ARCHIVES OF ACOUSTICS
32, 4, 895–901 (2007)

INFLUENCE OF PHASE CORRECTION ON THE ACOUSTIC IMPEDANCE OF CIRCULAR PLANAR SOURCES

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(*received July 15, 2007; accepted October 24, 2007*)

In this work, the influence of velocity distribution phase correction on the acoustic impedance of a circular source located in an infinite baffle is analyzed. It has been observed that by turning back a velocity distribution phase by modifying the radiating surface shape in a special way, obtaining a more coherent acoustic radiation is possible. This problem has been analyzed during earlier investigations, with attention focused on the directivity of some acoustic sources [1]. The aim of this paper is to calculate the acoustic impedance which allows the examination of such a "phase correction" effect on the acoustic radiation of the considered sound sources. The result was achieved by numerical calculation with the use of Hankel representation of the acoustic impedance.

Keywords: sources of high directivity, phase correction, acoustic impedance.

1. Introduction

There are many domains such as metrology, diagnostics, hydrolocation etc., in which the sources of high directivity are useful. These kinds of sources radiate only within a certain cone. It has been stated [1, 2] that this property is exhibited by the plane circular piston with a special velocity amplitude distribution, given by the first order Bessel function divided by the argument: $J_1(x)/x$. In fact, it is possible to apply methods of directivity improvement for some frequently used acoustic sources. One of these methods was originally proposed by BARONE and GALLEGO–JUAREZ [4] for a plate applied as a high frequency acoustic transducer. The method involves turning back the vibration velocity phase between chosen nodal lines. The improvement of the plate radiation conditions has been achieved by modifying the radiating surface shape in a special way, which is explained in Fig. 1.

A phenomenon of "turning back" a vibration velocity phase between two sides of a nodal circle has been called a *phase correction*. The research has proved that it is

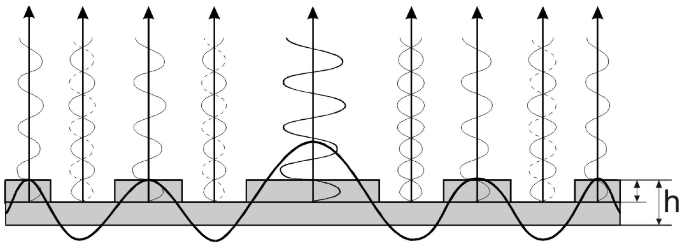

Fig. 1. Explanation of the phenomenon "turning back" the vibration velocity phase on the piston surface.

possible to turn back the phase of velocity distribution if zones vibrating in phase are displaced with respect to those in antiphase by a distance equal to half a sound wavelength in the propagation medium [4]. In paper [1] it has been shown analytically that the velocity distribution with correction of velocity phase improves directivity significantly in the case of source vibrating with some nodal lines. Still, in order to utilize such sources for designing acoustic devices requires identification of other important quantities that characterize an acoustic field.

This paper focuses on energetic properties of circular sound sources with high directivity. The derived formulas describe acoustic impedance for a source with velocity distribution phase correction. In order to investigate the correlation between the directivity improvement and the acoustic impedance of considered source, numerical calculations were made and discussed.

2. High directivity sources

Let us assume that a planar circular sound source of the radius a is set in a plane $z = 0$ which is a perfectly rigid baffle surrounded on both sides by a gas medium with the rest density ρ. The vibration of the source is time-harmonic and its amplitude is defined by the following surface distribution [1, 2]:

$$u(r) = \begin{cases} \dfrac{2u_0 J_1(\alpha_{1m} r/a)}{\alpha_{1m} r/a} & 0 < r < a, \\ 0 & r \geq a, \end{cases} \tag{1}$$

where u_0 – the amplitude of the midpoint, α_{1m} – zeros of the Bessel function of the first order for $m = 0, 1, 2, \ldots, r$ – radial variable. Nodal lines appear on the piston surface for $m = 2, 3, \ldots$. It is known from theoretical calculations [2] that the increasing m value in (1) expression worsen source directivity properties. It can be inferred that although velocity distribution (1) characterises sources with quite high directivity, it also could be improved further by "phase correction". This effect can be accomplished by using the following velocity distribution [1]:

$$u(r) = \begin{cases} \left| \dfrac{2u_0 J_1(\alpha_{1m} r/a)}{\alpha_{1m} r/a} \right| & 0 < r < a, \\ 0 & r \geq a. \end{cases} \tag{2}$$

In this case the vibration velocity changes its sign across the nodal circles where it oscillated in the opposite phase. This is equivalent to 180° velocity phase rotation (see Fig. 1).

3. Radiation impedance of a piston in infinite baffle

Consider a source located in an infinite baffle having the distribution of the normal component of vibration velocity amplitude defined by (1) or (2). Then, the acoustic impedance can be expressed using the definition [2]:

$$Z = \frac{1}{u_0^2} \int_\sigma p(\overline{r}) u^*(\overline{r}) \, \mathrm{d}\sigma = \frac{1}{u_0} \int_\sigma p(\overline{r}) f^*(\overline{r}) \, \mathrm{d}\sigma, \tag{3}$$

where $u^*(\overline{r}) = u_0 f^*(\overline{r})$, $f(r)$ is a function of velocity distribution and p is the amplitude of sound pressure. Formulae for the acoustic impedance can be obtained through the Fourier transform of acoustic pressure expressed by the Huygens–Rayleigh integral [2]. If the velocity distribution is axially-symmetric (like in our case), it is useful to define *characteristic function* $W(ka\xi)$ as follows:

$$W(ka\xi) = \int_0^a f(r) J_0(k\xi r) r \, \mathrm{d}r \tag{4}$$

and by the use of transformation shown in [2] one can get:

$$X = 2\pi\rho c k^2 \int_0^{\pi/2} W^2(ka \sin\psi) \sin\psi \, \mathrm{d}\psi, \tag{5}$$

$$Y = 2\pi\rho c k^2 \int_0^{\infty} W^2(ka \cosh\psi) \cosh\psi \, \mathrm{d}\psi, \tag{6}$$

where X and Y are real and imaginary parts of Z, respectively. For the considered surface distribution (1), the following characteristic function can be obtained:

$$\begin{aligned} W(ka\xi) &= \int_0^a \frac{J_1(\alpha_{1m} r/a)}{\alpha_{1m} r/a} J_0(k\xi r) r \, \mathrm{d}r \\ &= \frac{a}{\alpha_{1m}} \int_0^a J_1(\alpha_{1m} r/a) J_0(k\xi r) \, \mathrm{d}r. \end{aligned} \tag{7}$$

Finally, by the use of substitution described below:

$$R = \frac{r}{a}$$

function $W(ka\xi)$ takes the form:

- in the case without *phase correction*

$$W(ka\xi) = \frac{a^2}{\alpha_{1m}} \int_0^1 J_1(\alpha_{1m}R) J_0(ka\xi R)\,\mathrm{d}R, \tag{8$_1$}$$

- in the case with the *phase correction*

$$W_c(ka\xi) = \frac{a^2}{\alpha_{1m}} \int_0^1 \left|J_1(\alpha_{1m}R)\right| J_0(ka\xi R)\,\mathrm{d}R. \tag{8$_2$}$$

In order to extract real and imaginary part of impedance, $\xi = \sin\psi$ and $\xi = \cosh\psi$ were assumed. Usually, to draw characteristics of acoustic impedance of chosen source, its normalized form is used. Normalization process is based on the assumption that the real part of the impedance reaches the value of $+1$ as the parameter k approaches ∞. Thus, the normalized form of the acoustic impedance can be expressed as [2]:

$$\begin{gathered} \zeta = \frac{Z}{\pi A^2 \rho c} = \theta + i\chi\,, \\ \theta = \frac{X}{\pi A^2 \rho c}\,, \qquad \chi = \frac{Y}{\pi A^2 \rho c}\,, \end{gathered} \tag{9}$$

where $A^2 = \dfrac{1}{\pi} \displaystyle\int_\sigma f^2(r)\,\mathrm{d}\sigma$ is the mean square of vibration velocity distribution. In our case, where velocity distribution is given by (1) formula the coefficient A^2 equals:

$$A^2 = \frac{1}{\pi} \int_\sigma \left(\frac{J_1(\alpha_{1m}r/a)}{\alpha_{1m}r/a} \right)^2 \mathrm{d}\sigma = \frac{a}{\alpha_{1m}} \frac{1}{\pi} \int_0^{2\pi} \int_0^a \frac{J_1^2(\alpha_{1m}r/a)}{\alpha_{1m}r/a}\,\mathrm{d}\varphi\,\mathrm{d}r \tag{10}$$

and after simplifying we get:

$$A^2 = 2\left(\frac{a}{\alpha_{1m}}\right)^2 \int_0^{\alpha_{1m}} \frac{J_1^2(x)}{x}\,\mathrm{d}x. \tag{11}$$

The integral of this kind can be found in literature [3]:

$$\int_0^z \frac{1}{x} J_1^2(x)\,\mathrm{d}x = \frac{1}{2}\left[1 - J_0^2(z) - J_1^2(z)\right]. \tag{12}$$

Finally, the derived expressions look as follows:

$$A^2 = \left(\frac{a}{\alpha_{1m}}\right)^2 \left[1 - J_0^2(\alpha_{1m})\right] \tag{13}$$

and

$$\theta = \frac{2(ka)^2}{1 - J_0^2(\alpha_{1m})} \int\limits_0^{\pi/2} W^2(ka \sin\psi) \sin\psi \, \mathrm{d}\psi, \tag{14}$$

$$\chi = \frac{2(ka)^2}{1 - J_0^2(\alpha_{1m})} \int\limits_0^{\infty} W^2(ka \cosh\psi) \cosh\psi \, \mathrm{d}\psi. \tag{15}$$

The obtained integrals have no exact analytical solutions but it is possible to carry out detailed numerical calculations. Real and imaginary parts of acoustic impedance have been evaluated for considered source using wide range of the interference parameter ka values.

4. Result of investigations

Our research was focused on checking the influence of velocity distribution phase correction which improves source directivity on the acoustic impedance of the source in question. Because there are no analytical formulas for calculating the integrals used in expressions (14) and (15), result shown in the below figures was achieved by numerical calculation.

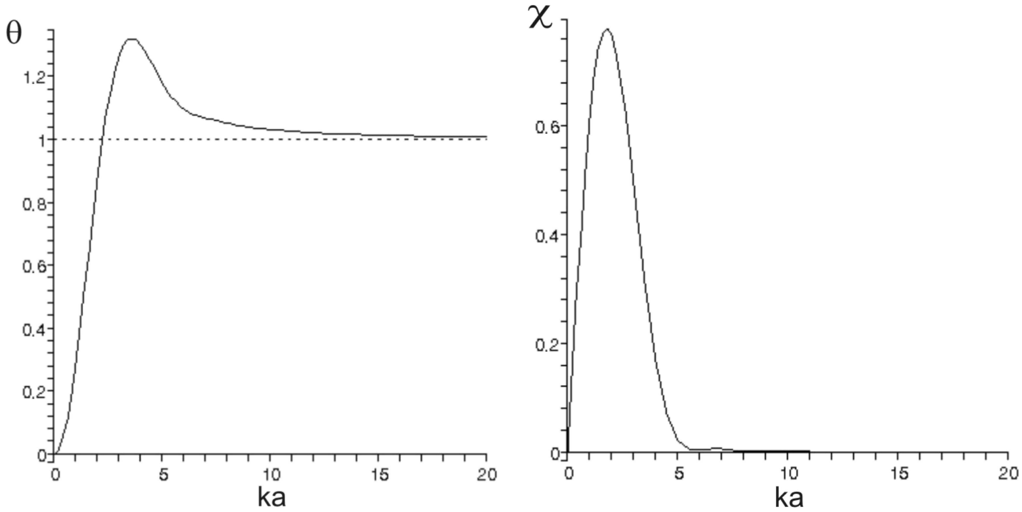

Fig. 2. Impedance of high directivity sound source for α_{11}.

In the above figure (Fig. 2) the curve with "correction" and the curve without "correction" are simply superimposed. This result is fully understood – there is no negative velocity distribution value for the considered piston because there are no nodal lines in this case and the addition of absolute value sign could not change the calculated characteristics. In the next two figures the "correction" effects can be easily noticed.

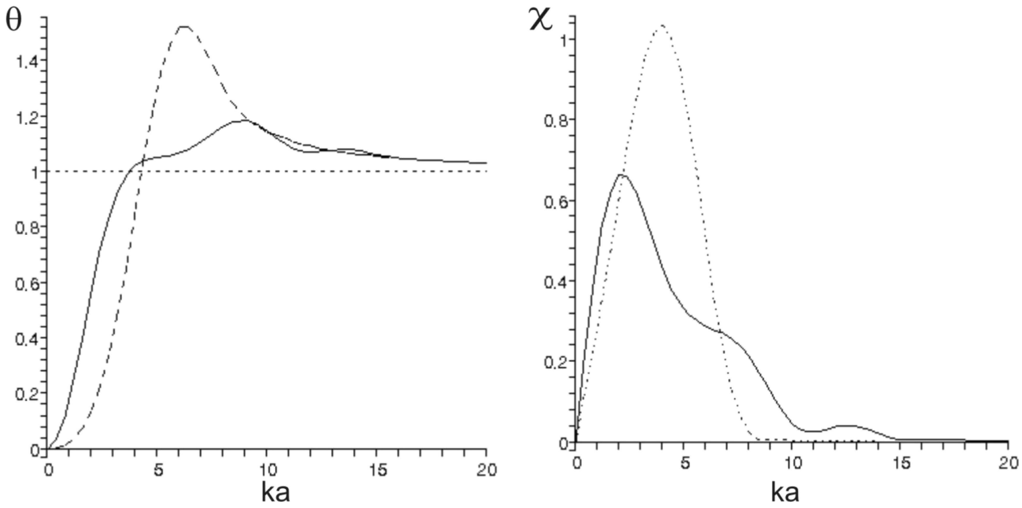

Fig. 3. Impedance of high directivity sound source for α_{12}: - - - without phase correction, — with phase correction.

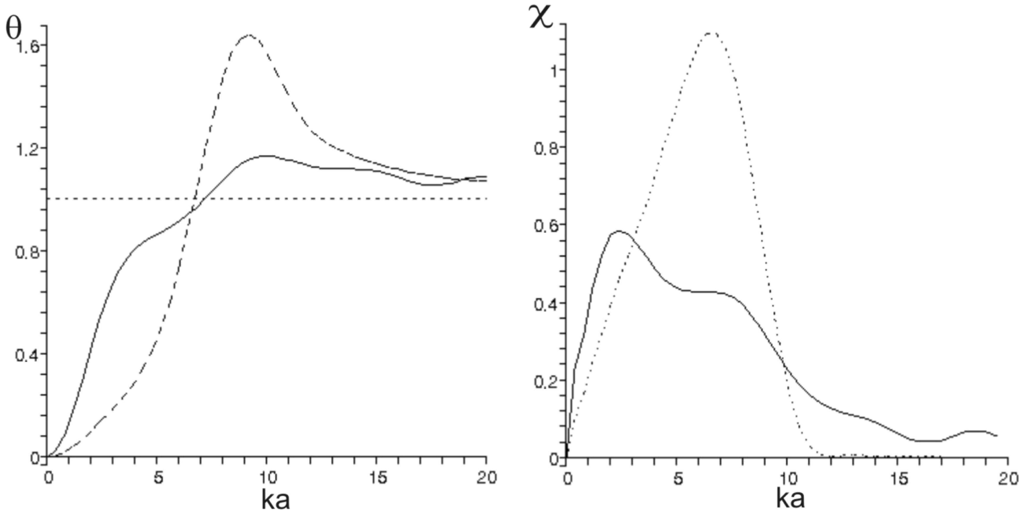

Fig. 4. Impedance of high directivity sound source for α_{13}: - - - without phase correction, — with phase correction.

5. Final remarks and conclusions

The aim of the considerations was to calculate the acoustic impedance which allows examining a "phase correction" effect on acoustic radiation of the considered sound sources. From our analysis it follows that in the wave length range, in which the considered sources with (2) amplitude distribution of the vibration velocity exhibit an adequately high directivity [1], they radiate effectively the energy vibrations into the Fraunhofer zone (i.e. with the power factor close to unity). There are no differences between characteristics with and without "correction" for the first zero of Bessel function. It is because there are no nodal lines on the surface of the source in this case. In all other cases under the analysis it has been observed that the betterment of the directivity of the source is accompanied by the decrease of the amount of vibration energy radiated by this source into the far field. In comparison with the distribution without phase correction it can be noticed that values of the acoustic impedance maxima decrease and are shifted towards lower frequencies. Result of investigations can be applied in the estimation of energetic properties of various real circular sources and acoustic systems.

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