least it is very low. The effect of energy flow between the structure elements is observed only for systems with high internal damping caused e.g. by local abrupt changes of properties or a localized damping force.

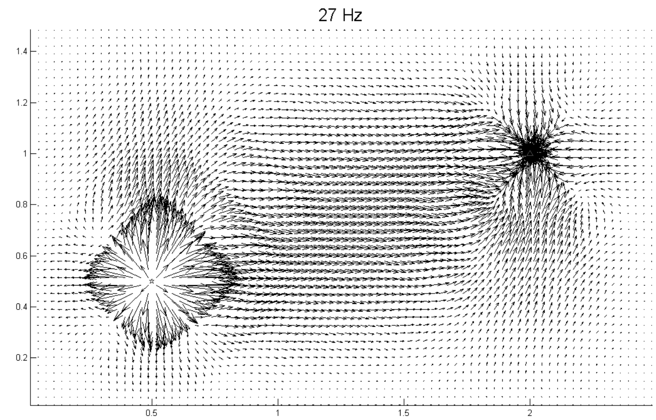

Fig. 1. Structural intensity distribution on plate. Excitation frequency 27 Hz.

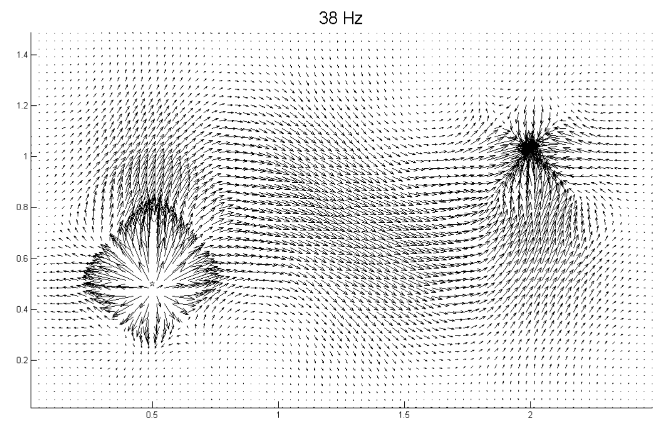

Fig. 2. Structural intensity distribution on plate. Excitation frequency 38 Hz.

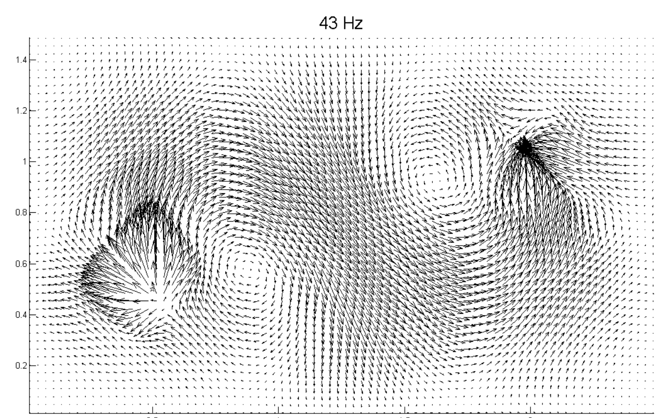

Fig. 3. Structural intensity distribution on plate. Excitation frequency 43 Hz.

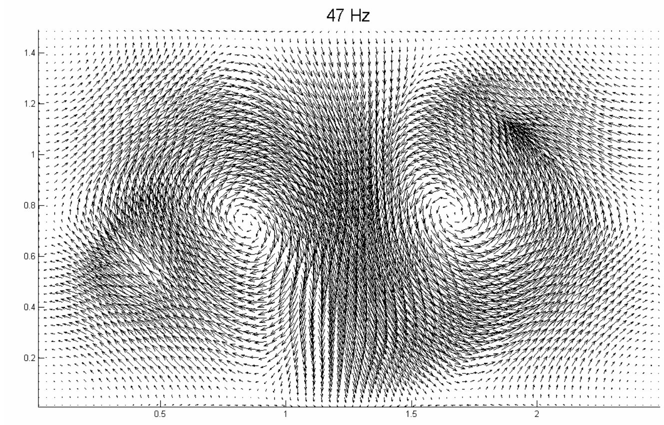

Fig. 4. Structural intensity distribution on plate. Excitation frequency 47 Hz.

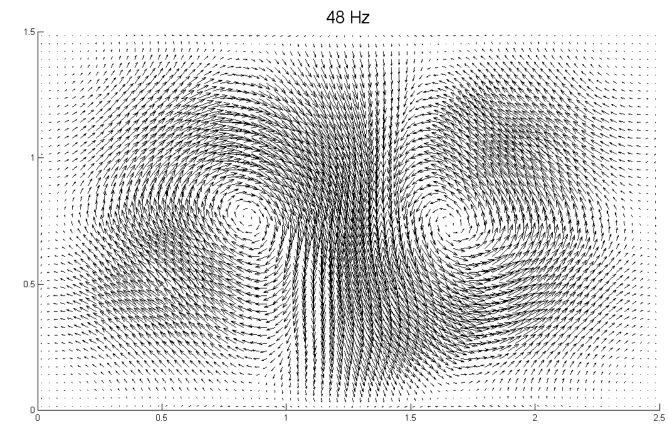

Fig. 5. Structural intensity distribution on plate. Excitation frequency 48 Hz.

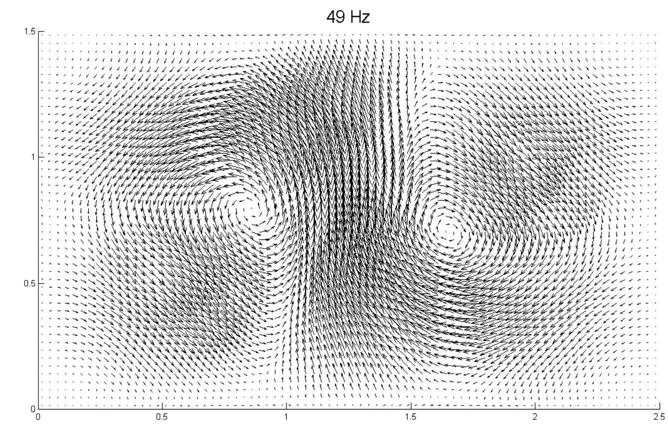

Fig. 6. Structural intensity distribution on plate. Excitation frequency 49 Hz.

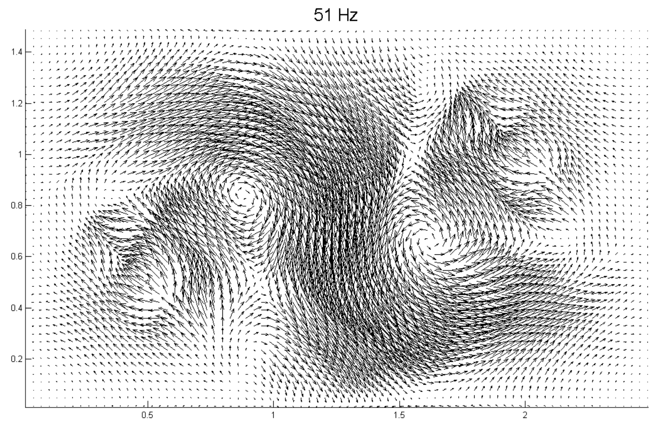

Fig. 7. Structural intensity distribution on plate. Excitation frequency 51 Hz.

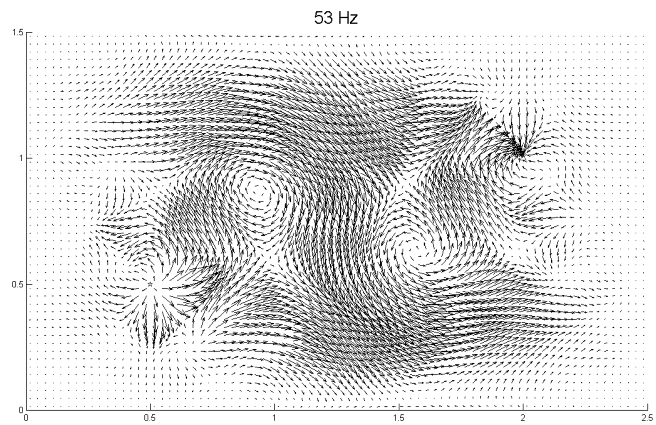

Fig. 8. Structural intensity distribution on plate. Excitation frequency 53 Hz.

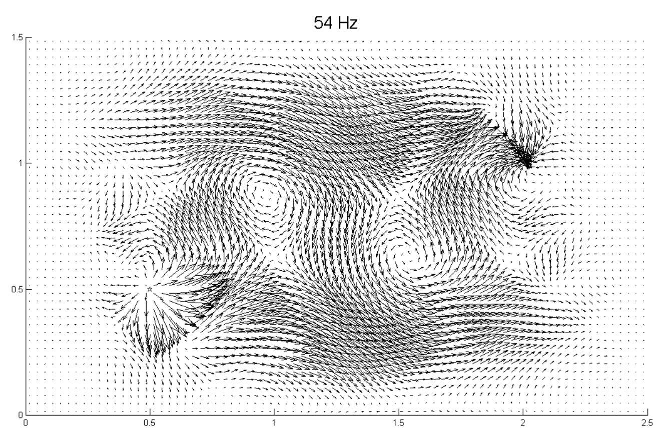

Fig. 9. Structural intensity distribution on plate. Excitation frequency 54 Hz.

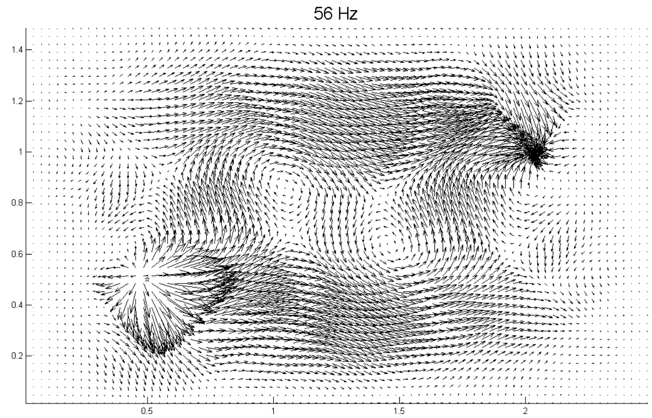

Fig. 10. Structural intensity distribution on plate. Excitation frequency 56 Hz.

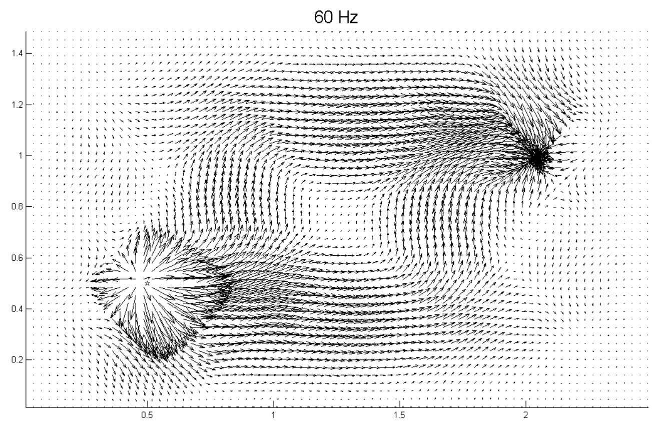

Fig. 11. Structural intensity distribution on plate. Excitation frequency 60 Hz.

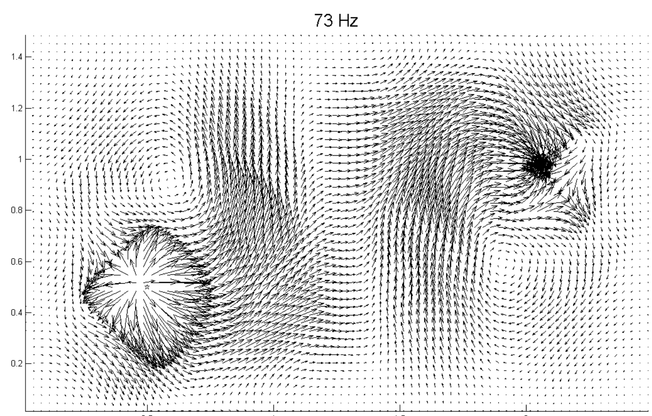

Fig. 12. Structural intensity distribution on plate. Excitation frequency 73 Hz.

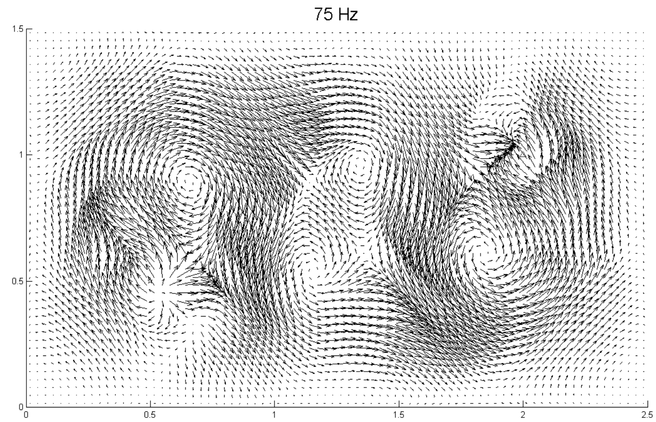

Fig. 13. Structural intensity distribution on plate. Excitation frequency 75 Hz.

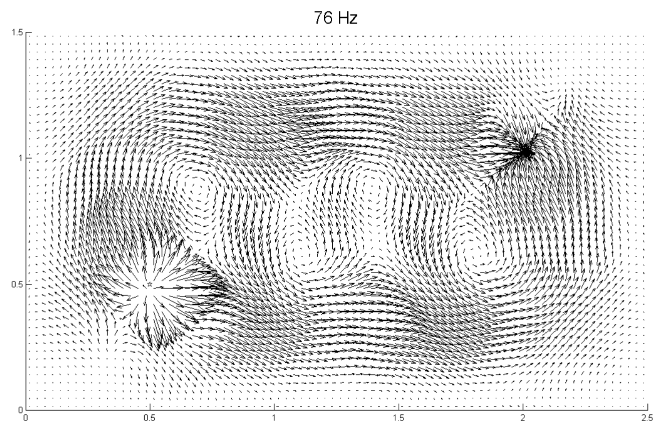

Fig. 14. Structural intensity distribution on plate. Excitation frequency 76 Hz.

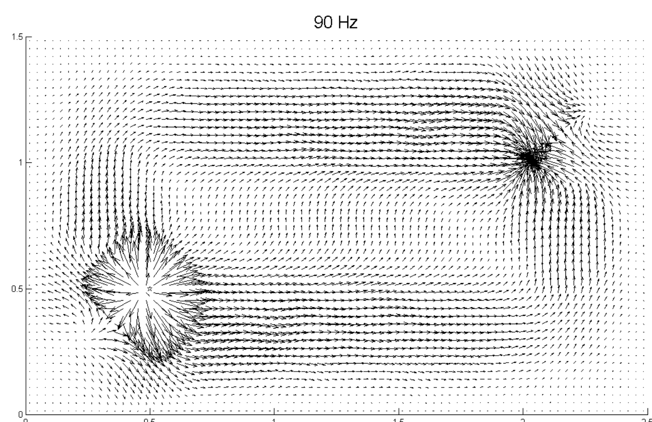

Fig. 15. Structural intensity distribution on plate. Excitation frequency 90 Hz.

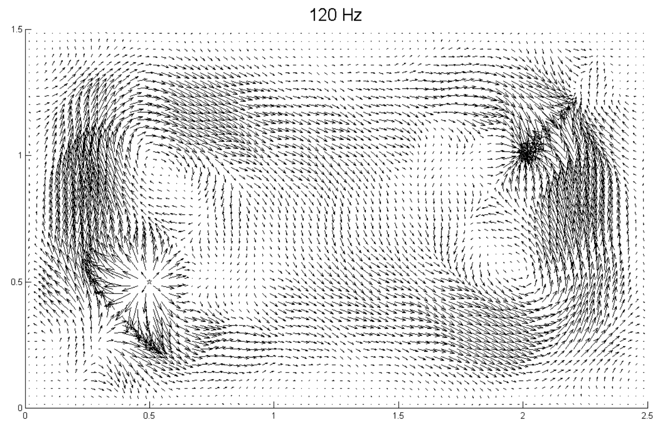

Fig. 16. Structural intensity distribution on plate. Excitation frequency 120 Hz.

6. Conclusions

The structural intensity pattern, which presents a vectorial nature of vibrational energy flow in structures, can be used to determine how energy is injected by mechanical excitations and to identify power transfer paths. The presented method of structural intensity vectors calculation enables the evaluation of structural intensity vectors for chosen frequency range and mode shapes [6]. The calculations are done with the application of complex modal parameters calculated by the use of the numerical modal analysis based on the finite element method.

From the results of the calculations we can find that despite the change of the excitation force frequency acting on the plate, the structural intensity fields can clearly indicate the source, the sink and the transmission of energy flow from the source of excitation to the sink through the plate. The patterns of structural intensity in the plate depend on many factors, such as loading characteristics, mode shapes of coupled structures, number and geometry shape of stiffeners attached to the plate, and many others. All of them have the influence upon the filed distribution and magnitude of structural intensity vectors. This also means that the nature of structural intensity is frequency dependent. The numerical experiment presented here has proved that the structural intensity is very sensitive in the range close to the mode frequency in which abrupt changes of vector field distribution can be observed though this distribution is not similar in shape to the suitable mode shape. When the excitation frequency is close to the structural natural frequencies, the magnitude (or pattern) of the structural intensity mainly depends on the relation between the structural mode shape and external loading characteristics. The calculation results show that, despite the different excitation force frequency acting on the plate, the structural intensity fields can still clearly indicate the source, the sink and the direction of energy flow from the source to the sink. Also, it has been observed that the change of the excitation force frequency changes the amount of energy flow in the plate.

The nature of the structural intensity is frequency dependent: the energy flow in plates will depend on the mode shapes that are dominant for a given frequency range. It has been shown that the structural intensity analysis can act as a new (maybe more reliable) criterion for a vehicle structural design. It can also be an effective tool of vibration control for vehicle structures provided the power flow pattern and energy density in plates can be controlled by, for example, proper arranging of stiffeners.

The main disadvantage of the applied method of structural intensity calculation is its poor convergence. In the case of the analyzed plates the number of mode shapes taken into consideration should be greater than 60. The obtained results have shown good convergence. The distribution of structural intensity vectors shows the qualitative characteristic of vibration energy flow in mechanical systems. The introduction of an additional measure in the form of an integral of magnitude structural intensity vector component perpendicular to a certain closed surface enables the quantitative assessment of energy transfer paths and energy balance in the structure.

The method of analysis of structural intensity distribution enables the investigation in the regions of high concentration of vibration energy flow which consequently is exposed to the risk of damage or is propagating the sound waves to the environment. It can be also considered as the identification of the regions for application of additional damping in purpose of lowering of vibration level and resulting noise radiation.

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