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DEMODULATION OF NON-STATIONARY MACHINE VIBRATION USING CYCLE-TIME SCALE

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Inner time-scale affects the operation of angle demodulator. Can linear approximation of monotonic variations in carrier-cycle improve the recovery of unknown low-energy signal modulating observable vibration processes? Authors make an attempt to answer using phase-locked loop PLL as demodulator. Simulations conducted for complex phase model indicate potentially vast possibilities of recognizing and selection of information component. Appropriate selection of approximation time-range enables to achieve acceptable RMS value loads and harmonics of rotational speed of outgoing estimates.

Key words: diagnostics, modulation, synchronism.

Notations

FDM	frequency demodulation,
DRP	dynamic residual process,
DPMS	demodulation procedure of machine signal,
MMV	modulation of machine vibrations,
PDM	phase demodulation,
PLD	procedure of linear decimation,
PLL	phase-locked loop,
PPM	pulse-position modulation,
SNR	signal to noise ratio,
VCO	voltage-controlled oscillator,
δ	relative deviation,
$\mathbf{\Phi}_C$	carrier phase,
$\mathbf{\Phi}_X$	value phase,
$\mathbf{\Phi}_F$	fluctuation phase,
$\mathbf{\Phi}_T$	trend phase,

$\mathbf{\Omega}_C$ carrier frequency,
$\mathbf{\Theta}_C$ carrier cycle,
$\mathbf{x}_O$ reference modulating signal,
$\mathbf{x}_2$ high-frequency excitement signal (modulated),
$\mathbf{x}_p$ phase demodulation result PLL,
$\mathbf{x}_h$ demodulation result of a phase as analytic signal argument,
$\mathbf{y}$ modulated vibrational signal,
$\mathbf{y}_C$ carrier signal.

1. Usefulness of local time scale

Local and global time scales are determined by sequences of reference events – clocks🕒. Role of a clock is generating ordering events, organizing other event sequences. Regular and irregular course of time depends on clock's scale choice, therefore it would be advisable to search for a clock representative for certain event-subset, e.g. vibroacoustical process containing information of diagnostic value.

Time scale in which:
- description of variability becomes simpler,
- significant characteristics of informative variability are preserved,
- non-informative variability is reduced,

can facilitate diagnosing in variable operating conditions. Can, but does not necessarily have to. In each case something is gained and something lost. The complexity extent of practical implementation, the method sensitivity to the selection of appropriate conversion parameters, flexibility in adaptation to an object – finally specifying, what in this case is the coveted informative variability, will decide about the usefulness of suggested time scale.

2. Cycle-determined time

We will take into consideration objects, in which repeating elements' interaction takes place, as well as media in motion. The sequence of the selected event-series leading to achieving basic goal [4, 6], repeats in time-intervals, also referred to as momentary cycle $\Theta_k(t)$. Consecutive realizations of Θ_k are not identical, even in good technical condition and constant machine-operating conditions (hence cyclic motion is not synonymous to periodic motion).

Cyclic motion makes an interesting possibility to distinguish, apart from running according to universal clock exploitation time ϑ and dynamic time t – differing only in scale multiplier, also of the third category – relative time scale "η", which rhythm is determined by cycle Θ – time interval of consecutive sequences [8]. Its relation with dynamic time t is approximated by the formula (1)

$$\eta = n - 1 + \frac{t - \sum\limits_{1}^{n-1} \Theta_j}{\Theta_n}. \tag{1}$$

For the interval T comprising 1,..n, ..M cycles: $\eta \in [0, mM]$, $t \in [0, T]$, m – number of distinguishable cycle moments.

Replacing, in transformation formula (1), the momentary cycle Θ by its characteristic $\varphi_k(\Theta)$ can facilitate scale "η" adaptation for specific method categories of value variability recovery (Fig. 2b and c). Relation (1) takes the form (2)

$$\eta = n - 1 + \frac{t - \sum_{k=1}^{n-1} \varphi_k(\Theta)}{\varphi_k(\Theta)}. \tag{2}$$

If $\varphi_k(\Theta)$ represents averaging operations, approximation of momentary-cycle scale takes place, performing:

- broken-line approximation for range averaging,
- smoothing for mobile average value.

Rule of creating scale for "$\eta(\Theta)$" with linear approximation within Θ_k is presented in Fig. 1

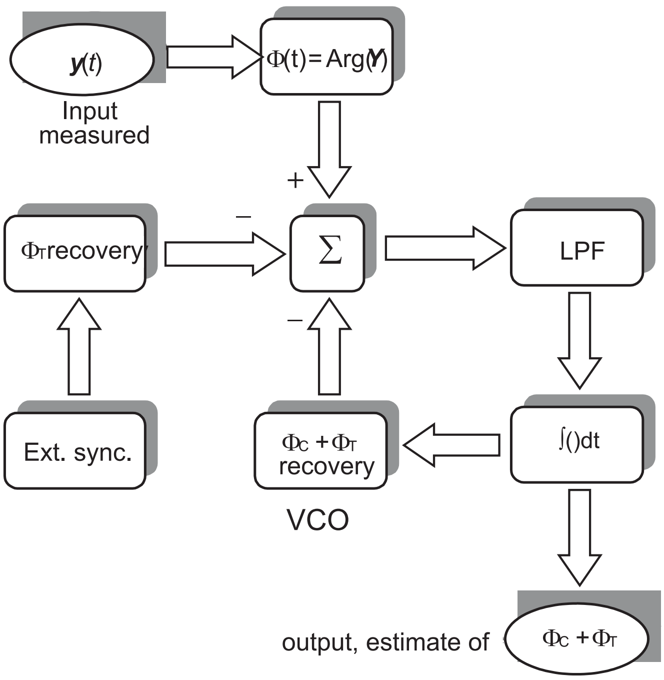

Fig. 1. Recovery of phase components by the use of modified PLL tracking filter.

It is remarkable, that the number of observable moments η is practically finite. Its increasing, not necessarily on account of real need, hinders the measurement and its results processing. Authors' experiments serve as confirmation [8].

Therefore consideration of continuous nature of η by means of interpolation seems useless, although theoretically possible, if momentary cycle, or its characteristic φ is depicted by function $\Gamma(t)$ continuous in observation-interval T according to relation (3)

$$\frac{\mathrm{d}\eta}{\mathrm{d}t} = \frac{1}{\Gamma(t)}. \tag{3}$$

Two cases seem to be interesting:

When $\varphi(\Theta)$ corresponds to momentary cycle Θ, signal description as a frequency function presents the row spectrum, applied also in diagnosing [12].

Row spectra feature good resolution and dynamic. However their estimation necessitate complex calculations and precise measurement of phase (location) [*ibidem*]. Furthermore, in thus defined scale "η" information about cycle fluctuations is lost, which fluctuations could be the consequence of vibration PPM by means of dynamic residual processes and constitute valuable source of early diagnostics information.

The mentioned before inconvenience can be avoided in many cases by linear approximation of monotonic cycle variations. Then in interval T:

$$\varphi(t) = \Theta_{\mathrm{ref}}\left(1 + \frac{\Delta_{\Theta}}{\Theta_{\mathrm{ref}}}t\right). \tag{4}$$

3. Demodulation of vibration angle in cyclic machines

3.1. Difficulties

Many detailed researches [1, 2, 3] indicate the occurrence of angle modulation and its impulse version PPM as early as in initial stages of damage evolution, whereas only advanced wear (e.g. due to friction or play in transmissions) results in noticeable general increase in spectrum power of the entire vibration signal [10]. Without going into details, in each case the demodulation results can constitute the reliable base for technical condition assessment (4) merely when its PDSM procedure is a non-load estimator and in various exploitation conditions.

However, if rotation speed and therefore carrier-frequency are not constant, angle-demodulation methods, created and optimized for carrier-signals of specified form and constant frequency, appear to be ineffective. Inappropriate recovering of momentary phase components is the most likely load source – Arg[$\mathbf{y}(t)$].

3.2. Phase-locked loop PLL

Potentially vast capability of adaptation to specific vibration demodulation requirements features phase tracking system (*Tracking filter*), also referred to as phase-locked loop PLL, for years commonly applied telecommunication and measuring technique, first of all for all kinds of *on-line* demodulation. Excepting vastly explained operation principle of PLL system and its dynamics analysis [1, 3, 9, 13], it is still worth to consider the effect on momentary phase of the signal ***y***, exerted by recovering manner applied by the subsystem traditionally referred to as VCO (Fig. 1).

3.3. Model of machine phase of angle modulation

Let us make an approach to considering, which requirements are to be fulfilled by the transformation of dynamic time scale in order to achieve the domination of information variability. Well, apart from the method, adapted model of signal ***y*** phase determines the correctness of demodulation results interpretation.

Real MMV is a series of unique impulses of modulated position [1, 3]. PPM model [1, 10] indicates that angle modulation recovers each component of impulse-series harmonics. Therefore further considerations will pertain to momentary phase of harmonic component of PPM – the first one as a rule.

Thus the appropriate modeling concerns the phase itself, in which generally four components must be distinguished (5), of qualitatively different variability and dissimilar, although not always unambiguous interpretation:

$$\mathrm{Arg}[\mathbf{Y}(jt)] = \mathbf{\Phi}(t) = \mathbf{\Phi}_C(t) + \mathbf{\Phi}_X(t) + \mathbf{\Phi}_F(t) + \mathbf{\Phi}_T(t) \tag{5}$$

where:

$\mathbf{\Phi}_C(t)$ – carrier-signal phase with the steady cycle Θ_C,
$\mathbf{\Phi}_T(t)$ – result of monotonous carrier-cycle variation,
$\mathbf{\Phi}_X(t)$ – informative component; here – variability recovering, DPR,
$\mathbf{\Phi}_F(t)$ – non-informative fluctuations connected with exploitation state.

How to distinguish and differentiate them? Which ones are the interference disturbing the diagnosing? How to contain the influence interfering with effective recovery of informative phase – unobservable directly?

In dynamic-time perspective, progress of wear processes is generally non-measurable, therefore monotonous resultant phase trend $\mathbf{\Phi}_C + \mathbf{\Phi}_T$ can be attributed to non-informative carrier-frequency variation.

Phase synchronization loop (Fig. 1) enables the approximation of real carrier phase of non-stationary $\mathbf{y}$ signal (e.g. at variable rotational speed). Such an action reduces demodulation to the removal of resultant non-linear phase trend $\mathbf{\Phi}_C + \mathbf{\Phi}_T$. It can be interpreted as a result of time-scale transformation, leading the carrying process to constancy in terms of cycle stability – $\Theta_C = \mathrm{const}$.

Differentiation fluctuation $\mathbf{\Phi}_X$ from $\mathbf{\Phi}_F$ requires individual treatment. High-fidelity recovery of $\mathbf{\Phi}_X$(t) is not always possible or even necessary, since there is a lack of shape reference, and also inclination to repeatability of consecutive processes, due to their random nature DPR [3].

To recapitulate, the base of useful time-scale transformation "t" $\Rightarrow$ "η" should be the carrier cycle Θ_C. Unfortunately, generally speaking, there is no vibration form generated especially for this purpose. Vibrational carrier-signal $\mathbf{y}_0$ does not need to be identical with kinematical, dynamic, or any other element of achieving basic goal of the machine.

3.4. Short-term trends Θ_C approximation

In practical applications linear cycle trend occurs quite rarely, yet it serves effectively as a model approximating monotonic variation $\Delta\Theta_C$ in observation interval T [1, 3, 4].

The alternative is utilizing range-average of momentary cycle for linear approximation of carrier frequency using monotonic line-segments (4, also Fig. 2c). Such a solution involves the risk of creating parasite fluctuations deforming original PPM, (which in case of narrow-band carrier signal corresponds to phase $\mathbf{\Phi}_F$)

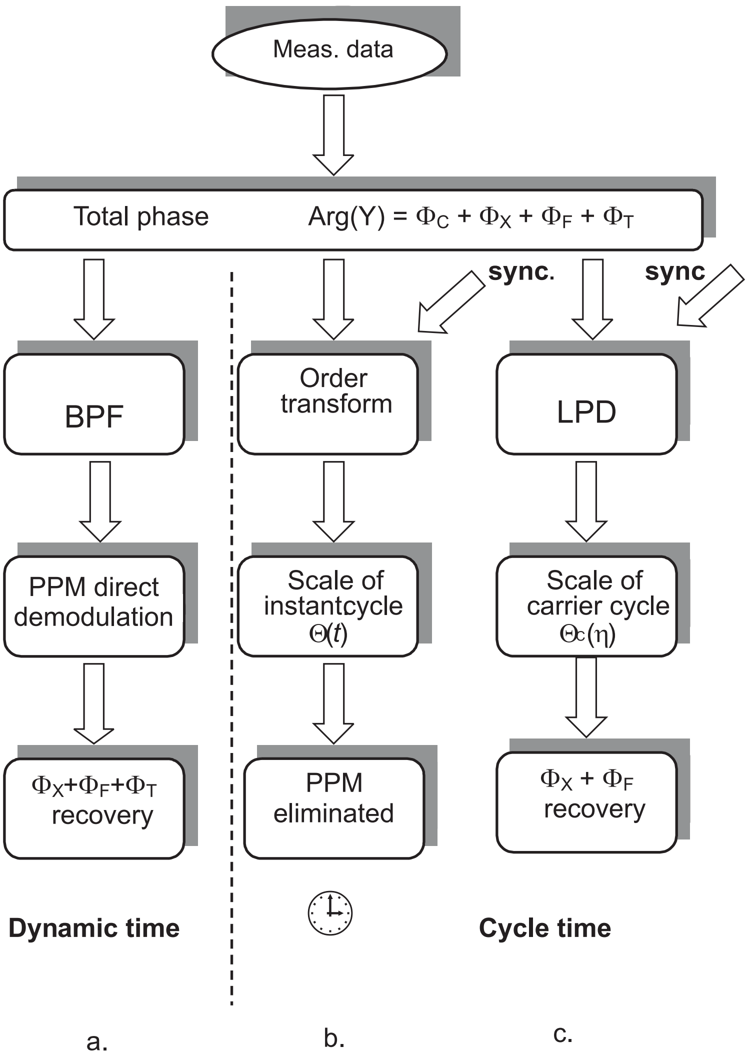

Fig. 2. Capability of vibration-signal angle demodulation depending on time-scale of recovered carrier wave: a) dynamic time scale "t", b) scale of rows time, or momentary cycle – "$\eta(\Theta)$", c) cycle-time scale with linear approximation – "$\eta(\Theta_T)$".

For preliminary distinguishing of components $\mathbf{\Phi}$, frequency-selection of at least partial DPR can be taken into consideration (Fig. 3, band F_X).

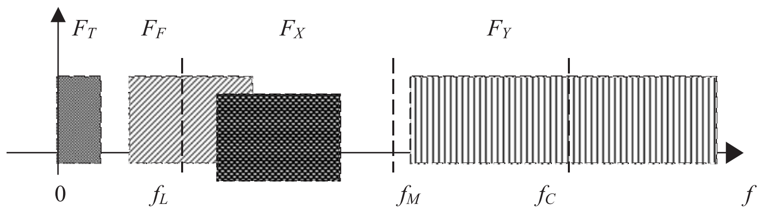

Fig. 3. Phase modulation – ranges of signals' frequencies of narrow-band carrier wave: F_T – rotational frequency trend (and carrier $\Rightarrow$ f_C) in interval T, F_F – frequency band corresponding to interfering phase fluctuations Φ_F, F_X – frequency band of modulating signal $\mathbf{\Phi}_X$, F_Y – modulation frequency band (maximum useful).

To trend Ω_C reduction can also lead the time-scale transformation "t" $\Rightarrow$ "η" through proportional specimen selection, as provided by the generally known in literature procedure of linear decimation – PLD [1, 4, 6].

The common area of F_F and F_X should be minimized by the appropriate selection of base characteristic of the cycle [3]. If band filter proves to be ineffective, new differentiating factors must be provided. As a result we should adapt the demodulation symptomatic band, in which the variability prevails caused by the change of technical state – and not the exploitation state.

4. Simulation tests

4.1. Range and purpose

The proposed model of modulated vibration-signal argument (3) enables to trace the phase components processing during demodulation and provisionally assesses the rightness of division criteria and selection possibilities of the phase regarded as informative.

Research included:

- adaptation and tuning of model demodulator PLL,
- range and effectiveness of carrier cycle approximation,
- effect of modulating-signal sort.

Quantitative criterion of the assessment was a relative deviation in recovering modulating-signal RMS value – $\mathbf{x}_0$

For rectangular wave, recovery of the first seven harmonic spectra RMS was assessed. The recovery was conducted by means of outgoing signal PLL – $\mathbf{x}_p$ and after band-filtering – $\mathbf{x}_{pf}$.

Alternative method of demodulation using analytic signal argument:

$$\mathbf{x}_h = \mathrm{detrend}\{\mathrm{Arg}[\mathbf{Y}(t)]\}$$

turned out to be ineffective in the research area.

4.2. *Examples*

During coasting or racing, the machine variations of rotational speed are approximated by an exponential curve, and in shorter time-span by a straight-line segment. In both cases the approximation with linear cycle increment will not be accurate. Will it still prove to be satisfactory?

Let's consider two cases:

1. Approximation of linear carrier frequency 10% increment by means of linear decrease of carrier cycle Θ_C.

$$\Omega_C(t) = \Omega_{CO}\left(1 + b\frac{t}{t_M}\right) \quad \Rightarrow \quad \frac{\Omega_{VCO}}{1 - \dfrac{t}{ct_M}}$$

2. Approximation of exponential 10% decrease of carrier frequency by means of linear decrease of carrier cycle Θ_C.

$$\Omega_C(t) = \Omega_{CO}\left[1 - \exp\left(-a\frac{t}{t_M}\right)\right] \quad \Rightarrow \quad \frac{\Omega_{VCO}}{1 + \dfrac{t}{ct_M}}$$

In both cases, changing the time scale eliminates only linear-segment variation of carrier cycle. Its effects on shape and spectrum of demodulation results are presented in Figs. 4, 5 and 6.

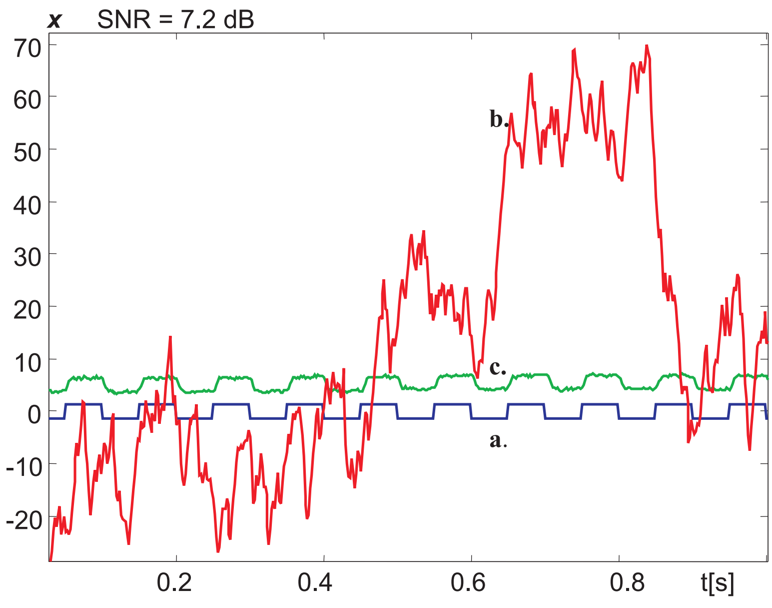

Fig. 4. Effect of various demodulation procedures (linear increase of carrier frequency) a) reference $\mathbf{x}_0$, original $\mathbf{x}$, b) $\mathbf{x}_H - \Phi$, linear trend removed, c) $\mathbf{x}_P$ – PLL output.

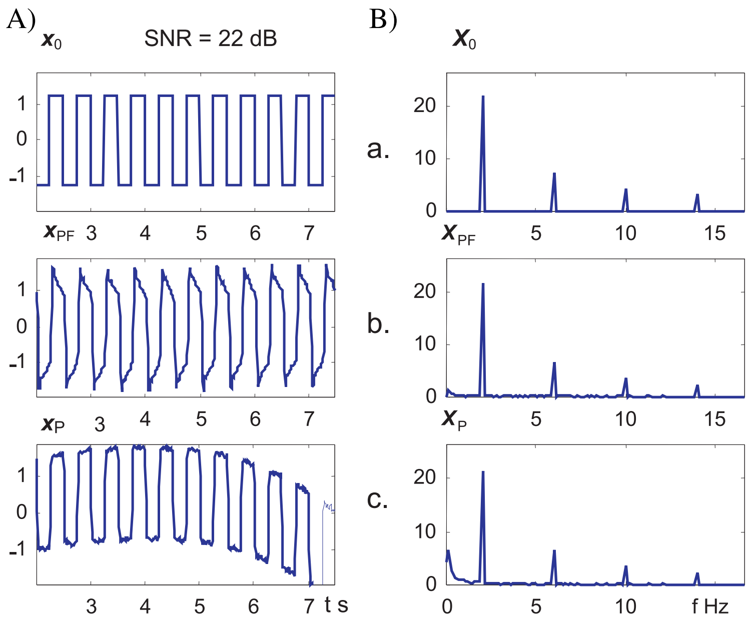

Fig. 5. PDM results – case of linear carrier frequency increase A) time domain, B) RMS spectra, a) $\mathbf{x}_0$ – modulating reference signal; b) $\mathbf{x}_P$ – PLL output, HP filter used; c) $\mathbf{x}_P$ – PLL output; $\delta\mathbf{x}_{pf} = -5.8\%$, SNR = 22 dB, $\delta\,\mathbf{x}_p$ = 1360%.

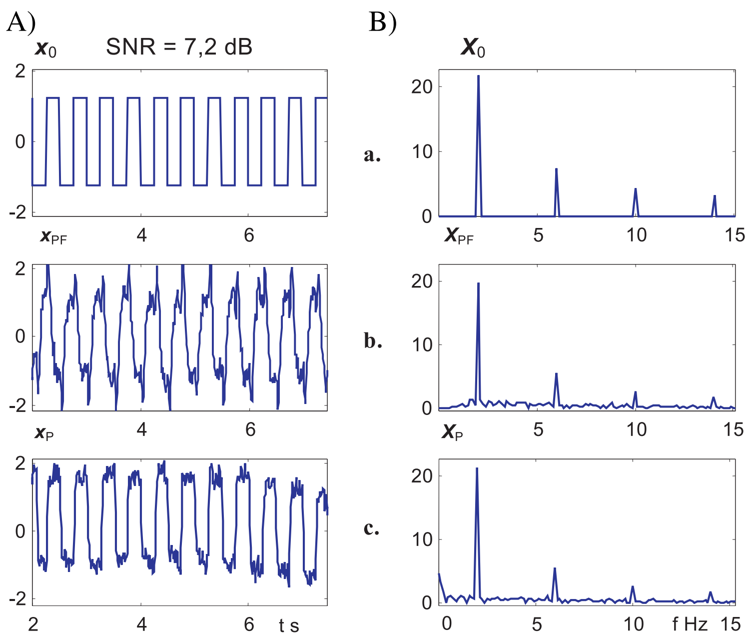

Fig. 6. PDM results – case of exponential carrier decrease A) time domain, B) RMS spectra, a) $\mathbf{x}_0$ – modulating reference signal; b) $\mathbf{x}_P$ – PLL output, HP filter used; c) $\mathbf{x}_P$ – PLL output; $\delta\mathbf{x}_{pf} = -6.85\%$, SNR = 7.2 dB, $\delta\mathbf{x}_p$ = over 2000%.

For the accepted approximation-span, PLL in both cases turns out to be significantly loaded (Fig. 5c and 6c) by fluctuations caused by the difference in variability of real and recovered carrier cycle. However, that does not disqualify either the method or the outcome.

In unintentional modulations the form of modulating signal is not precisely recognized generally. That also concerns shape evolution. Symptomatic are the changes of statistic characteristics of individual impulses, e.g. skewness or kurtosis [2].

However, in the figure enclosed, the shape of rectangular wave was recovered correctly and similarly was the RMS spectrum in harmonic range 1–7. Slow-variations fluctuations are HP filtered, which removes biasing (Fig. 6b), recovering correctly RMS value $\mathbf{x}_O$ – as well as potential carrier of diagnostic information.

Simulations also enable to determine the time intervals of acceptable approximation results with given relative variation of signal's $\boldsymbol{y}$ carrier-frequency. Therefore simulations should on every occasion precede testing of real vibrations signals, as calibrating procedures for synchronous demodulator.

5. Conclusions and remarks

- Time scale choice enables to enhance certain variability characteristics and reduce other ones.
- The indicator of the extent of adjusting generator PLL time-scale to real carrier cycle is similarity of certain qualitative and quantitative features (here: shape and effective value of shape or effective values of model- and recovered signal $\mathbf{\Phi}_X$).
- With recovery of carrier-signal of constant cycle-increment, simultaneous fulfilling both requirements proved to be not always possible.
- In cases when shape of informative phase is undetermined or a priori unfamiliar, its averaged energetic characteristics, like RMS, PSD, SNR could turn out to be more useful.
- In selection of time scale of carrier recovery, a compromise seems to be purposeful, so as possibly quick and stable demodulation procedure recovers the selected features of informative variability with the necessary accuracy.
- Distinction between informative and interfering phase-fluctuations is not always possible by means of simple band filtering.

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