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THE INFLUENCE OF CONSTRUCTIONAL PARAMETERS ON STIFFENED PLATES SOUND RADIATION

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This work concerns the method of the plate parameter selection and the choice of the stiffening set, assuming the minimization or maximization of the energy stored or radiated by a stiffened plate as a criterion. The energy balance equations based on the SEA method were used in order to quickly evaluate the energy flow as a function of chosen constructional parameters. Calculations of mechano-acoustic energy flow were performed in AutoSEA and Matlab environment.

Key words: stiffened plate, energy balance, SEA method, sound power.

1. Introduction

Among a large number of machines and industrial devices, plate elements are dominating sources of sound radiation and vibrations. One of the methods of modifying the acoustical energy radiated by a vibrating plate is a change in its structure. Such a modification can be carried out by varying the rigidity of the structure – for instance by introducing additional rigidity generated by a set of stiffeners. The appropriate adjustment of rigidity in the plate-set of stiffeners system may result in reduction of the acoustic energy radiated in a certain frequency band.

For the purposes of analysing the acoustical field around a flexuraly vibrating plate, complex systems of differential equations are usually used. In order to obtain the solution of such a system of equations by the use of numerical methods, complicated and time-consuming computations have to be carried out [6, 7, 10, 11]. The higher is the considered frequency, the more complicated and time-consuming are the computations. Very often the decrease in the accuracy of the obtained numerical results is also noticed. On the contrary, the SEA method is an effective tool for prediction of energy of vibrations and acoustical power radiated in the middle and high-frequency ranges, where the modal densities of modelled subsystems are relatively high [8, 10, 11]. Modal density, damping loss factor and the coupling loss factor are the main parameters that are required for modelling the SEA subsystem. Conservation of energy rule is then applied to each subsystem resulting in formulation of a system of power balance equations [7]. Therefore the predictive SEA method is the process of computing system response due to a known power input on the basis of the power balance equations [8]. Even complex mechanical systems consisting of many various subsystems can be easily described by energy equations.

In the SEA method, complex systems are modelled by dividing them into simple subsystems, such as a rods, beams, plates, acoustical cavities etc. under the assumption that these subsystems are weakly coupled [1, 7, 11] and their reciprocal interactions as well as vibroacoustical energy flow are well described by the Coupling Loss Factors (CLFs).

The aim of the following paper is to present the method for the matching plate parameters with the stiffeners set when the minimization or maximization of the energy stored or radiated by a stiffened plate is considered as a criterion of optimization.

The SEA method supply mathematical approach that enables to quickly evaluate the energy flow as a function of many constructional parameters. The obtained data basis is searched for the optimal set of parameters. Calculations of the acoustical energy flow were carried out in the AutoSEA and Maltab environment.

2. The model of a stiffened plate in the SEA system

Assembly of the stiffeners of the plate results in a change in the plate local stiffness which, in consequence, complicates the analysis and implies additional assumptions in order to facilitate the problem solution. Introduction of an equivalent flexural rigidity is one of the methods of simplifying the differential equations describing vibrations of a stiffened plate. Equation describing the flexural plate vibrations can be written in the form [9]:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + \rho h \frac{\partial^2 w}{\partial t^2} = q(x,y,t), \tag{1}$$

where D_x – average plate flexural rigidity in the x direction, D_y – average plate flexural rigidity in the y direction.

Equation (1) can be written in the modified form [9]:

$$\zeta^4 \frac{\partial^4 w}{\partial x^4} + 2\mu\zeta^2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} - \rho \frac{h}{D_y} \frac{\partial^2 w}{\partial t^2} = \frac{q}{D_y}\,, \tag{2}$$

where

$$\zeta^4 = \frac{D_x}{D_y}\,, \qquad \mu = \frac{D}{\sqrt{D_x D_y}}.$$

Natural frequencies of a stiffened plate with simply supported edges, obtained as a solution of the Eq. (2), are given by [9]:

$$\omega_{m,n} = \frac{\pi^2}{b^2}\sqrt{\frac{D_y}{\rho h}}\sqrt{m^4 + 2\mu\zeta^2\left(\frac{mnb}{a}\right)^2 + \zeta^4\left(\frac{nb}{a}\right)^2}. \tag{3}$$

It is convenient to show the influence of the stiffeners distribution on the natural frequencies by considering the relation between the frequency and mode shape index in the m-n system of coordinates. In such a case the frequency zone is divided into 4 regions.

The first region corresponds to low frequencies. The influence of stiffeners on the plate rigidity in the first region of natural frequencies can be assessed under the assumption that additional stiffness introduced by the set of stiffeners is uniformly distributed on the plate surface. In this region, a stiffened plate can be treated as a homogeneous plate of equivalent mass and stiffness, taking into account influence of the stiffeners.

In the second region the effect of homogenous stiffening concerns only the stiffeners spacing in the x direction. Half-wave length in this direction is smaller than the distance between stiffeners. In the second region, only the mode shapes the half-wave-length of which is an integer multiple of the subpanel length in the x direction, are observed.

Similarly, in the third region the influence of the stiffeners in the y direction is considered. Half-wavelength in this direction is smaller than the distance between stiffeners. Only the mode shapes, the half-wave-length of which is an integer multiple of the subpanel length measured in y direction, are observed.

The fourth region is the region of the highest frequencies. Half-wave in both directions is shorter than the distance between stiffeners. For instance, in Fig. 1 there are presented the frequency zones of the plate divided by stiffeners into 15 subpanels

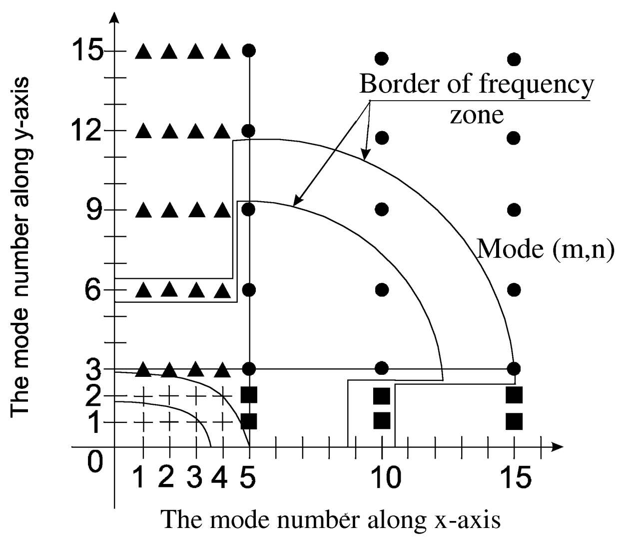

Fig. 1. Numbers of mode shapes excited in the individual frequency zones.

(2 stiffeners along the y-axis divide the plate into 3 panels and 4 stiffeners along the x-axis divide each panel into 5 subpanels).

It is easy to assign individual mode shapes to frequency regions in the domain of natural numbers corresponding to numbers of mode shapes (m, n). But the natural frequencies ordered increasingly belong to different frequency regions, which complicated analytical formulation of the optimization problem.

3. Vibroacoustical energy balance

The model of the investigated system (Fig. 2) is built on the basis of the SEA method and consists of two subsystems: the stiffened plate and a three-dimensional acoustical cavity. The plate is driven to vibrations by the concentrated force. This force is the only source of the input power supplied to the system.

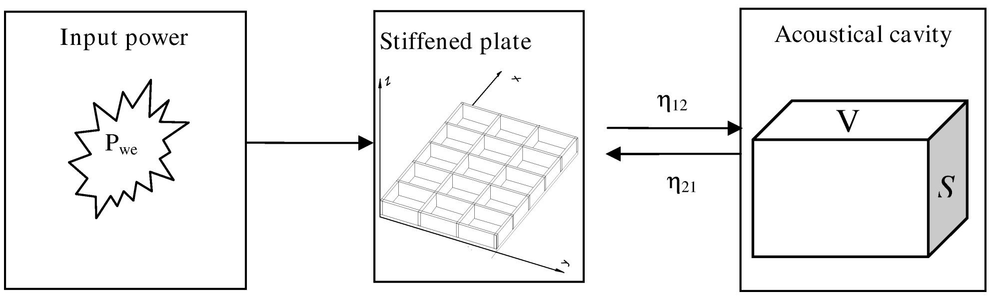

Fig. 2. Model of a stiffened plate-acoustical cavity system.

The energy flow is described by the equation which is classical for the SEA method [8]:

$$\begin{bmatrix} \eta_{1\,\mathrm{tot}} & -\eta_{12} \\ -\eta_{21} & \eta_{2\mathrm{tot}} \end{bmatrix} \begin{vmatrix} E_p \\ E_a \end{vmatrix} = \begin{vmatrix} W_{\mathrm{in}}/\omega \\ 0 \end{vmatrix}. \tag{4}$$

Time-averaged input power W_{in} is given by the following formula:

$$W_{\mathrm{in}} = \frac{1}{2} F_0^2(\omega) \mathrm{Re}\left[\frac{1}{Z_f(\omega)}\right]. \tag{5}$$

In practice, it is difficult to evaluate impedance Z_f for a plate of finite dimensions. Function Z_f is a rapidly varying function of frequency and its practical evaluations is a very difficult task. Cremer has shown that impedance averaged over the frequency bands can be approximated by the corresponding function for an infinite plate [1, 8, 11]:

$$Z_f = \frac{\pi\kappa}{A m_z}, \tag{6}$$

where κ – modal density and m_z – equivalent mass, calculated for each frequency zone.

Under such an assumption, the input power supplied to the plate is given by

$$\Pi_{\mathrm{in}} = \frac{\pi}{2Am_z}\kappa \cdot F_0^2. \tag{7}$$

Making use of the vibroacoustical energy balance (4) in the considered system, energy of the flexural vibrations of a stiffened plate is expressed by the following equation:

$$E_p = \frac{\Pi_{\mathrm{in}}}{\omega}\,\frac{(\eta_2+\eta_{12})}{(\eta_1+\eta_{12})(\eta_2+\eta_{21})-\eta_{21}\eta_{12}}\,. \tag{8}$$

In the same way, the acoustical energy stored in the three-dimensional acoustical cavity can be determined as follows:

$$E_a = \frac{\Pi_{\mathrm{in}}}{\omega}\,\frac{\eta_{21}}{(\eta_1+\eta_{12})(\eta_2+\eta_{21})-\eta_{21}\eta_{12}}\,. \tag{9}$$

Coupling loss factor η_{12} between the stiffened plates and 3D acoustic cavity is given by

$$\eta_{12} = \frac{\rho_a c_a}{\omega_i m_e}\sigma_{\mathrm{rad}}. \tag{10}$$

Radiation efficiency σ_{rad} was investigated by many authors, but for our purposes it can be expressed by the simplified formula:

$$\sigma_{\mathrm{rad}} = \frac{2A}{\pi}\cdot\frac{\omega_i^2}{c_a^2}J, \tag{11}$$

where J – connection acceptance, c_a – sound velocity in the plate material.

Energy transfer from the acoustical cavity to the stiffened plate can be estimated by introducing into Eq. (4) the modal density-dependent reciprocity equation:

$$N_1\eta_{21} = N_2\eta_{12}\,. \tag{12}$$

The physical quantities and coefficients mentioned above are calculated on the basis of dynamical parameters determined separately for each frequency zone. Natural frequencies of free vibrations ordered increasingly (from the lowest to the highest values) with respect to the magnitude belong to different frequency zones, which results in the difficulties with analytical formulation of the modal density, rigidity, equivalent mass etc. It is difficult to provide a simple form of analytical relations describing the influence of constructional parameters on the energy stored and radiated by the stiffened plate in a simple form. For the purposes of practical applications it may be helpful to perform numerical calculations for the interesting range of parameters, and to make the optimum choice of the plate and stiffeners set structure on the basis of the received results stored in the database.

4. An example of analysis of the stiffened plate sound radiation

An example of analysis was carried out for a model of a stiffened plate simply supported at four edges, located in a 3D acoustical cavity. The local change in homogenous plate stiffness was obtained by assembling various sets of steel stiffeners of rectangular cross-sections. In the considered examples, the symmetrical structure of stiffening set was assumed – in both directions from 1 to 8 stiffeners were equally spaced. The stiffeners width varied from 0 to $6 \cdot 10^{-3}$ [m] with $2 \cdot 10^{-3}$ [m] step while the stiffeners height was increased from 0 to $25 \cdot 10^{-3}$ [m] with $5 \cdot 10^{-3}$ [m] step. It resulted in 214 ways of changing the plate local stiffness by introducing different sets of stiffeners. The com-

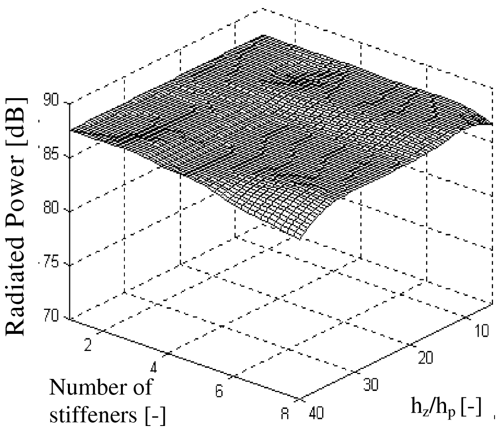

Fig. 3. The level of sound power radiated by the stiffened plate in the 125–250 [Hz] frequency band (stiffeners of 2 [mm] thickness).

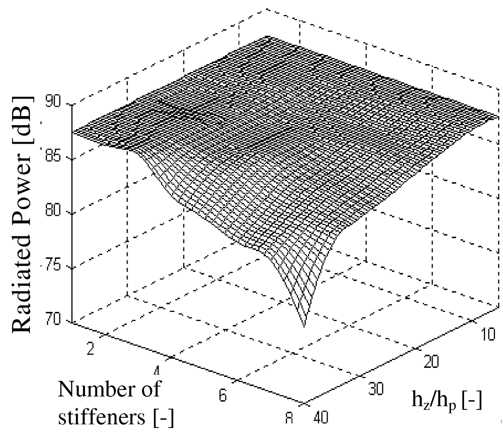

Fig. 4. The level of sound power radiated by the stiffened plate in the 125–250 [Hz] frequency band (stiffeners of 4 [mm] thickness).

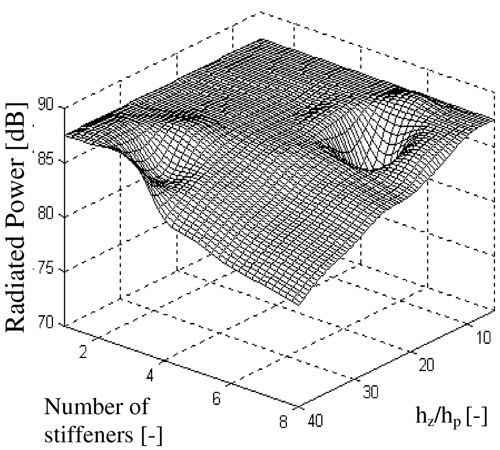

Fig. 5. The level of sound power radiated by the stiffened plate in the 125–250 [Hz] frequency band (stiffeners of 6 [mm] thickness).

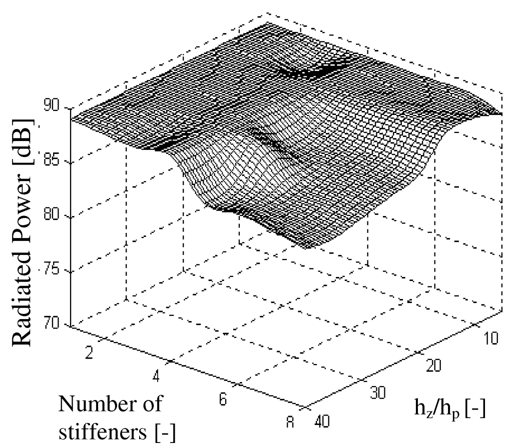

Fig. 6. The level of sound power radiated by the stiffened plate in the 250–500 [Hz] frequency band (stiffeners of 2 [mm] thickness).

puter simulations were carried out in octave bands of the frequency from 125 [Hz] to 3000 [Hz]. The results were linearly averaged and analyzed in the wider frequency bands: 125 Hz–250 [Hz], 250 [Hz]–500 Hz, 500 [Hz]–1000 Hz, 1000 [Hz]–2000 [Hz], 2000 [Hz]–3000 [Hz]. These bands were chosen subjectively after preliminary analysis of the results obtained for 1/3 octave bands. Extending the band range over 1/3 octave band makes it possible to reduce the number of required diagrams and to reduce the range of detailed analysis down to the wider frequency bands in which a significant influence of the stiffening effect was observed.

The computations aimed at assessing the influence of parameters of the set of stiffeners on plate dynamic parameters and energy flow in the system consisting of a stiffened plate and a three-dimensional acoustical cavity. In the figures presented below there are presented differences in the levels of acoustical power radiated by a stiffened

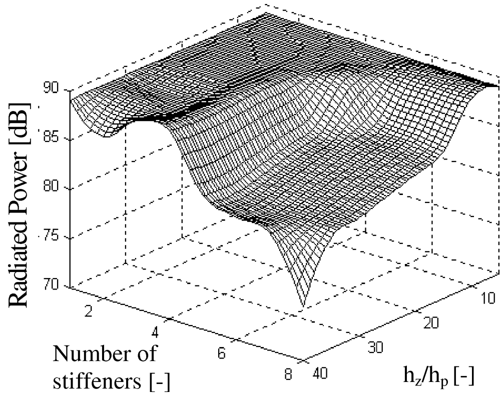

Fig. 7. The level of sound power radiated by the stiffened plate in the 250–500 [Hz] frequency band (stiffeners of 4 [mm] thickness).

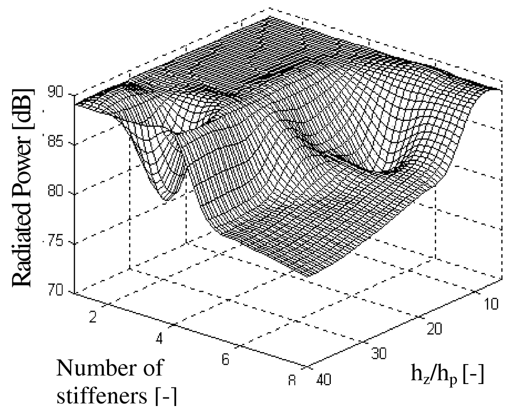

Fig. 8. The level of sound power radiated by the stiffened plate in the 250–500 [Hz] frequency band (stiffeners of 6 [mm] thickness).

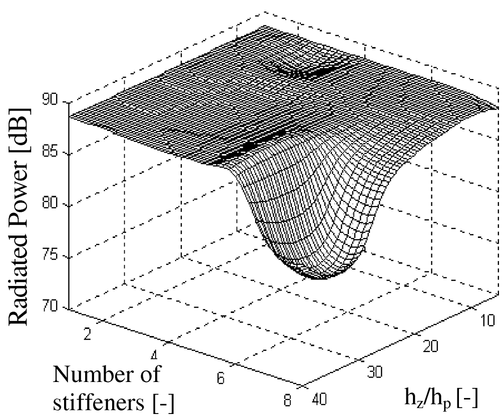

Fig. 9. The level of sound power radiated by the stiffened plate in the 500–1000 [Hz] frequency band (stiffeners of 2 [mm] thickness).

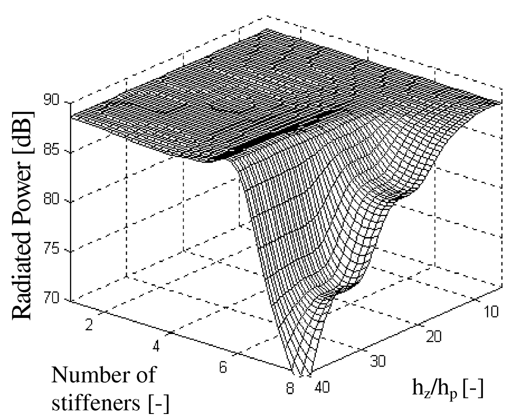

Fig. 10. The level of sound power radiated by the stiffened plate in the 500–1000 [Hz] frequency band (stiffeners of 4 [mm] thickness).

and homogenous plate as a function of the number of stiffeners, thickness and relative thickness h_z/h_p, (where h_z – stiffener height, h_p – plate thickness). In the Figs. 3–14 are presented some results of the radiated acoustical power obtained for a group of 214 plates.

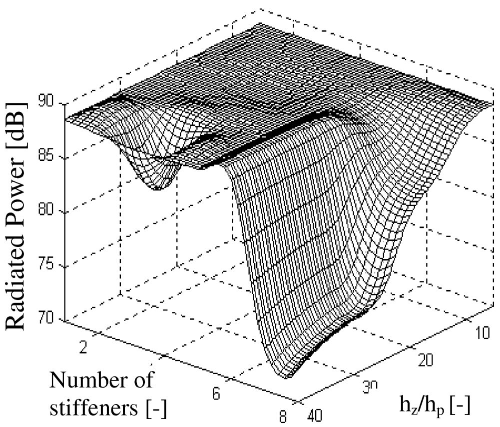

Fig. 11. The level of sound power radiated by the stiffened plate in the 500–1000 [Hz] frequency band (stiffeners of 6 [mm] thickness).

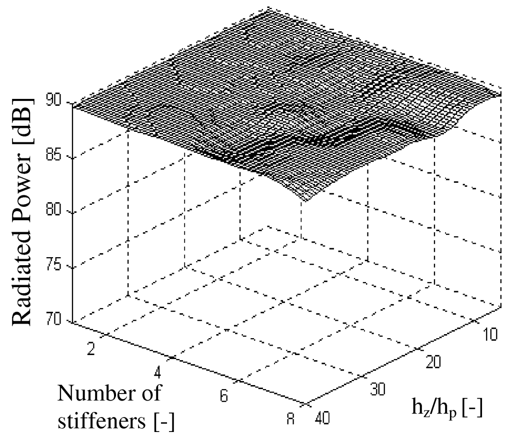

Fig. 12. The level of sound power radiated by the stiffened plate in the 1000–2000 [Hz] frequency band (stiffeners of 2 [mm] thickness).

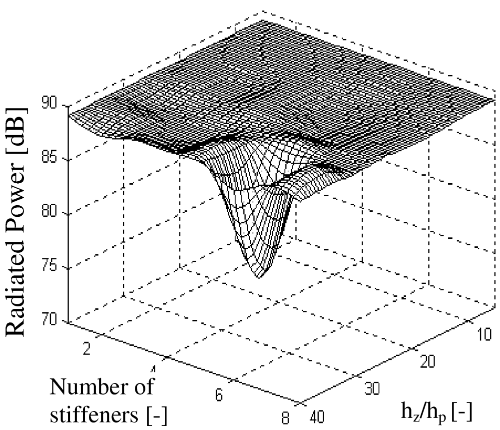

Fig. 13. The level of sound power radiated by the stiffened plate in the 1000–2000 [Hz] frequency band (stiffeners of 4 [mm] thickness).

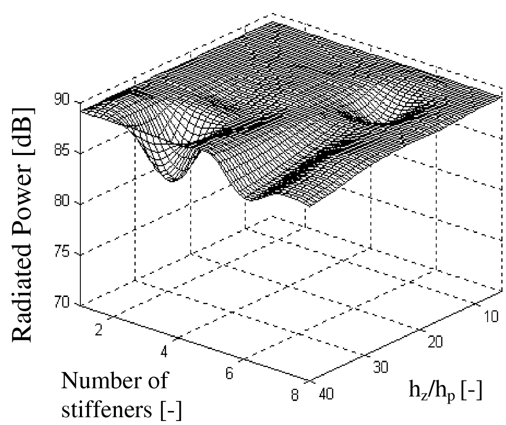

Fig. 14. The level of sound power radiated by the stiffened plate in the 1000–2000 [Hz] frequency band (stiffeners of 6 [mm] thickness).

5. Discussion of the results

In the Figs. 3–14 there are presented the results of prediction of acoustical power radiation in the wide frequency bands. In the first frequency band (from 125 [Hz] to

250 [Hz]), the most significant reduction of acoustical emission was observed for stiffeners of 4 [mm] and 6 [mm] thickness. In order to obtain the most noticeable reduction of sound power it is advantageous to use the set of stiffeners consisting of more than 4 stiffeners of 30–40 [mm] height.

In the 250–500 [Hz] frequency band, the increase in the thickness of stiffeners results in the decrease in the sound power. In the considered range of stiffeners parameters the most significant reduction of the sound power was observed for all the sets of stiffeners consisting of more than 4–5 stiffeners of height over 15 [mm]. For the stiffeners of 2 [mm] thickness the maximal reduction in the sound power amounted to about 5 dB, while for the stiffeners of 4 [mm] thickness the power reduction amounted to 8 dB. For the stiffeners of the 6 [mm] thickness the observed maximal reduction in the sound power was the highest and amounted to 11 dB. Moreover, for the stiffeners of 6 [mm] thickness, in the considered range of stiffeners parameters, two sharp minimums were observed: for a system consisting of 3 stiffeners of 35 [mm] thickness and a system consisting of 6 stiffeners of 15 [mm] thickness. In the 500 [Hz]–1000 [Hz] frequency band, the power radiated by a stiffened plate can by reduced even by 15 dB to 20 dB. Such a significant reduction is possible for a system of stiffeners consisting of 6 or more stiffeners. The decrease in the acoustical emission resulted from the increase in the stiffeners height. The influence of the stiffeners height is particularly noticeable for stiffeners of 4 and 6 [mm] height.

In the 1000–2000 [Hz] frequency band the decrease in the sound power is not so significant as in lower frequencies. They appear for all the considered thicknesses of stiffeners. In this frequency range the system is particularly sensitive to the selection of stiffeners parameters. It should be stressed that the reduction does not concern only the individual mode shapes, since the results are averaged over the whole frequency band.

In the 2000–3000 [Hz] frequency band, the reduction of sound power resulting from introduction of the set of stiffeners is insignificant, but even in this frequency range, for same numbers of stiffeners and their height, it is possible to obtain reduction of the sound power by 5–6 dB.

On the basis of the analysis of the results obtained for various stiffeners sets it can be stated that the best effect of stiffening of the considered plate was observed in the 125–1000 [Hz] frequency band. The most significant reduction of the radiated acoustical power level (even by 20 dB) was observed for the set of 6–8 stiffeners of 25–40 [mm] height and 4–6 [mm] thickness.

6. Experimental verification

Since the analysis was carried out for a large number of plates, the detailed experimental verification of all the obtained results is impossible, taking into account at least the economical aspects. The possible detailed and extended verification of the obtained results could be only viewed as another voice in the possible discussion concerning the field of applications and the accuracy of computations performed by means of the SEA

Table 1. Comparison of the sound power reduction estimated and determined experimentally [9].

Area of stiffener cross-section	Experiment		SEA method	
	125–500 [Hz]	500–1000 [Hz]	125–500 [Hz]	500–1000 [Hz]
[mm×mm]	[dB]	[dB]	[dB]	[dB]
$2 * 10$	3.6	1		
	4.4	–0.7		
	4.9	1.3		
Average	4.3	0.5	3.8	0.1
Standard deviation	0.6	1.1		
$2 * 25$	6.8	4.7		
	7.4	5.8		
	4.5	1.5		
	4.8	5.2		
	4.6	4.2		
Average	5.62	4.28	4.75	3.90
Standard deviation	1.37	1.66		
$2 * 40$	3.6	–0.1		
	3.5	0.6		
	3	1.9		
	3	1		
Average	3.28	0.85	3.9	–0.70
Standard deviation	0.88	0.83		
$4 * 30$	7.4	4.6		
	6.4	5.3		
	5.7	4.3		
	6.1	4		
Average	6.4	4.55	5.65	3.50
Standard deviation	0.73	0.56		
$4 * 40$	5.3	0.7		
	3.7	–0.2		
	5.4	–0.5		
	7.7	1.3		
Average	5.53	0.33	5.98	0.59
Standard deviation	1.65	0.83		
$4 * 50$	7.5	–1.3		
	11.2	5.4		
	12.1	6.5		
	5.5	–2.2		
Average	9.075	2.1	8.1	3.55
Standard deviation	3.11	4.48		

Table 1. [cont.]

Area of stiffener cross-section	Experiment		SEA method	
	125–500 [Hz]	500–1000 [Hz]	125–500 [Hz]	500–1000 [Hz]
[mm×mm]	[dB]	[dB]	[dB]	[dB]
$6 * 20$	6.6	7.9		
	7.3	8.2		
	6.4	4.7		
	6.8	8.1		
	7.8	10.1		
	5.6	9.4		
Average	6.75	8.07	5.32	5.70
Standard deviation	0.76	1.86		
$6 * 30$	9	9.2		
	6.9	10.7		
	10.9	12.6		
	7.1	10.7		
	4.6	9.6		
	5.4	9.8		
Average	7.32	10.43	6.1	6.69
Standard deviation	2.32	1.22		

method. The presented algorithm and software do not provide any additional constraints on the method. Above there is presented only a simple comparison of the results of the predicted reduction of the sound power level with the experimental results [9] published by other authors.

7. Conclusions

The analysis of the stiffened plate acoustical power radiation was carried out using the energy balance equations. Numerical calculations, taking into consideration the influence of the changes in rigidity caused by various configurations of stiffeners sets, were carried out for a wide range of constructional parameters.

The reduction of the energy radiated by the tested plates may reach even 20 dB. However, such a great reduction is only possible for the exactly specified parameters and the number of stiffeners. The increase in the stiffeners height usually results in the reduction of acoustical radiation, especially in low frequency bands. This influence is particularly visible for stiffeners of 4 and 6 [mm] thickness. In higher frequency range the influence of the parameters of stiffeners on reduction of the plate acoustical radiation is not so significant. Increase in the number of stiffeners had no particular effect in the high frequency range since the surfaces between the stiffeners radiate like separate plates. In the high frequency range the radiation of the stiffeners can not be neglected. Nevertheless, even in the frequency range from 2000 [Hz] to 3000 [Hz], for carefully adjusted parameters of the stiffeners, it is possible to achieve reduction of the radiated power (up to 5–6 dB).

On the basis of the analysis of the obtained results, the general trend is observed that the increase in the number and height of the stiffeners results in reduction of the averaged level of the sound power radiated by the plate. Such an effect was observed for all the examined frequency bands. However, the functions describing the power radiated and stored in the stiffened plate are not monotonically decreasing. Local extremes of the radiated energy may differ from the general trend even by several dB. A slight change in the parameters of the stiffeners may result in a considerable decrease or increase in the sound power. It provides a practical opportunity to reduce the radiated energy without a harmful increase in the plate mass or in the cost of additional stiffeners. Thus, searching for such a minimum may have practical significance in the sound and vibration control.

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