ARCHIVES OF ACOUSTICS
31, 2, 243–263 (2006)

ULTRASONIC SCATTERING FROM COMPRESSIBLE CYLINDERS INCLUDING MULTIPLE SCATTERING AND THERMOVISCOUS EFFECTS

SEYYED M. HASHEMINEJAD, M. A. ALIBAKHSHI

Iran University of Science and Technology
Department of Mechanical Engineering
Acoustics Laboratory
Narmak, Tehran 16844, Iran
e-mail: hashemi@iust.ac.ir

(*received November 25, 2005; accepted March 20, 2006*)

This paper presents a study of acoustic scattering by a pair of parallel circular thermoviscous fluid cylinders submerged in an unbounded viscous thermally conducting medium. The translational addition theorem for cylindrical wave functions, the appropriate wave field expansions and the pertinent boundary conditions are employed to develop a closed-form solution in the form of infinite series. The analytical results are illustrated with a numerical example in which two identical thermoviscous fluid cylinders are insonified by a nearby parallel acoustic line source at broadside/end-fire incidence. The backscattered pressure amplitude is numerically evaluated and discussed for representative values of the parameters characterizing the system. The effects of source position, transmission frequency and proximity of the two cylinders are examined. Particular attention has been focused on multiple scattering interactions as well as thermoviscous effects. The imperative influence of thermoviscosity on the analysed phenomena is revealed by notable reduction of backscattering amplitude at intermediate and high frequencies. The numerical results also show that the multiple scattering interaction effects are of great (moderate) consequence for end-fire (broadside) incidence at small separations of the cylinders. A limiting case involving a pair of ideal compressible fluid cylinders is considered and a fair agreement with preceding solutions is established.

Key words: line source, multiple scattering, thermoviscous cylinders, addition theorem.

1. Introduction

There exists a vast body of literature on scattering theory, extending back for more than a century and across the boundaries of many disciplines (acoustics, electromagnetics, quantum mechanics). Particularly, the scattering of waves by two-dimensional circular cylinders is a fundamental problem, which has attracted attention of great many researchers in both acoustics (MORSE and INGARD [32]) and electromagnetics (BALANIS [7]). In this section, we shall briefly review the research literature relevant to the present study, i.e., the investigations of sound scattering by fluid cylinders and also by cylindrical objects with allowance for different dissipation mechanisms such as viscothermal and viscoelastic losses. KOZHIN [19] studied sound propagation in a viscous medium containing cylindrical filaments by calculating the field scattered by a cylinder suspended freely in the viscous medium. LIN and RAPTIS [22] extended the so-called ECAH theory (i.e., the most well known acoustic theory for heterogeneous systems with allowance for viscous and thermal losses) to study the scattering by a free thermoelastic cylinder submerged in a thermoviscous fluid. Subsequently, the latter authors presented a general analysis for scattering of a plane sound wave obliquely incident upon a thin, elastic circular cylinder immersed in an unbounded viscous fluid (LIN and RAPTIS [24]). As a first step towards the numerical solution of the inverse scattering problem for acoustic and elastic waves, ALEMAR *et al.* [2, 3] investigated the scattering of acoustic waves from an infinite cylindrical fluid obstacle immersed in a fluid loading medium of greater/smaller density. They subsequently showed, in a step-by-step approach, how the Resonance Scattering Theory permits the simultaneous prediction of the size, orientation, and nature of a cylindrical fluid-scatterer (ALEMAR *et al.* [4]). CHANDRA and THOMPSON [10] employed the method of Padé approximants to determine the scattered pressure from a fluid cylinder having a strong compressibility contrast. BOAG *et al.* [9] used a multifilament source model to present the solution for the problem of two-dimensional acoustic scattering from homogeneous fluid cylinders. ROUSSELOT [35] also studied the acoustic field scattered by a fluid cylinder. LEE *et al.* [21] investigated the effects of material attenuation on acoustic resonance scattering from an air-filled cylindrical shell immersed in water. More recently, GINSBERG [13] studied the effect of viscosity in 2D scattering of a plane sound wave from a partially coated infinite cylinder. WEI *et al.* [42] expressed the radiation force-per-length on an infinitely long fluid cylinder in an acoustic plane standing wave in terms of partial-wave scattering coefficients for the corresponding travelling wave scattering problem. The latter authors obtained a simple long wavelength approximation of the radiation force for the following situations: a hot gas column (used to approximate a small flame), a compressible liquid bridge in a Plateau tank, a liquid bridge in air, and a cylindrical bubble of air in water. Also, SCOTTI and WIRGIN [37] studied the inverse medium problem (i.e., the retrieval of the material constants) for a generally lossy fluid-like circular cylindrical object in a lossless fluid-like host probed by plane-wave acoustic radiation. MITRI *et al.* [28] calculated acoustic backscattering from cylindrical targets, suspended in an inviscid fluid including the effects of absorption of shear and compressional waves in viscoelastic materials. Just recently, HASHEMINEJAD and SAFARI [17] employed the novel features of the Havriliak–Negami model (HARTMAN *et al.* [14]) to investigate the effects of dynamic viscoelastic properties on acoustic scattering by viscoelastically coated cylinders submerged in a viscous fluid. MITRI [29] presented theoretical calculations for the acoustic radiation force experienced by fluid, rigid, elastic and viscoelastic cylinders immersed in ideal fluids and placed in a standing or quasistanding wave field. The latter author also analyzed the effect of hysteresis-type of absorption on the frequency dependence of the acoustic radiation force for infinite and absorbing solid cylinders [30] and cylindrical shells (MITRI [31]) placed in a plane incident ultrasound field.

Many researchers have studied the mutual interaction between multiple objects subject to a primary acoustic field. In particular, exact and approximate solutions have been developed for scattering of plane waves by multiple parallel circular cylinders. One of the earliest solutions to this problem (for rigid cylinders) was developed by TWERSKY [41], who decomposed the total field into an incident field plus a higher order scattered field contributions whose coefficients were determined in an iterative manner. Additional solution techniques were later developed by YOUNG and BERTRAND [44] who examined both theoretically and experimentally the scattered field produced by a plane acoustic wave normal to the axis of two identical parallel, rigid cylinders. An analytical study of multiple scattering of a plane sound wave by a group of circular, rigid cylinders oscillating in an ideal fluid is performed by LIN and RAPTIS [23]. The solution of diffraction of acoustic waves by a system consisting of two circular, nonparallel, and nonintersecting cylinders is presented by KUBENKO [20]. ZHUK [46, 47] studied the interaction of acoustic waves with a pair of parallel impenetrable circular cylinders immersed in a viscous fluid medium. SCHARSTEIN [36] used the Graf's addition theorem for cylindrical harmonic to compute the scattered field due to two parallel soft circular cylinders illuminated by a plane acoustic wave for several geometries and incidence angles. A new approach based on the group theory in order to study acoustic scattering by a pair of identical parallel cylinders is developed by DECANINI [11]. Scattering of a plane acoustic wave by an infinite penetrable (impenetrable) circular cylinder, parallel with another one of acoustically small radius, is examined by ROUMELIOTIS *et al.* [34]. WU *et al.* [43] presented the acoustic band gap results for two-dimensional arrays of water (mercury) cylinders with circular cross-section in a mercury (water) host. Just recently, a general analytic method for evaluating the scattered fields created by multiple rigid circular cylinders arranged in an arbitrary parallel configuration is developed by SHERER [38].

The above review clearly indicates that while there exists a vast body of literature on acoustic wave scattering from cylindrical objects suspended in an ideal or a viscous fluid medium, rigorous analytic or numerical solutions including thermoviscous as well as multiple scattering effects seem to be nonexistent. Our primary objective is to fill this gap. Therefore, noting that the most fundamental problem of multiple scattering in two dimensions involves a cluster of two circular cylinders, we employ the translation addition theorem for cylindrical Bessel functions to study theoretically and numerically the scattering of compressional acoustic waves by a pair of interacting thermoviscous fluid cylinders due to a nearby acoustic line source suspended in an unbounded viscous thermally conducting fluid medium. The proposed model is of noble interest essentially due to its inherent value as a canonical problem in theoretical acoustics. It can form an invaluable guide in establishing proximity thresholds for the influence of multiple scattering in terms of thermoviscous fluid properties, average distance between the cylinders and the incident wave field. It has promising applications in a wide range of physical and technical fields including acoustic analysis of highly concentrated cylindrical emulsions

(PRINCEN [33]) and liquid composites (WU *et al.* [43]). It may also be of practical interest in passive acoustic stabilization and breakup control of long liquid bridges, small diffusion flames and hot cylindrical fluid objects under microgravity conditions (see WEI *et al.* [42]; MARR–LYON *et al.* [25]; MARSTON [26, 27]). Lastly, the presented exact solution can serve as the benchmark for comparison to other solutions obtained by strictly numerical or asymptotic approaches.

2. Formulation

2.1. Basic equations

There is now available a seemingly endless variety of models dealing with thermoviscous effects in acoustic wave propagation. The most exclusive model is based on a solution of the full set of basic governing equations, i.e., all terms in the linearized Navier Stokes equations are taken into account. This inclusive treatment of thermoviscosity can greatly complicate the analysis because the fluid medium can then support shear and thermal as well as compressional modes, both of which must be accounted for in satisfying the boundary conditions at the interfaces. The classical Helmholtz decomposition expansion may be employed advantageously to express fluid-particle velocity vector in the thermoviscous acoustic medium in terms of a compressional-wave scalar potential and a viscous-wave vector potential as (TEMKIN [40])

$$\mathbf{u} = -\nabla\varphi + \nabla \times \boldsymbol{\psi}. \tag{1}$$

The governing equations for φ, $\boldsymbol{\psi}$ and the excess temperature T is then written as (TEMKIN [40]; BELTMAN [8])

$$\begin{aligned} -\omega^2\varphi + \mathrm{i}\omega\frac{\eta c^2}{\gamma}T &= \left(\frac{c^2}{\gamma} - \mathrm{i}\omega\beta\right)\nabla^2\varphi, \\ -\mathrm{i}\omega T &= \frac{\kappa}{\rho C_v}\nabla^2 T + \frac{\gamma-1}{\eta}\nabla^2\varphi, \\ -\mathrm{i}\omega\boldsymbol{\psi} &= \nu\nabla^2\boldsymbol{\psi}, \end{aligned} \tag{2}$$

where $\mathrm{i} = \sqrt{-1}$, κ is the thermal conductivity, C_v is the specific heat at constant volume, ρ is the mass density, $\beta = 4\nu/3 + \mu_b/\rho$, $\nu = \mu/\rho$ is the kinematic viscosity, μ is shear (dynamic) viscosity, μ_b is the bulk (expansive) viscosity, c is the adiabatic speed of sound, η is the coefficient of thermal expansion, $\gamma = C_p/C_v$ is the specific heat ratio and C_p is the specific heat at constant pressure. In view of the fact that the incident wave is time-harmonic, with the circular frequency ω, we shall assume harmonic time variations throughout with the $e^{-\mathrm{i}\omega t}$ dependence suppressed for simplicity.

The above governing equations may be algebraically manipulated to yield Helmholtz-type equations (HASHEMINEJAD and GEERS [16])

$$\begin{aligned}\left(\nabla^2 + k_c^2\right)\, \varphi_c &= 0,\\ \left(\nabla^2 + k_t^2\right)\, \varphi_t &= 0,\\ \left(\nabla^2 + k_s^2\right)\, \boldsymbol{\psi} &= 0,\end{aligned} \tag{3}$$

where the subscripts c, t, and s denote compressional, thermal, and shear, respectively. In addition, $\varphi = \varphi_c + \varphi_t$, and accurate the approximations for k_c, k_t and k_s are given as

$$\begin{aligned}k_c &= \frac{\omega}{c}\,\left[1 + \mathrm{i}\frac{\omega\nu}{2c^2}\left(\frac{4}{3} + \frac{\mu_b}{\mu} + \frac{\gamma - 1}{\mathrm{Pr}}\right)\right],\\ k_t &= (1 + \mathrm{i})\,\sqrt{\omega/2\sigma},\\ k_s &= (1 + \mathrm{i})\,\sqrt{\omega/2\nu},\end{aligned} \tag{4}$$

where $\mathrm{Pr} = \mu C_p/\kappa$ is the Prandtl number and $\sigma = \kappa/\rho\, C_p$ is the thermal diffusivity.

The geometry problem and the coordinate systems used are depicted in Fig. 1. A cylindrical sound wave emerging from a nearby line source is normally incident on

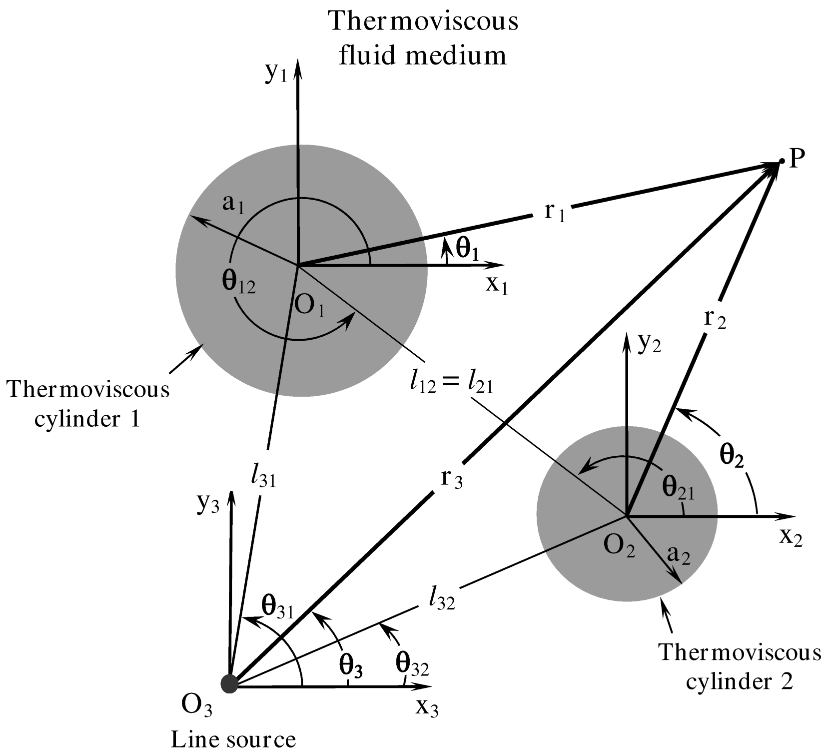

Fig. 1. Geometry problem.

the cylinders. Every origin O_i ($i = 1,\ 2,\ 3$) of the cylindrical coordinate system (r_i, θ_i) is placed at the center of the respective fluid cylinder and the line source as shown in the figure. Making use of (1) through (3), while keeping in mind the symmetry problem, $\boldsymbol{\psi} = (0, 0, \psi)$, the radial and tangential velocities and the excess temperature are determined as (HASHEMINEJAD and GEERS [16]):

$$\begin{aligned} u_r &= -\frac{\partial\varphi}{\partial r} + \frac{1}{r}\frac{\partial\psi}{\partial\theta}, \\ u_\theta &= -\frac{1}{r}\frac{\partial\varphi}{\partial\theta} - \frac{\partial\psi}{\partial r}, \\ T &= b_c\varphi_c + b_t\varphi_t, \end{aligned} \tag{5}$$

where

$$\begin{aligned} b_c &= \frac{\gamma}{\mathrm{i}\omega\eta c^2}\left[\omega^2 - k_c^2\left(\frac{c^2}{\gamma} - \mathrm{i}\omega\beta\right)\right], \\ b_t &= \frac{\gamma}{\mathrm{i}\omega\eta c^2}\left[\omega^2 - k_t^2\left(\frac{c^2}{\gamma} - \mathrm{i}\omega\beta\right)\right]. \end{aligned} \tag{6}$$

In addition, the classical relations for radial and tangential stresses, heat flux, and acoustic pressure may be employed to yield the following potential-based expressions (LIN and RAPTIS [22]):

$$\begin{aligned} \sigma_{rr} &= -p - \left(\mu_b - \frac{2}{3}\mu\right)\nabla^2\varphi - 2\mu\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r^2}\frac{\partial\psi}{\partial\theta} - \frac{1}{r}\frac{\partial^2\psi}{\partial r\partial\theta}\right), \\ \sigma_{r\theta} &= \mu\left(-\frac{2}{r}\frac{\partial^2\varphi}{\partial r\partial\theta} + \frac{2}{r^2}\frac{\partial\varphi}{\partial\theta} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} - \frac{\partial^2\psi}{\partial r^2}\right), \\ q &= -\kappa\left(b_c\frac{\partial\varphi_c}{\partial r} + b_t\frac{\partial\varphi_t}{\partial r}\right), \end{aligned} \tag{7}$$

where

$$p = -\mathrm{i}\omega\rho\varphi - \left(\mu_b + \frac{4}{3}\mu\right)\nabla^2\varphi. \tag{8}$$

2.2. Field expansions and boundary conditions

Following the standard methods in theoretical acoustics, the dynamics of the multi-scattering problem may be expressed in terms of appropriate scalar potentials. In the surrounding thermoviscous fluid medium, the possibility of outgoing waves exists, while in the fluid cylinders only incoming waves are possible. Therefore, keeping in mind the radiation condition, the outgoing solutions can be expressed as

$$\begin{aligned}
\varphi_{cM}^{(i)}(r_i,\theta_i,\omega) &= \sum_{n=-\infty}^{\infty} a_n^{(i)}(\omega) H_n(k_{cM} r_i) e^{\mathrm{i}n\theta_i},\\
\varphi_{tM}^{(i)}(r_i,\theta_i,\omega) &= \sum_{n=-\infty}^{\infty} b_n^{(i)}(\omega)\, H_n(k_{tM} r_i) e^{\mathrm{i}n\theta_i},\\
\psi_{M}^{(i)}(r_i,\theta_i,\omega) &= \sum_{n=-\infty}^{\infty} c_n^{(i)}(\omega)\; H_n(k_{sM} r_i) e^{\mathrm{i}n\theta_i},
\end{aligned} \tag{9}$$

with $i = 1$ and 2 corresponding to the first and second cylindrical coordinates, respectively (Fig. 1), and the subscript M refers to the boundless thermoviscous medium, $H_n(x) = J_n(x) + \mathrm{i}Y_n(x)$ is the cylindrical Hankel function of the first kind of order n (ABRAMOWITZ and STEGUN [1]), and $a_n^{(i)}(\omega)$, $b_n^{(i)}(\omega)$ and $c_n^{(i)}(\omega)$ are unknown scattering coefficients. Similarly, the solution of the Helmholtz equations for the acoustic velocity potentials inside each fluid cylinder may be represented by

$$\begin{aligned}
\varphi_{cB}^{(i)}(r_i,\theta_i,\omega) &= \sum_{n=-\infty}^{\infty} d_n^{(i)}(\omega)\, J_n(k_{cB} r_i) e^{\mathrm{i}n\theta_i},\\
\varphi_{tB}^{(i)}(r_i,\theta_i,\omega) &= \sum_{n=-\infty}^{\infty} e_n^{(i)}(\omega)\, J_n(k_{tB} r_i) e^{\mathrm{i}n\theta_i},\\
\psi_{B}^{(i)}(r_i,\theta_i,\omega) &= \sum_{n=-\infty}^{\infty} f_n^{(i)}(\omega)\, J_n(k_{sB} r_i) e^{\mathrm{i}n\theta_i},
\end{aligned} \tag{10}$$

in which $J_n(x)$ is the cylindrical Bessel function of the first kind (ABRAMOWITZ and STEGUN [1]) and the subscript B refers to the cylindrical bodies. Moreover, the velocity potential associated with the line source is written as (SKUDRZYK [39])

$$\phi_s = \phi_0 H_0(k_{cM} r_3) \tag{11}$$

in which ϕ_0 is the line source amplitude. Consequently, the total field potentials in the presence of the cylindrical scatterers, after accounting for all multiple scattering interactions, may initially be expressed as

$$\begin{aligned}
\phi_M = \phi_s + \varphi_{cM}^{(i)} + \varphi_{tM}^{(i)} + \varphi_{cM}^{(j)} + \varphi_{tM}^{(j)} &= \phi_0 H_0(k_{cM} r_3)\\
&+ \sum_{n=-\infty}^{\infty} \left[a_n^{(i)}(\omega)\; H_n(k_{cM} r_i) + b_n^{(i)}(\omega)\; H_n(k_{tM} r_i)\right] e^{\mathrm{i}n\,\theta_i}\\
&+ \sum_{n=-\infty}^{\infty} \left[a_n^{(j)}(\omega)\, H_n(k_{cM} r_j) + b_n^{(j)}(\omega)\; H_n(k_{tM} r_j)\right] e^{\mathrm{i}n\theta_j},
\end{aligned} \tag{12}$$

$$\psi_M = \psi_M^{(i)} + \psi_M^{(j)} = \sum_{n=-\infty}^{\infty} c_n^{(i)}(\omega)\, H_n(k_{sM} r_i) e^{\mathrm{i}n\theta_i} + \sum_{n=-\infty}^{\infty} c_n^{(j)}(\omega)\, H_n(k_{sM} r_j) e^{\mathrm{i}n\theta_j}, \tag{13}$$

where $i, j = 1, 2\, (i \neq j)$. Here we note that the last terms in the above equations, which represent the scattered field from the j-th cylinder, are expressed in the j-th coordinate system. These terms have to be transformed to the cylindrical coordinate system centered at the i-th cylinder before imposing the boundary conditions.

The unknown scattering coefficients $a_n^{(i)}(\omega)$ through $f_n^{(i)}(\omega)$ must be determined by imposing the suitable boundary conditions. The continuity of the normal and tangential velocity components, normal and tangential stress components, temperature and heat flux at the interface of each cylinder demands that (HASHEMINEJAD and GEERS [16]).

$$\begin{aligned} u_{rM}^{(i)}\Big|_{r_i=a_i} &= u_{rB}^{(i)}\Big|_{r_i=a_i}, \quad \sigma_{rrM}^{(i)}\Big|_{r_i=a_i} = \sigma_{rrB}^{(i)}\Big|_{r_i=a_i}, \quad T_M^{(i)}\Big|_{r_i=a_i} = T_B^{(i)}\Big|_{r_i=a_i},\\ u_{\theta M}^{(i)}\Big|_{r_i=a_i} &= u_{\theta B}^{(i)}\Big|_{r_i=a_i}, \quad \sigma_{r\theta M}^{(i)}\Big|_{r_i=a_i} = \sigma_{r\theta B}^{(i)}\Big|_{r_i=a_i}, \quad q_M^{(i)}\Big|_{r_i=a_i} = q_B^{(i)}\Big|_{r_i=a_i}, \end{aligned} \tag{14}$$

2.3. Wave transformations and series representations

Many radiation and scattering problems involve waves of one characteristic shape (coordinate system) that are incident upon a boundary of some other shape (coordinate system). There exists, however, a class of mathematical relationships called wave transformations that circumvent this difficulty in many cases by allowing one to study the fields scattered by the various bodies, all referred to a common origin. Accordingly, to fulfil orthogonality in the current problem, we need to express the cylindrical wave functions of the (r_j, θ_j) coordinate system in terms of cylindrical wave functions of the (r_i, θ_i) coordinate system by application of the classical form of the translational addition theorem for cylindrical coordinates (IVANOV [18]):

$$H_n(kr_j)\, e^{\mathrm{i}\,n\theta_j} = \sum_{m=-\infty}^{\infty} H_{n-m}(kl_{ji}) J_m(kr_i)\, e^{\mathrm{i}\,(n-m)\theta_{ji} + \mathrm{i}\,m\theta_i}, \tag{15}$$

where $l_{ji}\, (i, j = 1, 2, 3,\ i \neq j)$ is the distance between the centers of the j-th and i-th coordinate systems, θ_{ji} is the angle between the O_jO_i line and the x_j axis.

At this point, we can utilize the addition theorem (15) in (11) through (13) to obtain the total acoustic field potentials with respect to the coordinate system of the i-th cylinder as

$$
\begin{aligned}
\phi_M(r_i,\theta_i) &= \phi_s(r_i,\theta_i) + \varphi^{(i)}_{cM}(r_i,\theta_i) + \varphi^{(i)}_{tM}(r_i,\theta_i) + \varphi^{(j)}_{cM}(r_i,\theta_i) + \varphi^{(j)}_{tM}(r_i,\theta_i) \\
&= \sum_{n=-\infty}^{\infty} \left[\lambda^{(i)}_n J_n(k_{cM} r_i) + a^{(i)}_n(\omega)\, H_n(k_{cM} r_i) + b^{(i)}_n(\omega)\, H_n(k_{tM} r_i)\right] e^{\mathrm{i}\,n\,\theta_i} \\
&+ \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} a^{(j)}_m(\omega)\, H_{m-n}(k_{cM} l_{ji})\, J_n(k_{cM} r_i)\, e^{\mathrm{i}(m-n)\theta_{ji}} e^{\mathrm{i}\,n\,\theta_i} \\
&+ \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} b^{(j)}_m(\omega)\, H_{m-n}(k_{tM} l_{ji})\, J_n(k_{tM} r_i)\, e^{\mathrm{i}(m-n)\theta_{ji}} e^{\mathrm{i}\,n\,\theta_i}, \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
\psi_M(r_i,\theta_i) &= \psi^{(i)}_M(r_i,\theta_i) + \psi^{(j)}_M(r_i,\theta_i) = \sum_{n=-\infty}^{\infty} c^{(i)}_n(\omega)\, H_n(k_{sM} r_i) e^{\mathrm{i}n\theta_i} \\
&+ \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} c^{(j)}_m(\omega)\, H_{m-n}(k_{sM} l_{ji})\, J_n(k_{sM} r_i)\, e^{\mathrm{i}(m-n)\theta_{ji}} e^{\mathrm{i}n\theta_i}. \qquad (17)
\end{aligned}
$$

where $i, j = 1, 2\,(i \neq j)$ and $\lambda^{(i)}_n = \phi_0\, H_{-n}(k_{cM} l_{3i}) e^{-\mathrm{i}\,n\,\theta_{3i}}$. A subsequent application of the expansions (9), (10), (16) and (17) in the boundary conditions (14), leads to a linear systems of equations:

$$
\begin{aligned}
&T^{(1)}_1(k_{cM}a_i)\, a^{(i)}_n + T^{(1)}_1(k_{tM}a_i)\, b^{(i)}_n + T^{(1)}_2(k_{sM}a_i)\, c^{(i)}_n - T^{(2)}_1(k_{cB}a_i)\, d^{(i)}_n \\
&\quad - T^{(2)}_1(k_{tB}a_i)\, e^{(i)}_n - T^{(2)}_2(k_{sB}a_i)\, f^{(i)}_n + T^{(2)}_1(k_{cM}a_i) A^{(j)}_{mn} \\
&\quad + T^{(2)}_1(k_{tM}a_i) B^{(j)}_{mn}, + T^{(2)}_2(k_{sM}a_i) C^{(j)}_{mn} = -\lambda^{(i)}_n\, T^{(2)}_1(k_{cM}a_i), \qquad (18)
\end{aligned}
$$

$$
\begin{aligned}
&T^{(1)}_2(k_{cM}a_i)\, a^{(i)}_n + T^{(1)}_2(k_{tM}a_i)\, b^{(i)}_n - T^{(1)}_1(k_{sM}a_i)\, c^{(i)}_n - T^{(2)}_2(k_{cB}a_i)\, d^{(i)}_n \\
&\quad - T^{(2)}_2(k_{tB}a_i)\, e^{(i)}_n + T^{(2)}_1(k_{sB}a_i)\, f^{(i)}_n + T^{(2)}_2(k_{cM}a_i) A^{(j)}_{mn} \\
&\quad + T^{(2)}_2(k_{tM}a_i) B^{(j)}_{mn} - T^{(2)}_1(k_{sM}a_i) C^{(j)}_{mn} = -\lambda^{(i)}_n\, T^{(2)}_2(k_{cM}a_i), \qquad (19)
\end{aligned}
$$

$$
\begin{aligned}
&T^{(1)}_3(k_{cM}a_i)\, a^{(i)}_n + T^{(1)}_3(k_{tM}a_i)\, b^{(i)}_n + T^{(1)}_4(k_{sM}a_i) c^{(i)}_n - T^{(2)}_3(k_{cB}a_i)\, d^{(i)}_n \\
&\quad - T^{(2)}_3(k_{tB}a_i)\, e^{(i)}_n - T^{(2)}_4(k_{sB}a_i) f^{(i)}_n + T^{(2)}_3(k_{cM}a_i)\, A^{(j)}_{mn} \\
&\quad + T^{(2)}_3(k_{tM}a_i)\, B^{(j)}_{mn} + T^{(2)}_4(k_{sM}a_i) C^{(j)}_{mn} = -\lambda^{(i)}_n\, T^{(2)}_3(k_{cM}a_i), \qquad (20)
\end{aligned}
$$

$$
\begin{aligned}
&T^{(1)}_4(k_{cM}a_i) a^{(i)}_n + T^{(1)}_4(k_{tM}a_i) b^{(i)}_n - T^{(1)}_5(k_{sM}a_i) c^{(i)}_n - T^{(2)}_4(k_{cB}a_i) d^{(i)}_n \\
&\quad - T^{(2)}_4(k_{tB}a_i) e^{(i)}_n + T^{(2)}_5(k_{sB}a_i) f^{(i)}_n + T^{(2)}_4(k_{cM}a_i) A^{(j)}_{mn} \\
&\quad + T^{(2)}_4(k_{tM}a_i) B^{(j)}_{mn} - T^{(2)}_5(k_{sM}a_i) C^{(j)}_{mn} = -\lambda^{(i)}_n T^{(2)}_4(k_{cM}a_i), \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
T_6^{(1)}(k_{cM}a_i)a_n^{(i)} + T_7^{(1)}(k_{tM}a_i)b_n^{(i)} - T_6^{(2)}(k_{cB}a_i)d_n^{(i)} - T_7^{(2)}(k_{tB}a_i)e_n^{(i)} \\
+ T_6^{(2)}(k_{cM}a_i)A_{mn}^{(j)} + T_7^{(2)}(k_{tM}a_i)B_{mn}^{(j)} = -\lambda_n^{(i)}\, T_6^{(2)}(k_{cM}a_i),
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
T_8^{(1)}(k_{cM}a_i)a_n^{(i)} + T_9^{(1)}(k_{tM}a_i)b_n^{(i)} - T_8^{(2)}(k_{cB}a_i)d_n^{(i)} - T_9^{(2)}(k_{tB}a_i)e_n^{(i)} \\
+ T_8^{(2)}(k_{cM}a_i)A_{mn}^{(j)} + T_9^{(2)}(k_{tM}a_i)B_{mn}^{(j)} = -\lambda_n^{(i)}\, T_8^{(2)}(k_{cM}a_i),
\end{aligned}
\tag{23}
$$

where

$$
\begin{aligned}
A_{mn}^{(j)}(\omega) &= \sum_{m=-\infty}^{\infty} a_m^{(j)}(\omega)\, H_{m-n}(k_{cM}l_{ji})e^{\mathrm{i}(m-n)\theta_{ji}}, \\
B_{mn}^{(j)}(\omega) &= \sum_{m=-\infty}^{\infty} b_m^{(j)}(\omega)\, H_{m-n}(k_{tM}l_{ji})e^{\mathrm{i}(m-n)\theta_{ji}}, \\
C_{mn}^{(j)}(\omega) &= \sum_{m=-\infty}^{\infty} c_m^{(j)}(\omega)\, H_{m-n}(k_{sM}l_{ji})e^{\mathrm{i}(m-n)\theta_{ji}},
\end{aligned}
\tag{24}
$$

and

$$
\begin{aligned}
&T_1^{(l)}(kr) = -kZ_n^{(l)\prime}(kr), \qquad T_2^{(l)}(kr) = (\mathrm{i}n/r)Z_n^{(l)}(kr), \\
&T_3^{(l)}(kr) = (\mathrm{i}\omega\,\rho_l - 2\mu_l k^2)Z_n^{(l)}(kr) - 2\mu_l k^2 Z_n^{(l)\prime\prime}(kr), \\
&T_4^{(l)}(kr) = (2\mu_l \mathrm{i}n/r^2)\left[krZ_n^{(l)\prime}(kr) - Z_n^{(l)}(kr)\right], \\
&T_5^{(l)}(kr) = (\mu_l/r^2)\left[-n^2 Z_n^{(l)}(kr) + krZ_n^{(l)\prime}(kr) - k^2r^2 Z_n^{(l)\prime\prime}(kr)\right], \\
&T_6^{(l)}(kr) = b_c^{(l)} Z_n^{(l)}(kr), \qquad T_7^{(l)}(kr) = b_t^{(l)} Z_n^{(l)}(kr), \\
&T_8^{(l)}(kr) = -b_c^{(l)}\kappa_l k Z_n^{(l)\prime}(kr), \qquad T_9^{(l)}(kr) = -b_t^{(l)}\kappa_l k Z_n^{(l)\prime}(kr),
\end{aligned}
\tag{25}
$$

in which $Z_n^{(1)}(kr) = H_n(kr)$, $Z_n^{(2)}(kr) = J_n(kr)$, and the primes denote differentiation with respect to the argument, while $l = 1$ ($l = 2$) refers to the parameters associated with the surrounding ambient medium (fluid cylinders). This completes the necessary background required for the exact acoustic analysis of the problem. Next, we consider some numerical examples.

3. Numerical results

Realizing the crowd of parameters and the relatively intense computations involved here, no attempt is made to exhaustively evaluate the effect of varying each of them. The intent of the collection of data presented here is merely to illustrate the kinds of results to be expected from some representative choices of values for these parameters. From these data certain trends are noted and general conclusions are made about the relative importance of specific parameters. Accordingly, we confine our attention to a particular model. First, to better examine the thermoviscous effects, the fluid surrounding the two cylinders is assumed to be glycerine at atmospheric pressure and 300 Kelvin. The fluid cylinders are supposed to be identical ($a_1 = a_2 = a$ =0.001 m), positioned alongside each other (i.e., $\theta_{12} = 0$ and $\theta_{21} = \pi$ in Fig. 1), and made of olive oil, with their physical properties as given in Table 1 (BABICK *et al.* [5]).

Table 1. The input parameter values used in calculations.

Parameter	Glycerine	Olive oil	Water	FC-75
μ (kg/m.s)	0.95	0.084	0.000894	0.00079
μ_b (kg/m.s)	0.95	0.084	0.00250	0.00079
c (m/s)	1910	1440	1497	613.77
κ (N/s.K)	0.286	0.19	0.5950	0.06398
η (K^{-1})	6.1×10^{-4}	7.2×10^{-8}	2.57×10^{-4}	1.799×10^{-3}
C_p (J/kg.K)	2427	2000	4179	1044
C_v (J/kg.K)	2427	2000	4138	1038
ρ (kg/m^3)	1250	900	1000	1730

A FORTRAN code was constructed for treating boundary conditions and to calculate the unknown scattering coefficients and the relevant acoustic field quantities as functions of the nondimensional frequency $ka = \mathrm{Re}\{k_{cM}a\}$, source position $(r_s(= l_{31}), \theta_s(= \theta_{31}))$, and the center-to-center distance $d(= l_{12} = l_{21})$ (see Fig. 1). Accurate computations of cylindrical Bessel functions of complex arguments and their derivatives were accomplished by utilizing the module CH12N described in the monograph of ZHANG and JIN [45]. The precision of the values calculated were checked against Maple specialized math functions “HankelH1” and “BesselJ”, and also the printed tabulations in the handbook of ABRAMOVITZ and STEGUN [1]. Matrix inversions were carried out using the subroutine ZGEFA from the portable numerical software library LINPAK (DONGARRA *et al.* [12]). The computations were performed on a Pentium IV personal computer with truncation constants of $n_{\max} = m_{\max} = 35$ to assure convergence in the high frequency range, and also in the case of closeness of the cylinders. In addition to avoid numerical overflow/underflow problems at high wave numbers (i.e., for large complex arguments), an extension of the range of the floating-point numbers was pursued. This would enable us to make the computation using a numeric precision beyond the single or double precisions ordinarily provided in hardware. Accordingly, a very powerful multi-precision FORTRAN software package MPFUN developed by David H. BAILEY [6] from NASA was employed to compute mathematical functions on floating point numbers of arbitrarily high precision.

The most relevant acoustic field quantity is the scattered pressure amplitude. Using (8) and keeping (16) in mind, the scattered pressure amplitude with respect to the first coordinate system may be written as

$$|p_{\mathrm{scat}}(r_1,\theta_1,\omega)| = \left| -\mathrm{i}\omega\,\rho_1\overline{\phi}_M(r_1,\theta_1,\omega) + \left(\mu_{b1}+\frac{4}{3}\mu_1\right)\left[k_{cM}^2\phi_{cM}(r_1,\theta_1,\omega) + k_{tM}^2\phi_{tM}(r_1,\theta_1,\omega)\right]\right|, \tag{26}$$

where

$$\begin{aligned}\phi_{cM}(r_1,\theta_1,\omega) &= \varphi_{cM}^{(1)}(r_1,\theta_1,\omega)+\varphi_{cM}^{(2)}(r_1,\theta_1,\omega),\\ \phi_{tM}(r_1,\theta_1,\omega) &= \varphi_{tM}^{(1)}(r_1,\theta_1,\omega)+\varphi_{tM}^{(2)}(r_1,\theta_1,\omega),\\ \overline{\phi}_M(r_1,\theta_1,\omega) &= \varphi_{cM}(r_1,\theta_1,\omega)+\varphi_{tM}(r_1,\theta_1,\omega).\end{aligned} \tag{27}$$

The two most important source angles are $\theta_s = 0$ (end-on incidence) and $\theta_s = \pi/2$ (broadside incidence), as they help best to expose the physics of the problem. Figure 2 displays the variation of the normalized backscattered pressure magnitude at a far-field point ($|p_{\mathrm{scat}}(r_1 = 100a, \theta_1 = \pi + \theta_s, \omega)|/\rho_1 c_1^2$) with ka for a unit amplitude line source ($\phi_0 = 1$) at selected source positions ($r_s = 2a,\ 5a;\ \theta_s = 0,\ \pi/2$) and distance parameters ($d/a = 2, 4, 10$). We have also generated the backscattered amplitude curves for two identical ideal fluid cylinders by using an independently developed FORTRAN code. The main observations are as follows. At very low incident wave frequencies ($ka \ll 1$), all lines roughly coincide and thus both the thermoviscous and source proximity effects are very insignificant regardless of the center-to-center distance of the fluid cylinders. Thermoviscosity has a depressing effect on the backscattered pressure amplitudes mainly at intermediate and high frequencies. In particular, we see a relatively rapid decrease in the overall pressure magnitudes for the thermoviscous cylinders in comparison with those of the ideal fluid cylinders as the frequency increases. Furthermore, we generally observe a somewhat larger thermoviscous effects in the end-on incidence ($\theta_s = 0$) case in comparison with the broadside incidence ($\theta_s = \pi/2$) situation, especially at higher frequencies. As the distance parameter (d/a) increases, the overall pressure magnitudes appear to decrease, more rapidly in the end-on incidence ($\theta_s = 0$) situation. Also, the backscattered pressure peaks become more densely packed as the separation grows. This behaviour is principally caused by the interference of the fields scattered by the two cylinders. More exactly, it is associated with fact that the phase difference between the scattering cylinders oscillates with increasing frequency as the separation between the cylinders grows (HASHEMINEJAD and BADSAR [15]). This becomes clearer if we note that the oscillating nature of the phase factors $H_{n-m}(k_{cM}d)$, $H_{n-m}(k_{tM}d)$ and $H_{n-m}(k_{sM}d)$ present in the final equations imply different arrival times for these waves, which form an oscillation pattern in the latter terms at large arguments. Also, the increasing of the line source proximity, r_s, seems to have a larger effect on the backscattered pressure magnitudes in the broadside incidence ($\theta_s = \pi/2$) situation, especially for small distance parameters.

To best observe the multiple scattering effects, we favourably suppose the fluid cylinders to be made of 3M “Fluorinert” chemical FC-75 (see http://mmm.com)

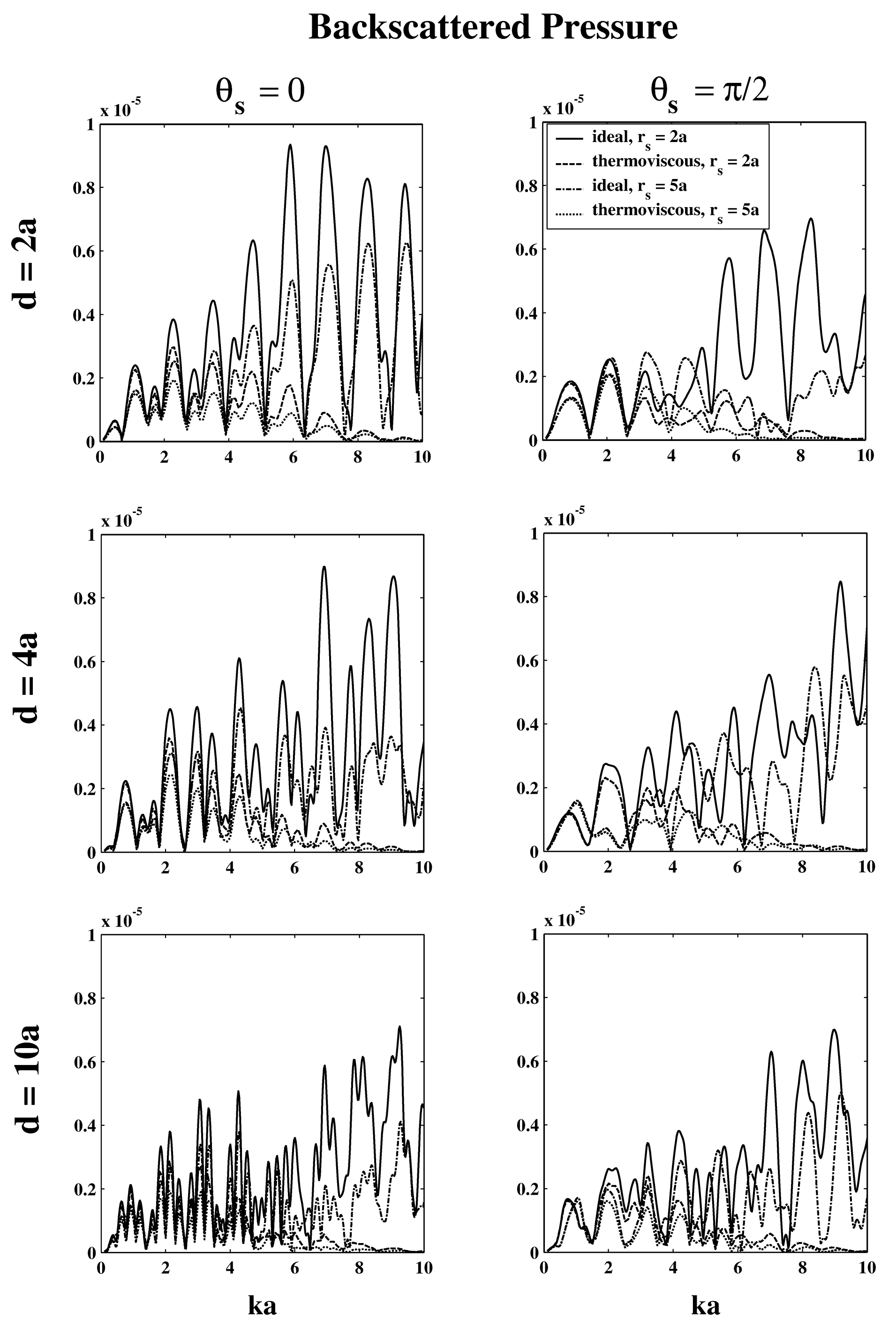

Fig. 2. Plot of the amplitude of the back-scattered form function versus nondimensional frequency for end-on/broadside incidence upon the fluid cylinders at selected distance parameters.

immersed in water, with their physical properties summarized in Table 1 (note the very low values of the thermoviscous parameters). Figures 3 through 5 examine the multiple scattering effects on the variation of the normalized backscattered pressure amplitude with the nondimensional frequency at selected source positions ($r_s = 2a,\ 10a, 50a; \theta_s = 0,\ \pi/2$) and distance parameters ($d/a = 2, 4, 10$). In order to best understand these effects, we show (in dashed lines) the backscattered pressure plots for a pair of "non-interacting" fluid cylinders. Here, the interaction effects are wholly omitted by making separate computations for the scattered field associated with each individual cylinder and subsequently adding the results to obtain the modified (non-interacting) backscattered pressure amplitude, i.e., we have essentially ignored all cylindrical wave transformations introduced by the application of the translational addition theorem in the entire formulation. We further display (in dotted lines) the corresponding plots of twice the backscattered pressure amplitude for a single fluid cylinder submerged in an infinite unbounded medium. A careful examination of the figures leads to the following important observations. The broadside incidence ($\theta_s = \pi/2$) case is studied first with the aim of best isolating the two cylinders' interaction effects. The difference between the full interaction results (solid line) from twice that of a single cylinder (dotted line) reveals the important features of this interaction. In the case of two nearly touching fluid cylinders ($d/a = 2$), the solid line exhibits noticeable oscillations about the dashed line and thus the multiple-scattering interactions between the two cylinders are significant. Also, when the line source is positioned relatively far away from the fluid cylinders ($r_s = 50a$), there is a nearly zero phase difference for broadside incidence (i.e., the backscattering echoes reach the far-field observer nearly at the same time), the backscattered pressure curves associated with the non-interacting cylinders (dashed line) almost perfectly coincide with twice that of a single cylinder (dotted line) at this separation distance (see Fig. 5). As the separation grows to $d/a = 4$, the dashed and the solid lines begin to overlap roughly at most frequencies which implies a decrease of multiple scattering effects for larger separations. When the separation grows to $d/a = 10$, there is a very small trace of multiple-scattering effects, since the dashed line follow very closely the solid line in the frequency range of interest, especially for the line source being located relatively far away ($r_s = 50a$). This further points to the insignificant interaction effects at broadside incidence for relatively large separations. The end-fire incidence ($\theta_s = 0$) case reveals more interesting features of the multiple-scattering interactions. For two nearly touching cylinders ($d/a = 2$), the relatively large differences observed between the solid line and the dashed line indicate the importance of multiple scattering effects for end-on incidence at small separations. When the separation parameter is increased to $d/a = 10$, multiple-scattering interactions decrease drastically at low and intermediate frequencies, as the solid line begins to more closely follow the dashed line at these frequencies. Also, the dotted line (i.e., twice the value of form function for a single cylinder) seems to become roughly the envelope of the peaks of the backscattered pressure (solid lines), especially when the line source is located relatively far away ($r_s = 50a$).

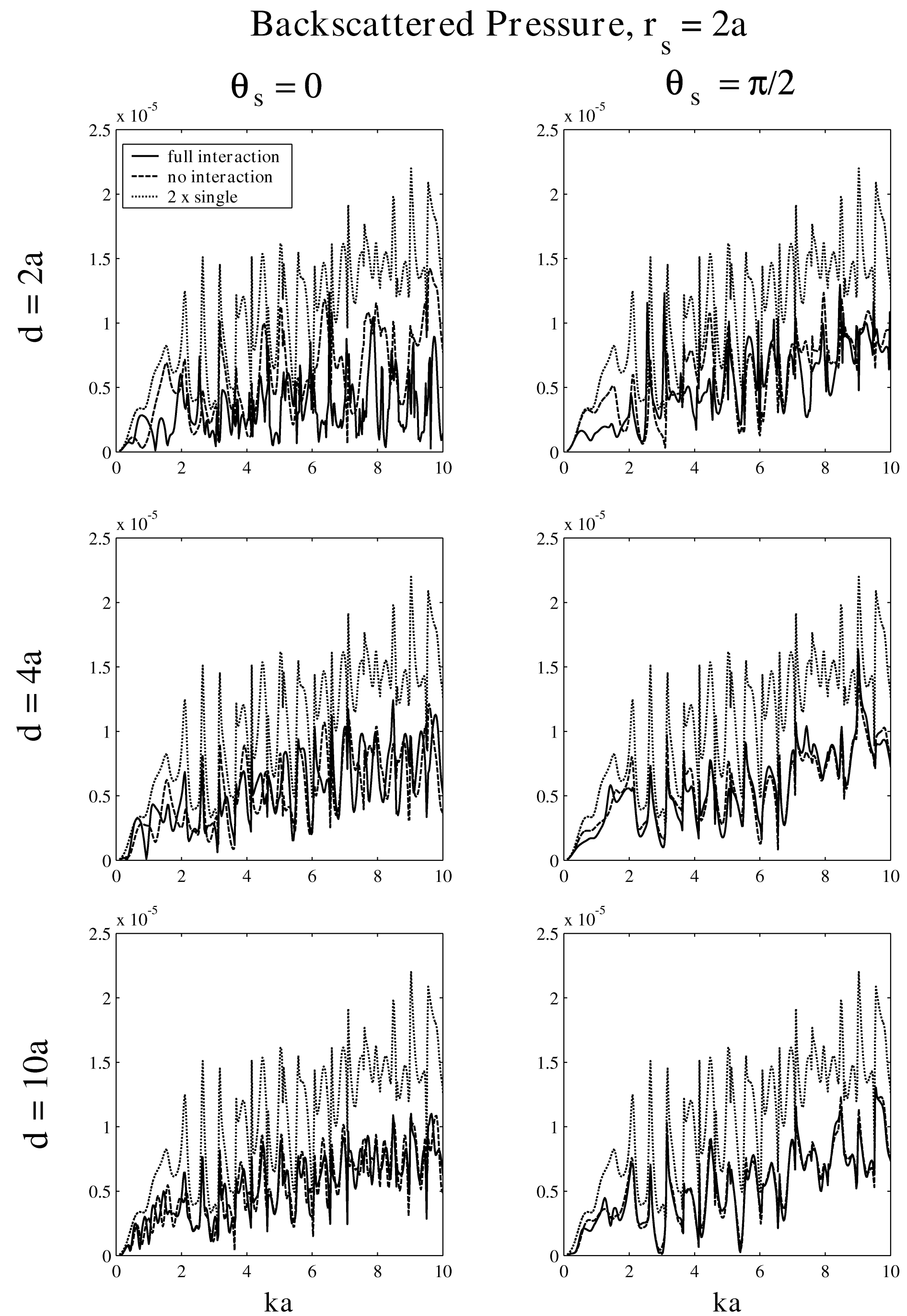

Fig. 3. Plot of the amplitude of the back-scattered form function versus nondimensional frequency for end-on/broadside incidence upon the thermoviscous cylinders at selected source position ($r_s = 2a$) and distance parameters.

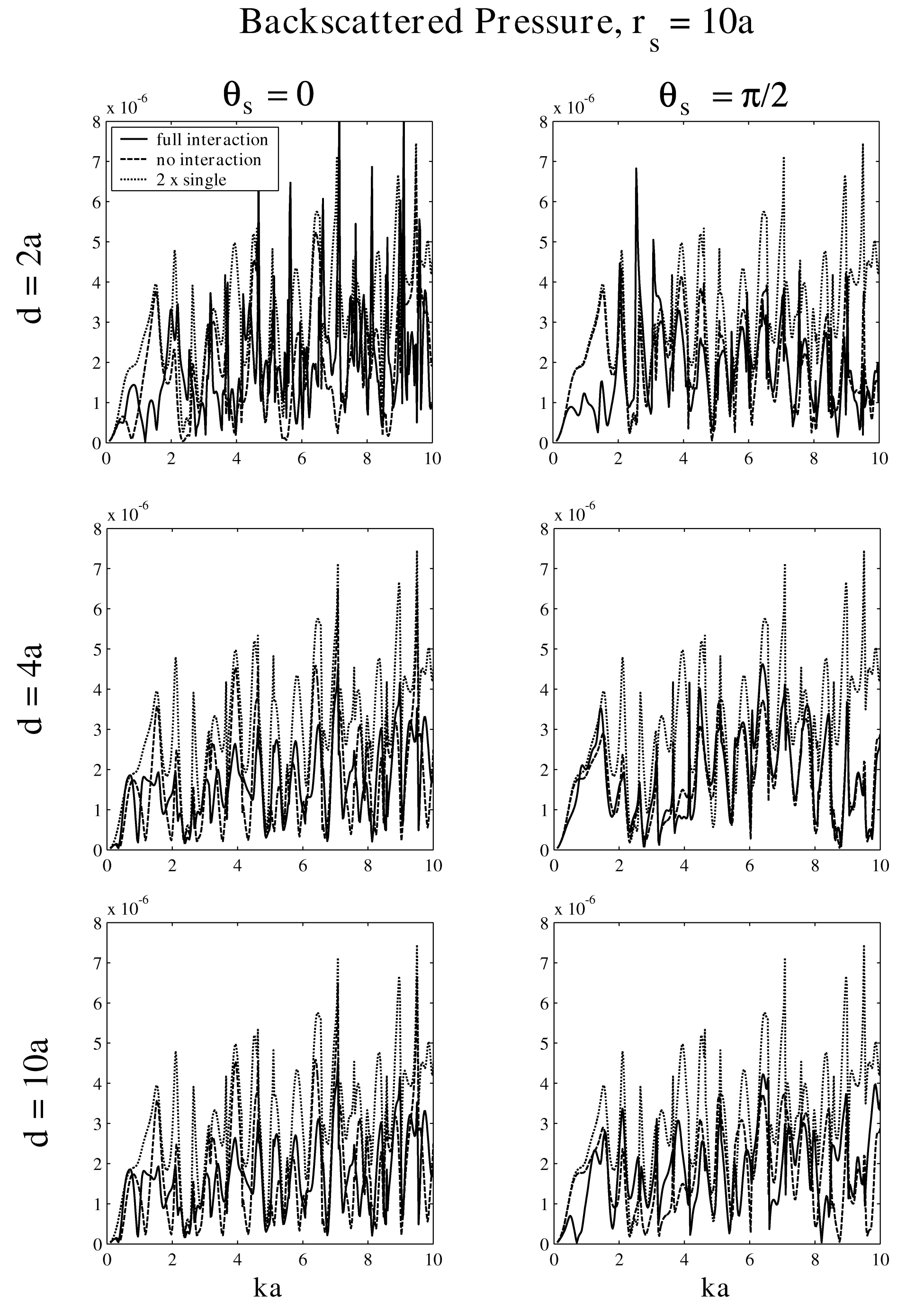

Fig. 4. Plot of the amplitude of the back-scattered form function versus nondimensional frequency for end-on/broadside incidence upon the thermoviscous cylinders at selected source position ($r_s = 10a$) and distance parameters.

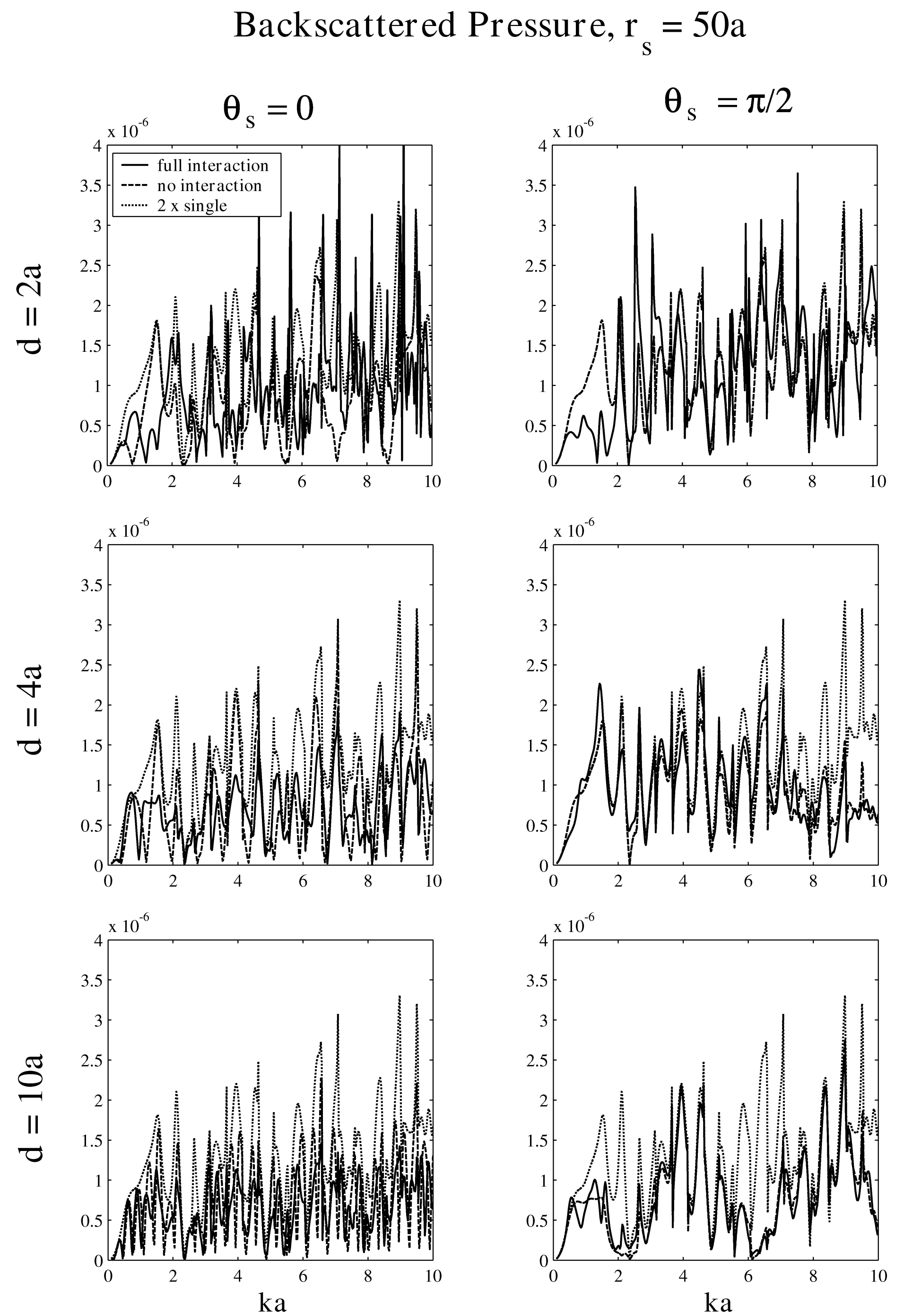

Fig. 5. Plot of the amplitude of the back-scattered form function versus nondimensional frequency for end-on/broadside incidence upon the thermoviscous cylinders at selected source position ($r_s = 50a$) and distance parameters.

Finally, in order to check the overall validity of the calculations, we replaced the line source in our formulation with an incident plane wave of the general form $\phi_{\mathrm{inc}} = \sum_{n=-\infty}^{\infty} \phi_0 \, \mathrm{i}^n J_n(k_{cM} r_1) \, e^{\mathrm{i}n(\theta_1 - \theta_s)}$, and computed the total scattering cross-section (see equation 34 in the work by ROUMELIOTIS *et al.* [34]) versus angle of plane wave incidence (θ_s) at selected radii ratios for a pair of nearly inviscid and relatively large fluid cylinders immersed in an infinite fluid medium by setting $\mu \to 0$ in our general FORTRAN code. The numerical results, as shown in Fig. 6, accurately reproduce the curves displayed in Fig. 4 of ROUMELIOTIS *et al.* [34] paper.

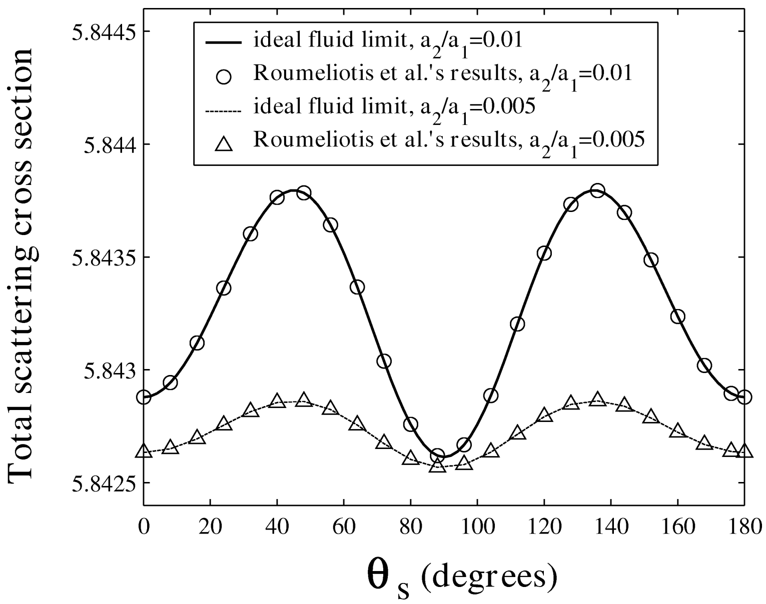

Fig. 6. Total scattering cross-section versus angle of wave incidence for a pair of nearly inviscid fluid cylinders at selected radii ratios.

4. Conclusions

This study solves the very important problem of acoustic multiple scattering in two dimensions. It treats the interaction of a nearby acoustic line source with a pair of parallel viscous thermally conducting fluid cylinders submerged in a boundless lossy fluid medium. The solution is based on the linearized coupled equations of motion for a dynamic description of thermoviscous fluid behaviour and the translational addition theorem for cylindrical wave functions. The backscattered pressure amplitudes are plotted for end-on/broadside incidence at selected frequencies and separations. At very low incident wave frequencies ($ka \ll 1$), both the thermoviscous and source proximity effects are very insignificant, regardless of the closeness of the fluid cylinders. Thermoviscosity has a remarkable depressing effect on the backscattered pressure amplitudes mainly at intermediate and high frequencies. In particular, a relatively rapid decrease in the overall backscattered pressure magnitudes for the thermoviscous cylinders in comparison with those of the ideal fluid cylinders at these frequencies is observed, especially in the end-on incidence situation. Also, the multiple-scattering interactions are found to be very significant for end-on incidence upon two closely positioned fluid cylinders. In the broadside incidence case, on the other hand, there is a very small trace of multiple-scattering effects, especially as the separation of the cylinders increases. The presented work demonstrates the call for consideration of multiple scattering interactions as well as the thermoviscous loss effects in problems involving multiple fluid cylinders suspended in an absorptive fluid medium. It is of practical interest in acoustic analysis and characterization of cylindrical emulsions, periodic liquid composites, and passive acoustic stabilization of long liquid bridges, small diffusion flames and hot cylindrical fluid objects under microgravity conditions.

References

[1] ABRAMOWITZ, M., STEGUN, I.A., *Handbook of mathematical functions*, National Bureau of Standards, Washington 1965.

[2] ALEMAR, J.D., DELSANTO, P.P., ROSARIO, E., NAGEL, A., UBERAL, H., *Spectral analysis of the scattering of acoustic waves from a fluid cylinder I: Denser fluid loading*, Acustica, **61,** 1–6 (1986).

[3] ALEMAR, J.D., DELSANTO, P.P., ROSARIO, E., NAGEL, A., UBERAL, H., *Spectral analysis of the scattering of acoustic waves from a fluid cylinder, II: Denser fluid inside*, Acustica, **61,** 7–13 (1986).

[4] ALEMAR, J.D., DELSANTO, P.P., ROSARIO, E., NAGEL, A., UBERAL, H., *Spectral analysis of the scattering of acoustic waves from a fluid cylinder. III: solution of the inverse scattering problem*, Acustica, **61,** 14–20 (1986).

[5] BABICK, F., HINZE, F., RIPPERGER, S., *Dependence of ultrasonic attenuation on the material properties*, Colloids Surf., **172,** 33–46 (2000).

[6] BAILEY, D.H., *A Fortran-90 based multiprecision system*, ACM Trans. Math. Soft., **21,** 379–387 (1995).

[7] BALANIS, C.A., *Advanced engineering electromagnetics*, Wiley, New York 1989.

[8] BELTMAN, W.M., *Viscothermal wave propagation including acousto-elastic interaction*, Enschede, PhD-thesis, chapter 2, 1998. [www.dinkel.utwente.nl/webdocs/wb/1/t000000b.pdf].

[9] BOAG, A., LEVIATAN, Y., BOAG, A., *Analysis of acoustic scattering from fluid cylinders using a multifilament source model*, J. Acoust. Soc. Am., **83,** 1–8 (1988).

[10] CHANDRA, K., THOMPSON, C., *Improved perturbation method for scattering from a fluid cylinder*, J. Acoust. Soc. Am., **92,** 1047–1055 (1992).

[11] DECANINI, Y., *Algebraic aspects of multiple scattering by two parallel cylinders: Classification and physical interpretation of scattering resonances*, J. Sound Vib., **221,** 785–804 (1999).

[12] DONGARRA, J., BUNCH, J., MOLER, C. STEWART, G.W., LINPACK user's guide, SIAM, Philadelphia, PA, 1979.

[13] GINSBERG, J.H, *On the effect of viscosity in scattering from partially coated infinite cylinders*, J. Acoust. Soc. Am., **112,** 46–54 (2002).

[14] HARTMAN, B., LEE, G.F., LEE, J.D., *Loss factor height and width limits for polymer relaxations*, J. Acoust. Soc. Am., **95,** 226–223 (1994).

[15] HASHEMINEJAD, M., BADSAR, A., *Acoustic scattering by a pair of poroelastic spheres*, Quart. J. Mech. Appl. Math., **57,** 95–113 (2004).

[16] HASHEMINEJAD, M., GEERS, T.L., *Modal impedance for two spheres in a thermoviscous fluid*, J. Acoust. Soc. Am., **94,** 2205–2214 (1993).

[17] HASHEMINEJAD, S.M., SAFARI, N., *Acoustic scattering from viscoelastically coated spheres and cylinders in viscous fluids*, J. Sound Vib., **280,** 101–125 (2005).

[18] IVANOV, Y.A., *Diffraction of electromagnetic waves on two bodies*, National Aeronautics and Space Administration, Washington, DC, 1968.

[19] KOZHIN, V.N., *Sound propagation in a viscous medium containing cylindrical filaments*, Akust. Zh., **16,** 542–547 (1971).

[20] KUBENKO, V.D., *Diffraction of acoustic waves by two nonparallel circular cylinders*, Sov. Appl. Mech., **25,** 34–41 (1989).

[21] LEE, J.P., SONG, J.H., CHOI, M.S., *The effects of material attenuation on acoustic resonance scattering from cylindrical tubes*, Ultrasonics, **34,** 737–745 (1996).

[22] LIN, W.H., RAPITS, A.C., *Thermoviscous effects on acoustic scattering by thermoelastic solid cylinders and spheres*, J. Acoust. Soc. Am., **74,** 1542–1554 (1983).

[23] LIN, W.H., RAPTIS, A.C., *Sound scattering by a group of oscillatory cylinders*, J. Acoust. Soc. Am., **77**, 15–28 (1985).

[24] LIN, W.H., RAPTIS, A.C., *Sound scattering from a thin rod in a viscous medium*, J. Acoust. Soc. Am., **79,** 1693–1700 (1986).

[25] MARR-LYON, M.J., THIESSEN, D.B., MARSTON, P.L., *Passive stabilization of capillary bridges in air with acoustic radiation pressure*, Phys. Rev. Lett., **86,** 2293–2296 (2001).

[26] MARSTON, P.L., *Capillary bridge stability in an acoustic standing wave: Linearized analysis of passive stabilization with radiation pressure*, J. Acoust. Soc. Am., **97,** 3377 (1995).

[27] MARSTON, P.L., *Born approximation for scattering by evanescent waves: Comparison with exact scattering by an infinite fluid cylinder*, J. Acoust. Soc. Am., **115,** 2473 (2004).

[28] MITRI, F.G., *Acoustic backscattering form function of absorbing cylinder targets (L)*, J. Acoust. Soc. Am., **115,** 1411–1413 (2004).

[29] MITRI, F.G., *Theoretical calculation of the acoustic radiation force acting on elastic and viscoelastic cylinders placed in a plane standing or quasistanding wave field*, Eur. Phys. J., **B44,** 71–78 (2005).

[30] MITRI, F.G., *Radiation Force acting on an absorbing cylinder placed in an incident plane progressive acoustic field*, J. Sound Vib., **284,** 494–502 (2005).

[31] MITRI, F.G., *Frequency dependence of the acoustic radiation force acting on absorbing cylindrical shells*, Ultrasonics, **43,** 271–277 (2005).

[32] MORSE, P., INGARD, K., *Theoretical acoustics*, McGraw-Hill, New York 1968.

[33] PRINCEN, H.M., *Highly concentrated emulsions. I. Cylindrical systems*, J. Colloid Interface Sci., **71,** 55–66 (1979).

[34] ROUMELIOTIS, J.A., ZIOTOPOULOS, A.P., KOKKORAKIS, G.C., *Acoustic scattering by a circular cylinder parallel with another of small radius*, J. Acoust. Soc. Am., **109,** 870–877 (2001).

[35] ROUSSELOT, J.L., *Field diffracted by a fluid cylinder. Comparison between the geometrical theory of diffraction and a model solution*, Acustica, **80,** 14–20 (1994).

[36] SCHARSTEIN, R.W., *Acoustic scattering from two parallel soft cylinders*, Proc. IEEE conf., **2,** 534–537 (1992).

[37] SCOTTI, T., WIRGIN, A., *Reconstruction of the three mechanical material constants of a lossy fluid-like cylinder from low-frequency scattered acoustic fields*, Comptes Rendus Mec., **332,** 717–724 (2004).

[38] SHERER, S.E., *Scattering of sound from axisymetric sources by multiple circular cylinders*, J. Acoust. Soc. Am., **115,** 488–496 (2004).

[39] SKUDRZYK, E., *The Foundations of acoustics*, Springer-Verlag, New York 1971.

[40] TEMKIN, S., *Elements of acoustics*, Wiley, New York 1981.

[41] TWERSKY, V., *Multiple scattering of radiation by an arbitrary configuration of parallel cylinders*, J. Acoust. Soc. Am., **24,** 42–45 (1952).

[42] WEI, W., THIESSEN, D.B., MARSTON, P.L., *Acoustic radiation force on a compressible cylinder in a standing wave*, J. Acoust. Soc. Am., **116,** 201–208 (2004).

[43] WU, F., HOU, Z., LIU, Z., LIU, Y., *Acoustic band gaps in two-dimensional rectangular arrays of liquid cylinders*, Solid State Commun., **123,** 239–242 (2002).

[44] YOUNG, J.W., BERRAND, J.C., *Multiple scattering by two cylinders*, J. Acoust. Soc. Am., 1190–1193 (1975).

[45] ZHANG, S., JIN, J., *Computation of special functions*, Wiley, New York 1996.

[46] ZHUK, A.P. , *Action of an acoustic wave on a system of two parallel circular cylinders in an ideal fluid*, Sov. Appl. Mech., **27,** 101–106 (1991).

[47] ZHUK, A.P., *Study of the interaction of an acoustic wave in a viscous liquid with two cylinders placed in parallel*, Sov. Appl. Mech., **27,** 321–327 (1991).