APPLICATION OF GENETIC ALGORITHMS TO THE EVALUATION OF THE FUNDAMENTAL FREQUENCY OF STIFFENED PLATES

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In this paper, the analytical and numerical procedures for multi-stiffened plate free vibration analysis are presented. The plate and stiffeners are treated as an assembled plate and stiffening elements. The energy analysis leads to the nonlinear formulation of the non-dimensional frequency function formulated for a group of similar palates. The generalized optimization problem was solved by the use of the Nelder-Mead and genetic algorithms. The verification of the results obtained was performed on FEM models created for the same examples of the physical representation of the considered group of stiffened plates. This paper also aims the study of the influence of geometrical and material parameters of the stiffeners as well as the plate proportions on the fundamental frequency.

Key words: stiffened plates, energy analysis, fundamental frequency estimation.

1. Introduction

Changes in the rigidity of the plate elements generated by inserting a system of stiffeners has a slight influence on the mass of the whole system and a considerable impact on the natural frequency as well as on the magnitude and distribution of the vibration energy. The stiffeners can also be considered as an additional subsystem, which not only changes the rigidity of the plate but also stores and dissipates a part of the energy of vibration. The influence of the set of stiffeners on the rigidity of the stiffened plate and on the energy of static mode and vibration modes is particularly important within the range of low frequencies. The analysis of the dynamic behaviour of the structure in several of the first natural frequencies as well as the introduction of changes in their magnitude can result in a profitable structure modification. In consequence, a decrease in the amplitude, dynamic stresses and the energy of vibration can be obtained. The optimal design and choice of all the plate-and-stiffeners system properties stand for an additional goal.

Nondimensional properties [2] of the plate and stiffeners used in this analysis and the proposed algorithm of calculations make it possible to evaluate the impact of the geometrical and material parameters on the natural frequency of the group of related plates. On this basis it is possible to calculate parameters of any practical realization of a stiffened plate representative for a considered group. The analytical results of the influence of the chosen geometrical and material properties of the stiffeners set on the natural frequency were verified on models created by the use of the finite elements method and were compared to the data reproted in literature [1, 7–9].

2. Analysis of the influence of the stiffeners parameters on the plate vibrations

The energy analysis of a ribbed plate was carried out by considering the vibrations of the whole system treated as a combination of a homogeneous plate and a set of beams [1, 2, 7]. The natural frequency of a group of stiffened plates can be estimated by comparing the maximum kinetic energy with the maximum potential energy of a system consisting of stiffeners and a homogenous plate.

The potential energy of a stiffened plate related to the field of stresses and strains can be computed after a separate analysis of the energy accumulated by the homogenous plate and a set of stiffeners. The strain energy of a homogenous plate is described by the relation:

$$U_p = \frac{1}{2} \iiint\limits_V \sigma_{ij}^p \varepsilon_{ij}^p \, \mathrm{d}V, \tag{1}$$

where σ_{ij} – components of stresses in the plate, ε_{ij} – components of strains.

The x-wise stiffeners overall strain energy consists of its energy of torsion, bending and longitudinal strain components and is described by the formula:

$$U_z^{xi} = \frac{E_z}{2} \iiint\limits_V \left(\varepsilon_x^{xi}\right)^2 \mathrm{d}V + \frac{GJ^{xi}a}{2} \int\limits_0^1 \left(\frac{\partial \theta}{\partial \zeta}\right)^2 \mathrm{d}\zeta, \tag{2}$$

where E – Young's moduli, G – Kirchoff's moduli, J – torsional rigidity of the stiffeners, ε – strain composed of three components due to the plate bending and stiffeners bending about the major and minor axes, ζ, η – non dimensional coordinates equal respectively x/a and y/b.

The strain energy of the y-wise stiffeners can be determined in the similar way. The kinetic energy of a stiffened plate can be computed as a sum of the kinetic energy of a homogenous plate and that one of a set of stiffeners. The kinetic energy of a homogenous plate can be estimated on the basis of the equation:

$$T_p = \frac{\rho_p a b h_p}{2} \int\limits_0^1 \int\limits_0^1 \left(\frac{\partial w}{\partial t}\right)^2 \mathrm{d}\zeta \mathrm{d}\eta, \tag{3}$$

where ρ – density of the plate material, a, b – dimensions of the plates.

The kinetic energy of the stiffeners consists of the kinetic energy of the out-of-plane and rotational motions. The kinetic energy of a typical stiffener along the x axis is described by the equation:

$$T_z^{xi} = \frac{m_z^{xi}}{2} A^{xi} a \int\limits_0^1 \left(\frac{\partial w}{\partial t}\right)^2 \mathrm{d}\zeta + \frac{m_z^{xi}}{2} I^{xi} a \int\limits_0^1 \left(\frac{\partial^2 w}{\partial t \partial \eta}\right)^2 \mathrm{d}\zeta, \tag{4}$$

where m_z – mass density of the stiffener, I – second moment of inertia about the major axis.

Introducing separations of the coordinates, the out-of-plane $W(\xi, \eta)$ and in-plane displacement functions $U(\xi, \eta)$, $V(\xi, \eta)$ are given by:

$$W(\zeta, \eta) = \sum_{i=1}^{M_1} \sum_{j=1}^{N_1} w_{ij} F_i(\zeta) G_j(\eta),$$

$$U(\zeta, \eta) = \sum_{m=1}^{M_2} \sum_{n=1}^{N_2} u_{mn} B_m(\zeta) D_n(\eta), \tag{5}$$

$$V(\zeta, \eta) = \sum_{r=1}^{M_3} \sum_{s=1}^{N_3} v_{rs} E_r(\zeta) H_s(\eta).$$

As the generalised functions F_i, G_j, B_m, D_n, E_r, H_s, arbitrary functions satisfying the boundary conditions at the plate edges can be selected. The coefficients w_{ij}, u_{mn}, v_{rs} determine the participation of basic functions in the displacements $W(\zeta, \eta)$, $U(\zeta, \eta)$, $V(\zeta, \eta)$.

For a thin rectangular plate simply supported along four edges, considering the admissible displacement functions [2] and the partial derivative $\partial W/\partial t$, the kinetic energy (3) of the plate obtained for the (i, j) mode, is given by:

$$T_p = \frac{\rho_p abh_p \omega_{ij}^2}{2} w_{ij}^2 \int\limits_0^1 \int\limits_0^1 \int\limits_0^{2\pi/\omega} \sin^2(i\pi\zeta) \sin^2(j\pi\eta) \cos^2(\omega_{ij}t) \, \mathrm{d}t \, \mathrm{d}\zeta \, \mathrm{d}\eta. \tag{6}$$

Then, introducing the partial derivatives $\partial W/\partial t$ and $\partial^2 W/\partial t \partial \eta$ in Eq. (4), the kinetic energy of stiffeners along the x axis for the (i, j) mode shape, is given by

$$T_z^{xk} = \frac{\rho_z^{xk} A_z^{xk} a}{2} \omega_{ij}^2 w_{ij}^2 \int\limits_0^1 \int\limits_0^{2\pi/\omega} \sin^2(i\pi\zeta) \sin^2(j\pi\eta_{kz}) \cos^2(\omega_{ij}t) \, \mathrm{d}t \, \mathrm{d}\zeta$$

$$+ \frac{\rho_z^{xk} I_z^{xk} a}{2} \omega_{ij}^2 w_{ij}^2 \int\limits_0^1 \int\limits_0^{2\pi/\omega} \sin^2(i\pi\zeta) \cos^2(j\pi\eta_{kz}) \cos^2(\omega_{ij}t) \, \mathrm{d}t \, \mathrm{d}\zeta. \tag{7}$$

The stiffening of a simple-supported plate results in the fact that the first vibration mode shape consists of many harmonic functions, the participation of which in the mode shape and the natural frequency of a stiffened plate is defined by the share coefficients. The fundamental frequency of the stiffened plate can be defined [2, 7] in the form of a nondimensional parameter Ω, which depends on the generalised material and geometrical proportions of the plate and a stiffeners set. It is a very convenient method of calculations, which makes it possible to compute on the basis of a single frequency parameter Ω the fundamental frequency Ω of any plate representative for the group of similar plates.

$$\Omega = \pi^2 \left\{ \frac{A + B}{C} \right\}^{1/2},\tag{8}$$

where

$$A = \beta^{-2} P_1 + 0.12 \left(\frac{b}{t} \right)^2 P_2 + \sum_{i=1}^{Ns^x} \left[0.2\gamma_1, \ \beta^{-2}\gamma_2, \ 0.4\beta^{-1}\gamma_3, \ 2\gamma_4 \right] \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix},$$

$$B = \sum_{i=1}^{Ns^y} \left[0.2\beta\gamma_5, \ 2\beta\gamma_6, \ 0.4\beta\gamma_7, \ 2\beta^{-1}\gamma_8 \right] \begin{bmatrix} I_5 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix},$$

$$C = \beta^{-2} \sum_{i=1}^{M_1} \sum_{i=1}^{M_1} w_{ij}^2 + 2 \sum_{i=1}^{Ns^x} \left[\beta^{-2} I_9 \, \delta_1^{xi} + \pi^2 \beta I_{10} \, \delta_2^{xi} \right]$$

$$+ 2 \sum_{i=1}^{Ns^y} \left[\beta^{-3} I_{11} \, \delta_1^{yi} + \pi^2 \beta^{-2} I_{12} \, \delta_2^{yi} \right],$$

where

$$\left\{ \begin{array}{c} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \\ \delta_1^{xi} \\ \delta_2^{xi} \end{array} \right\} = E_z \left\{ \begin{array}{c} A^{xi} b D^{-1} \\ I_y^{xi} (Db)^{-1} \\ A^{xi} e^{xi} D^{-1} \\ (1+v)(2J^{xi}(Db)^{-1}) \\ A^{xi}(E_z b t_p) \\ I_0^{xi}(E_z a^3 t_p) \end{array} \right\} \left\{ \begin{array}{c} \gamma_5 \\ \gamma_6 \\ \gamma_7 \\ \gamma_8 \\ \delta_1^{yi} \\ \delta_2^{yi} \end{array} \right\} = E_z \left\{ \begin{array}{c} A^{yi} b D^{-1} \\ I_x^{yi} (Db)^{-1} \\ A^{yi} e^{yi} D^{-1} \\ (1+v)(2J^{yi}(Db)^{-1}) \\ A^{yi}(E_z b t_p) \\ I_0^{yi}(E_z a^3 t_p) \end{array} \right\}$$

E – Young's moduli, ν – Poisson's coefficient, t_p – thickness of the plate, I_x, I_y – second moments of inertia of the cross-section of the stiffeners along the x and y axes, D – plate flexural rigidity, N^{xy} – number of x- and y-wise stiffeners, a, b – plate dimension. The functions P_1, P_2 and I_1–I_{12} were defined by O.K. BEDAIR [2].

3. Calculation procedures

The natural frequency parameter Ω given by the formula (1) can be expressed shortly as a function depending on the physical parameters (8) and the coefficients w_{ij}, u_{mn}, v_{rs} (5):

$$\Omega_{mn} = f\left\{\gamma_1, \gamma_2, \gamma_3\gamma_4, \delta_1^{xi}, \delta_2^{xi}\gamma_5, \gamma_6, \gamma_7, \gamma_8, \delta_1^{yi}, \delta_2^{yi}N_xN_yw_{ij}, u_{mn}, v_{rs}\right\}. \quad (9)$$

The object of the optimization process is to find the coefficients w_{ij}, u_{mn}, v_{rs} for the functions (8) that minimize the natural frequency parameter Ω. To this end the Nelder–Mead's optimization algorithm and the genetic algorithm [5] were used. In both cases, the calculations were carried out in the Matlab environment. In the case of the Nelder–Mead's algorithm, the standard "*fmins*" function ($[x] =$*fmins* (function, vector, options)) was used. As criterion of the termination of the calculations, the author assumed the decrease (below 0.001) in the distance between the symmetry centre of the simplex and its points. In some cases the constant number of iterations was established – always above 15000.

4. Genetic algorithm

Genetic algorithms are written on the basis of the imitation of nature, which is an inspiration in many fields of science. The idea of genetic algorithms consists in developing a group of potential solutions, which become more accurate by using such genetic operators as selection, cross-over, mutation and others. The better the solution, the more likely it is to remain, just as in nature, the best-adapted individuals survive. While looking for a maximum in a system, we should take into account that a better-adapted individual is that one with the highest value of the adaptation function (each individual undergoes a natural evaluation).

The minimum searched for (9) is a function of fourteen parameters, which are represented by chromosomes in the form of binary codes. The numbers of individuals, length of the binary code for each parameter, the time of calculations and other basic values of the GAs are chosen arbitrarily. Each chromosome is decoded into the space of variation of individual variables and afterwards an evaluation of the usefulness of a given solution is made. The genetic algorithm aims at the minimization of the value of the whole function for the searched vector of parameters. To this end, the method of ranking was used. The most important operator of the genetic algorithm is the operator of selection. In the considered problem, the universal method of selection was used. In this method, the whole parents' population in one turn of the roulette wheel is chosen. Another genetic operator is the cross-over of the individuals, which guarantees the exchange of information between them. The multipoint cross-over, where the number of points of the cross-breeding was equal to the length of the chromosome minus one, was used. The mutation operator in the classical form with a small coefficient of probability was applied. Additionally, the operator of longevity was employed; a few of the best-adapted

individuals from the parents' population were shifted to the descendants' population. While optimizing the fundamental frequency parameter of a stiffened plate, the genetic algorithm turned out to be twice faster than the Nelder–Mead's method. Moreover, the advantage of GA is that it leaves the local optima and searches for the global optimum. Other advantages of the genetic algorithms [5] arise from the features that distinguish them from traditional optimizing methods: they do not process the parameters of the problem directly but use their coded form, they start searching from a group of points (not from a single point), they use only the goal function and not the derivatives or other auxiliary information, they use probabilistic instead of deterministic rules of choice. The simple diagram of the used genetic algorithm is shown in Fig. 1.
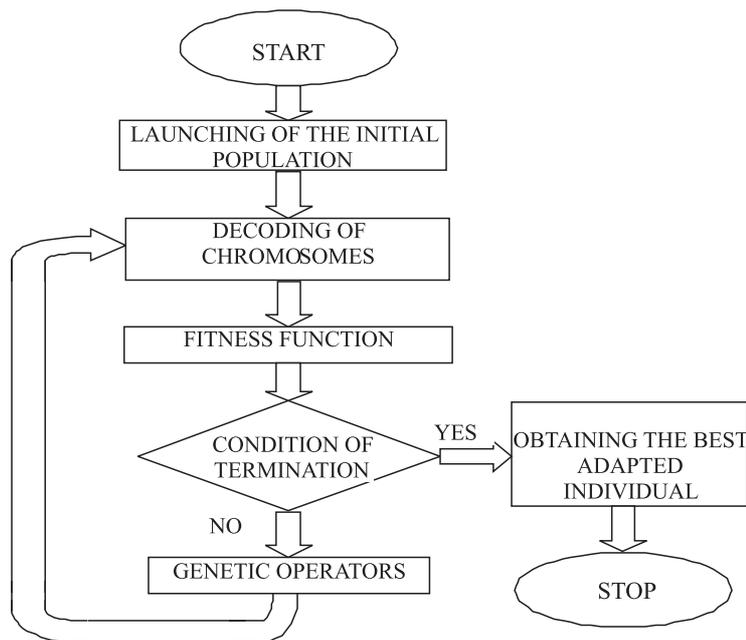
Fig. 1. Simplified block diagram of the applied genetic algorithm.

The first step of evaluation of the accuracy of the mathematical model and the operating of both the programs consisted in the estimation of the fundamental frequency of the homogenous plate. In Table 1, there are presented selected calculation results achieved by different authors and methods. The evaluation of the accuracy of the fundamental frequency parameter is of utmost importance as this value is computed for the whole system and the result depends on many optimized coefficients. These coefficients describe the participation of basic functions composing the complex mode shape of the stiffened plate. The errors of calculations of these coefficients add up and result in the summary error of the calculated natural frequency of a stiffened plate. A careful evaluation of these errors is possible when the precision of the natural frequency computations is assessed.

Table 1. Comparison of the fundamental frequency of a homogenous plate made of typical constructional steel (700 [mm] $\times$ 500 [mm] $\times$ 1.2 [mm]) estimated by different authors and methods.

Author	Frequency [Hz]
BEDAIR [2]	17.65
LEISSA [9]	17.69
FEM 20×20 elements	17.51
FEM 40×40 elements	17.68
Algorithm Nelder–Mead	17.75
Genetic Algorithm	17.7

In the case of a homogenous plate, the results of calculations are obtained very quickly and their precision, compared to results obtained by other methods, is entirely sufficient for engineering purposes.

5. Examples of the natural frequency of stiffened plate computations

Evaluation of the dimensionless coefficient of the fundamental frequency Ω allows to determine quickly the fundamental frequency ω of a given stiffened plate belonging to the group of plates of similar physical properties described by the Eq. (8). Also at this stage, the accuracy of computations carried out by the use of the optimizing algorithms was verified on the basis of the nondimensional coefficient of the fundamental frequency Ω determined for a specified stiffened plate. In the next step, the estimated value of the fundamental frequency was compared with the fundamental frequency value obtained by the use of the finite elements method.

A. The plate and stiffeners made of the same material

The approach presented allows to compute the fundamental frequency of a plate stiffened with any system of ribs. In the case of a plate with identical and equally spaced stiffeners, the calculations become much more simple. The following example deals with a stiffener placed symmetrically (Fig. 2). The system consists of a homogenous plate supported simply at the four edges and the stiffener with the following parameters:

- side dimensions ratio $\beta = a/b = 1.4$;
- Young's modulus ratio of the used materials $E_p/E_s = 1$;
- Poisson's coefficient 0.29;
- the dimensionless parameters of the stiffeners second moment of inertia I_s of the cross-section and the plate flexural rigidity I_p per unit width I_p/I_s were changed in a wide range presented in Fig. 3.

The results of calculations carried out using the optimization algorithm were verified by calculating the fundamental frequency of the plate representative for the group of plates analysed by the use of the FEM method. The FEM model of the stiffened plate with dimensions: $700 \times 500 \times 1.2$ [mm] consisted of 400 surface elements of the type
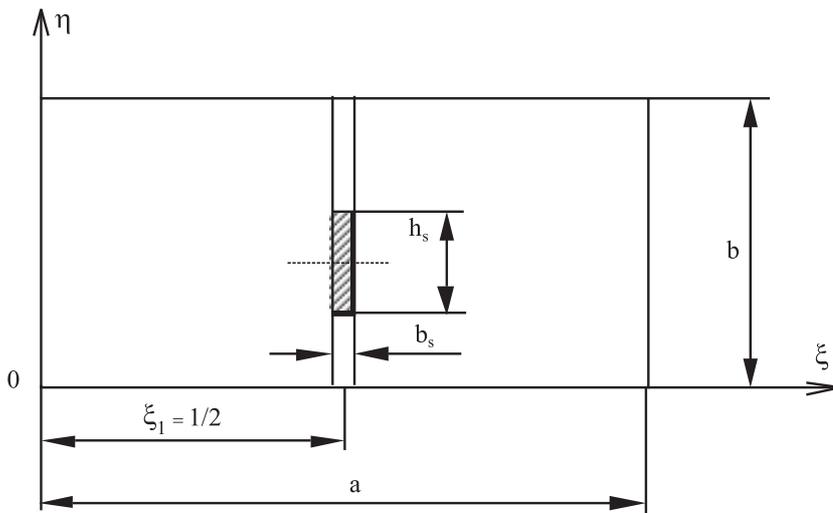
Fig. 2. The scheme of a symmetrically stiffened plate with a cross section of the stiffener $b_s \times h_s$.

QUAD4 and 20 elements of the BEAM modelling the stiffeners. The degrees of freedom at the plate edges were defined by limiting three linear displacements of the nodes. The height of the 4 [mm] wide stiffener was changed from 0 to 40 mm in of 5 mm steps. The scheme of the stiffened plate is presented in Fig. 2.
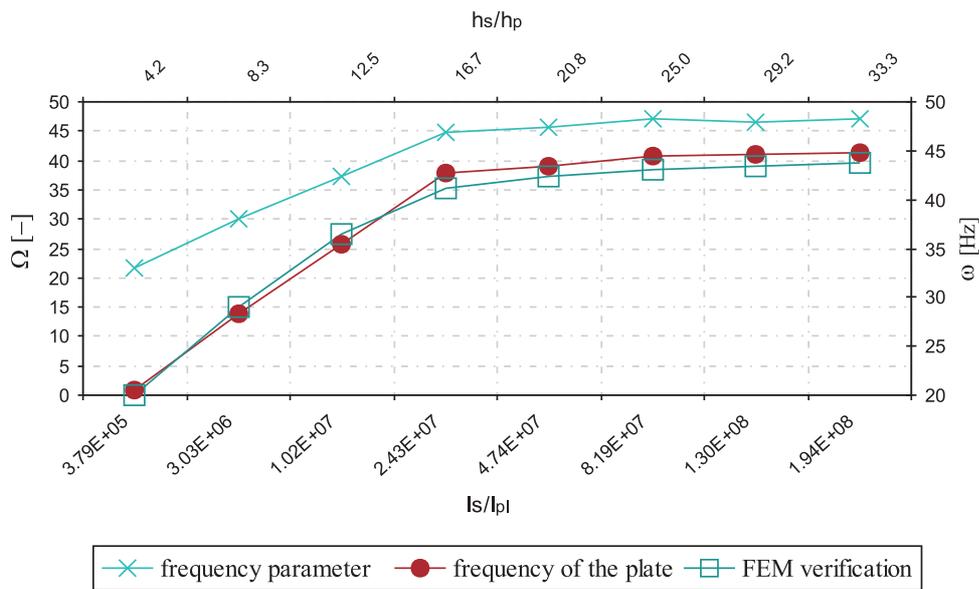
Fig. 3. The non-dimensional frequency parameter Ω computed for the group of similar plates and the first natural frequency Ω of a group of representative stiffened plates ($700 \times 500 \times 1.2$ [mm]) made of typical constructional steel.

The results of the influence of the stiffeners relative rigidity I_s/I_p on the natural frequency magnitude are presented in the Fig. 3. The non-dimensional frequency parameter Ω computed for a group of plates with the parameters given above is presented on the left scale. On the right scale of Fig. 3, there is the magnitude of the fundamental frequency Ω of the physical realization representative for the group of similar plates with one stiffener of a rectangular cross section. The curve * indicates the natural frequency Ω of a given plate computed upon the non-dimensional frequency parameter Ω, while the curve □ denotes the results obtained using the FEM method. For orientation, the stiffener height-to-plate thickness ratio is shown on the upper scale.

In the case considered above, the magnitude of the fundamental frequency is established at an almost constant level, while the second moment of inertia I_s of stiffeners in relation to the flexural rigidity of the plate I_p per unit width achieves about $3.4 \cdot 10^7$.

B. The plate and stiffeners made of different materials

The calculations of the non-dimensional frequency parameter Ω were also carried out for a group of simple-supported plates with a stiffener of varied stiffness placed at the centre of the plate as shown in Fig. 2. The plate was made of steel, while the stiffener is of aluminium alloy. The relative parameters were chosen as follows:

- side dimensions ratio $\beta = a/b = 1.4$;
- ratio of the plate Young's modulus to that of the stiffener material $E_p/E_s = 2.9$;
- Poisson's coefficient of the plate material 0.29;
- Poisson's coefficient of the stiffeners material 0.33.

The frequency parameter and the fundamental frequency of the steel plate stiffened with one central stiffener made of aluminium alloy are presented in Fig. 4. The fundamental frequency of the plate stops growing with the increase in the second moment of inertia along the major axis of the stiffener I_s in relation to the rigidity of the plate I_p approximately above $3.8 \cdot 10^7$.

C. The stiffened plate with variable number of stiffeners

The calculations of the dimensionless frequency parameter Ω were carried out for a group of the previously considered simple-supported plates stiffened with maximum 5 stiffeners of 4 mm × 10 mm which were parallel and evenly located perpendicular to the x axis (ζ).

The following parameters of the plate and stiffeners were assumed:

- side dimensions ratio $\beta = a/b = 1.4$;
- number of stiffeners $1 \div 5$;
- Young's modulus ratio $E_p/E_s = 1$;
- Poisson's coefficient of the plate material 0.29;
- Poisson's coefficient of the stiffeners material 0.29;
- cross-section of the stiffeners $4 \cdot 10^{-3} \times 10 \cdot 10^{-3}$ [m×m];
- relative rigidity of the stiffeners I_s/I_p $3.03 \cdot 10^6$

(defined by the second moment of inertia I_s along the $\xi\,(x)$ axis and the plate flexural rigidity I_p per unit width I_s/I_p).
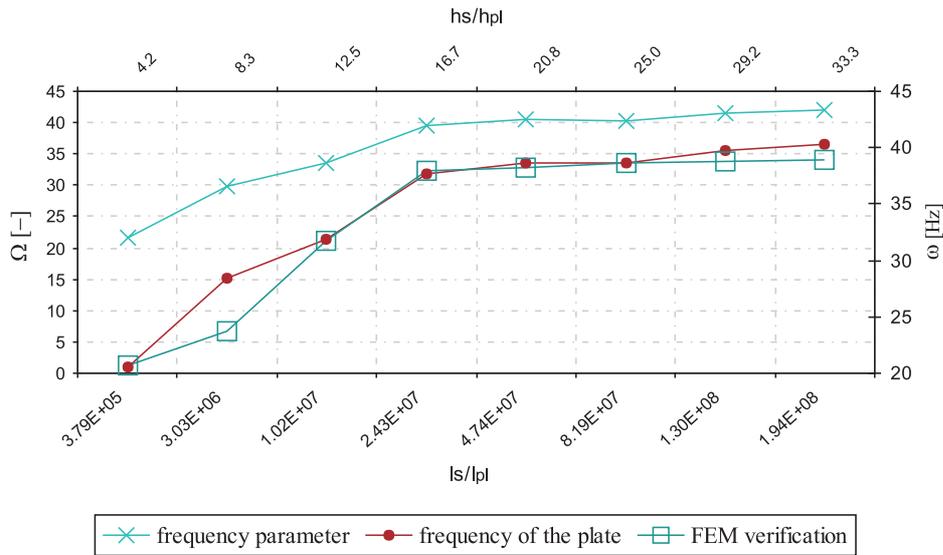
Fig. 4. The non- dimensional frequency parameter Ω for the group of similar plates and the first natural frequency ω of a group of representative stiffened plate made of typical constructional steel with aluminium alloy stiffeners.

The result were achieved analytically in the general form of the frequency parameter Ω. and then the fundamental frequencies of representative plates were calculated and referred to the values obtained by the use of the FEM method. The comparison of the achieved results is shown in Fig. 5.

$$\omega = 16.794 Ln(N) + 1.95.$$
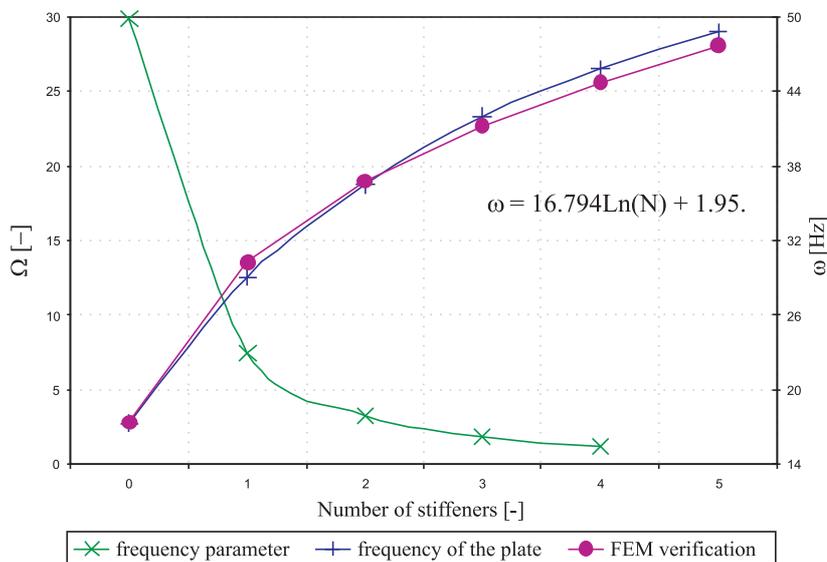
Fig. 5. The frequency parameter Ω and the natural frequencies ω of a steel plate with a variable number of ribs.

As shown in Fig. 5, the introduction of additional stiffeners results generally in an increase the fundamental frequency magnitude. The frequency parameter (left scale $\times$) and the fundamental frequency ω (right scale) of a representative steel plate is shown in Fig. 5 as function of the number of stiffeners. For the number of examined stiffeners N, the natural frequency of the tested panel can be approximated by the function: $\omega = 16.794 \text{Ln}(N) + 17.95$.

In Fig. 6, the results of the fundamental frequency calculated for plates stiffened with a variable number of stiffeners with growing cross-section area are presented. The thicknesses of the stiffeners/beams were assumed to be constant but the height has been changed in the range from 0 mm to 40 mm.
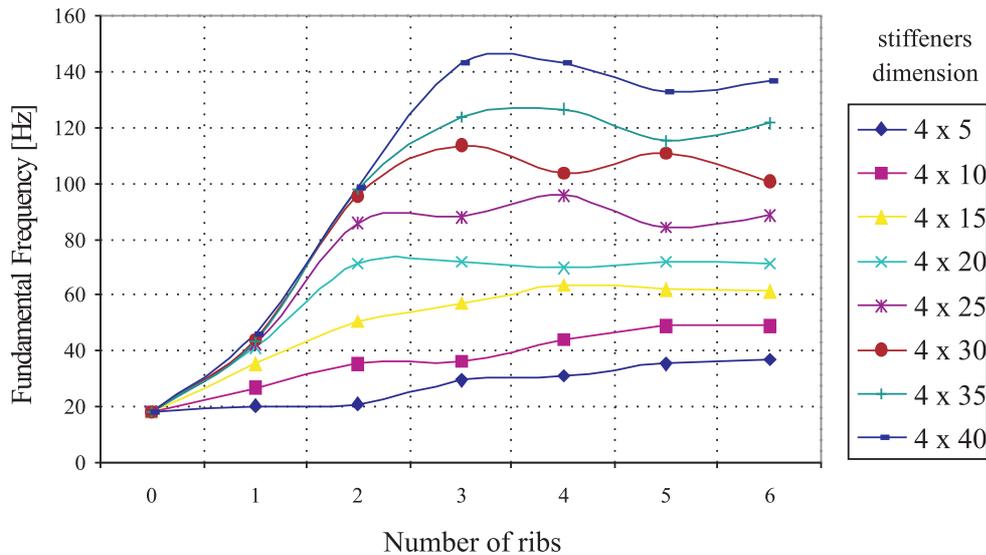
Fig. 6. The fundamental frequency computed for plates stiffened with a variable number of stiffeners.

For each system of stiffeners, there is a certain limit of rigidity. After going beyond this limit no significant increase in the natural frequency of a ribbed plate can be observed.

6. Conclusions

The application of energy analysis of a stiffened plate considered as a combination of a homogenous plate and a set of beams allows the calculation of the natural frequency and the distribution of energy stored and dissipated by the plate and by the stiffeners. In the model, plates with various numbers of ribs could be taken into account, as well as various cross-sections of the ribs and different materials that the stiffeners and the plates are made of.

The evaluation of the accuracy of the genetic algorithm was carried out by comparing the results obtained with those achieved by other authors and with the method of finite elements. The accuracies of both the methods are similar and sufficient for technical uses.

Comparing the times of computation using the genetic algorithm and the method of finite elements, we are led to conclude that the program here presented has several advantages; the calculations are conducted simultaneously for a whole group of similar plates and the time needed for the preparation of the model is much shorter than that in FEM.

The analysis of the stiffened plates' vibration applying mathematical equations in the analytical form makes it easier to solve technical problems and to arrive at optimal constructional solutions.

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