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A TWO-STAGE WIENER FILTER BASED MULTI-CHANNEL FEEDBACK VIRTUAL MICROPHONE ACOUSTIC NOISE REDUCING SYSTEM

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An optimal fixed-parameter control system capable to generate zones of quiet at a number of desired locations is designed and analysed in this paper. Although it is of feedback structure the design problem is reformulated so that the Wiener filter approach can be applied. Polynomial technique is used. Due to non-minimum phase character of the plant including delays it requires performing the inner-outer factorization of a polynomial matrix and the causal-anticausal decomposition. For this control system two stages of operation can be distinguished. Experimental verification confirms that the zones of quiet are correctly distributed. Moreover, the control system remains stable for significant changes of the plant.

Keywords: active noise control, multi-channel feedback control, optimal control, virtual microphone.

1. Introduction

For many acousto-electric applications there is a need to design control systems capable to shift the zones of quiet to desired locations [1–6]. Such systems are referred to as the Virtual-Microphone Control systems. They employ different techniques to estimate the residual noise at the desired location and then minimise it using a relevant criterion. The system considered in this paper does not require identification of the virtual path, usually time-variant. Two assumptions are made. One of them requires the distance between the desired location (the so-called virtual microphone, E_v) and the corresponding error (real) microphone, E_r, to be much smaller than the smallest wavelength in the acoustic noise. Such assumption is satisfied for many applications where local control is considered. Another assumption requires the noise to be stationary. Then, corresponding disturbances at the real and virtual microphones can be considered equivalent [3, 4].

The system has two stages of operation. In the first stage knowledge of the system state at the desired location is collected, whereas in the second stage it is used to synthesize a command signal for the measured/controlled signal.

The use of a single pair of microphone and loudspeaker does not frequently suffice to obtain satisfactory performance, i.e. generate a zone of quiet of acceptable dimension [3, 4]. Moreover, for some applications, presence of an obstacle, e.g. the head for an active headrest system, constitutes a barrier for the zone of quiet at one side to propagate to the other side. Therefore, more microphones and loudspeakers are often necessary. In the most general case a coupling between subsequent channels should be taken into account resulting in a fully coupled multi-channel system.

In this paper a formal derivation and analysis of the control algorithm is presented. Let G be the number of real and virtual microphones (plant outputs), and I be the number of secondary sources (plant inputs). Rational transfer functions of the real and virtual paths can be grouped together in polynomial matrices, $\mathbf{S}_r(z^{-1})$ and $\mathbf{S}_v(z^{-1})$, respectively, of dimension $G \times I$, e.g.

$$\mathbf{S}_r(z^{-1}) = \begin{bmatrix} S_{r11}(z^{-1}) & S_{r12}(z^{-1}) & \cdots & S_{r1I}(z^{-1}) \\ S_{r21}(z^{-1}) & S_{r22}(z^{-1}) & \cdots & S_{r2I}(z^{-1}) \\ \vdots & \vdots & \ddots & \vdots \\ S_{r\,G1}(z^{-1}) & S_{r\,G2}(z^{-1}) & \cdots & S_{r\,GI}(z^{-1}) \end{bmatrix}. \tag{1}$$

Estimated models will be noted with hats. Control filters are grouped together in a polynomial matrix $\mathbf{W}(z^{-1})$ of dimension $I \times G$, built in a similar way to the above plant matrix, so that

$$\mathbf{W}(z^{-1}) = \begin{bmatrix} W_{11}(z^{-1}) & W_{12}(z^{-1}) & \cdots & W_{1G}(z^{-1}) \\ W_{21}(z^{-1}) & W_{22}(z^{-1}) & \cdots & W_{2G}(z^{-1}) \\ \vdots & \vdots & \ddots & \vdots \\ W_{I1}(z^{-1}) & W_{I2}(z^{-1}) & \cdots & W_{IG}(z^{-1}) \end{bmatrix}. \tag{2}$$

The disturbances, if they are stochastic and wide-sense stationary, can be modelled as uncorrelated wide-sense stationary white noise sequences of unity variances, filtered by a shaping filters matrix, [7], i.e.

$$\mathbf{d}(i) = \mathbf{F}(z^{-1})\,\mathbf{e}(i). \tag{3}$$

The matrix $\mathbf{F}(z^{-1})$ can be found by performing spectral factorisation of the matrix of the disturbance Power Spectrum Density (PSD)

$$\mathbf{S}_{dd}(z^{-1}) = \mathbf{F}(z^{-1})\,\mathbf{F}^T(z)\big|_{z^{-1}=e^{-j\omega T_S}}, \tag{4}$$

where the matrix dimension is $G \times G$, and $\mathbf{S}_{dd}(e^{-j\omega T_S})$ is analytic and positive definite for all ωT_S. For notational convenience it is assumed that the shaping filters in $\mathbf{F}(z^{-1})$ have finite impulse responses (FIR structure), what does not restrict the considerations.

All signals are grouped in corresponding vectors, e.g.

$$\mathbf{d}(i) = [d_1(i),\ d_2(i),\ ...,\ d_G(i)]^T. \tag{5}$$

In the above equations ω is the angular frequency, T_S is the sampling period, and z^{-1} is a complex variable if present in a polynomial or transfer function, or a one-step backward time-shift operator if present in a difference equation. The variable/operator z^{-1} will be dropped in the sequel, where it will not lead to confusion.

2. Algorithm design

In the proposed system the control algorithm is composed of two stages – see Figs. 1 and 2.

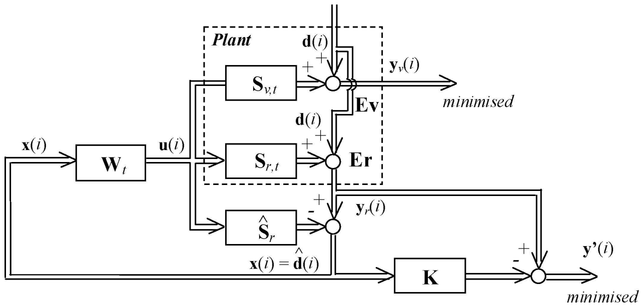

Fig. 1. First stage of the control system.

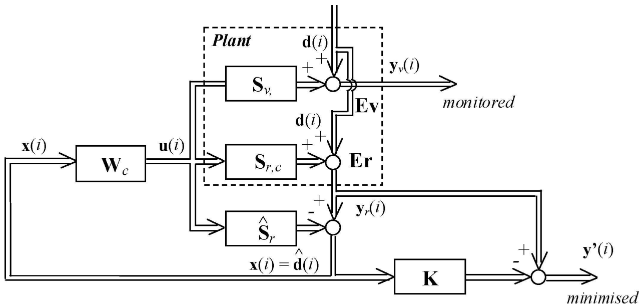

Fig. 2. Second stage of the control system.

In the first stage physical microphones are placed at positions of corresponding virtual microphones to provide the so-called virtual-microphone signals $\mathbf{y}_v(i)$ (Fig. 1). At the same time the filter matrix **K** is designed so that the vector signal $\mathbf{y}'(i)$ is minimised in the mean-square sense. In the second stage the virtual-microphone signals are not necessary for control (Fig. 2). The filter matrix **K** is used to work out command signals for the real-microphone signals $\mathbf{y}_r(i)$ [2]. For both stages the estimates of the disturbances are the control filter inputs.

The vector signal minimised in the mean-square sense by filters in matrix **K** in the first stage (see subscript t) can be obtained after some algebra from the block diagram in Fig. 1:

$$\mathbf{y}'(i) = \left[\mathbf{I}_G + \widehat{\mathbf{S}}_r \, \mathbf{W}_t - \mathbf{K}\right] \left[\mathbf{I}_G + \left(\widehat{\mathbf{S}}_r - \mathbf{S}_{r,t}\right) \mathbf{W}_t\right]^{-1} \mathbf{Fe}(i). \tag{6}$$

Hence, assuming that the control filters in matrix $\mathbf{W}_t$ are optimal and causal, the optimal solution for **K** is

$$\mathbf{K}_{\text{opt}} = \mathbf{I}_G + \widehat{\mathbf{S}}_r \, \mathbf{W}_{t,\,\text{opt}+}. \tag{7}$$

The minimised vector signal in this stage can be expressed as

$$\mathbf{y}_v(i) = [\mathbf{I}_G + \mathbf{S}_1\,\mathbf{W}]\,[\mathbf{I}_G + \mathbf{S}_2\mathbf{W}]^{-1}\,\mathbf{Fe}(i), \tag{8}$$

where

$$\begin{aligned} \mathbf{S}_1 &= \widehat{\mathbf{S}}_r - \mathbf{S}_{r,t} + \mathbf{S}_{v,t}\,, \\ \mathbf{S}_2 &= \widehat{\mathbf{S}}_r - \mathbf{S}_{r,t}\,. \end{aligned} \tag{9}$$

In the subspace of stable solutions the optimal control filter matrix minimising variance of $\mathbf{y}_v(i)$ can be obtained by considering the first square bracket of (8) and applying the polynomial-based method for Wiener filter matrix design, originally developed for feedforward systems. Then, for $G \geq I$ the matrix of stable and causal control filters has the form:

$$\mathbf{W}_{t,\,\mathrm{opt}+}(z^{-1}) = -\left[\mathbf{S}_1^{(o)}(z^{-1})\right]^{-1}\left\{\left[\mathbf{S}_1^{(i)}(z)\right]^T\mathbf{F}(z^{-1})\right\}_+\mathbf{F}^{-1}(z^{-1}), \tag{10}$$

where $\{.\}_+$ stands for the causal part of $\{.\}$. To calculate this matrix the following inner-outer factorisation should be performed

$$\mathbf{S}_1(z^{-1}) = \mathbf{S}_1^{(i)}(z^{-1})\,\mathbf{S}_1^{(o)}(z^{-1}), \tag{11}$$

where $\mathbf{S}_1^{(i)}(z^{-1})$, $\mathbf{S}_1^{(o)}(z^{-1})$ and $[\mathbf{S}_1^{(o)}(z^{-1})]^{-1}$ are stable, i.e. elements of the matrices are stable polynomials,

$$\begin{aligned} \dim\left(\mathbf{S}_1^{(i)}(z^{-1})\right) &= \dim\left(\mathbf{S}_1(z^{-1})\right) = G \times I, \\ \dim\left(\mathbf{S}_1^{(o)}(z^{-1})\right) &= I \times I, \end{aligned} \tag{12}$$

$$\left[\mathbf{S}_1^{(i)}(z)\right]^T\mathbf{S}_1^{(i)}(z^{-1}) = \mathbf{I}_I\,, \tag{13}$$

$$\left[\mathbf{S}_1^{(o)}(z)\right]^T\mathbf{S}_1^{(o)}(z^{-1}) = \mathbf{S}_1^T(z)\,\mathbf{S}_1(z^{-1}). \tag{14}$$

Analogous design can be performed for the case of $G < I$. However, such case is less justified for active noise control [7]. From the mathematical point of view the co-inner-outer factorization would then be required instead of the inner-outer factorization since the plant matrices would be of the horizontal layout.

In the second stage of the control system operation (see subscript c) the following vector signal is controlled

$$\mathbf{y}'(i) = \widehat{\mathbf{S}}_r\,(\mathbf{W}_c - \mathbf{W}_{t,\,\mathrm{opt}+})\left[\mathbf{I}_G + \left(\widehat{\mathbf{S}}_r - \mathbf{S}_{r,c}\right)\mathbf{W}_c\right]^{-1}\mathbf{Fe}(i). \tag{15}$$

Hence, the matrix of optimal single-sided control filters takes the form

$$\mathbf{W}_{c,\,\mathrm{opt}+} = \mathbf{W}_{t,\,\mathrm{opt}+}\,. \tag{16}$$

So it is exactly the same as the matrix of optimal filters minimising mean-square values of $\mathbf{y}_v(i)$ in the first stage, regardless of $\mathbf{d}(i)$ and properties of the plant and modelling errors.

The residual signal at the virtual microphone becomes

$$\mathbf{y}_v(i) = \left[\mathbf{I}_G + \left(\widehat{\mathbf{S}}_r - \mathbf{S}_{r,c} + \mathbf{S}_{v,c}\right)\mathbf{W}_c\right]\left[\mathbf{I}_G + \left(\widehat{\mathbf{S}}_r - \mathbf{S}_{r,c}\right)\mathbf{W}_c\right]^{-1}\mathbf{F}\mathbf{e}(i). \qquad (17)$$

Polynomial matrices $\mathbf{K}$, $\mathbf{W}_t$ and $\mathbf{W}_c$ can also be estimated using an adaptive procedure, e.g. based on the LMS algorithm. It can be proven that the estimates will converge to the solutions derived above.

The control system is of the feedback nature and therefore it is subject to stability constraints. The stability can be thus analysed using techniques relevant for a multi-channel feedback system [8]. Robustness can be obtained and adjusted by applying the following factorization of matrix PSD of the disturbance and tuning parameter β:

$$\mathbf{F}(z^{-1})\,\mathbf{F}^T(z) = \mathbf{S}_{dd}(z^{-1}) + \beta\,\mathbf{I}_G\big|_{z^{-1}=e^{-j\omega T_S}}\ . \qquad (18)$$

3. Simulation analysis

The presented control system has been experimentally verified to reduce acoustic noise at the user's ears in the active headrest system [1, 9]. Results of control for a tone of 250 Hz and a real broadband noise of frequencies 120–300 Hz, obtained in a double-input double-output system operating with the sampling frequency of 2 kHz are presented in Figs. 3 and 4 (4-th order Butterworth anti-aliasing and reconstruction filters were used for the experiments providing data for plant modelling and control system simulation). Geometrical arrangement of the headrest components can also be concluded from the figures as any dashed square is of dimensions 7×7 cm. The measurements have been performed for different positions of the head at the right ear (hence the shift in the figures). Results for the left ear were similar. Therefore, robustness of

	3L	2L	1L	0	1R	2R	3R
f	0.9	4.3	7.4	4.8	8.2	3.4	-0.3
e	2.1	4.1	8.6	7.8	13.0	4.3	1.2
d	4.0	6.9	12.4	15.1	26.8	7.5	2.4
c	8.1	9.9	18.1	25.8	27.1	8.4	3.3
b	10.5	14.2	23.6	93.6	22.7	7.8	4.2
a	17.7	24.9	18.4	9.4	8.3	4.8	3.1

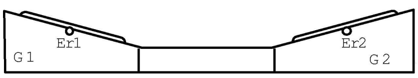

Fig. 3. SPL reduction [dB] of the 250 Hz tone.

	3L	2L	1L	0	1R	2R	3R
f	1.9	1.6	2.6	1.8	1.7	1.0	1.1
e	1.2	1.7	2.9	2.4	2.6	1.9	0.9
d	1.8	2.5	3.1	3.3	3.1	2.4	1.5
c	2.7	2.1	3.0	3.9	3.4	2.2	1.2
b	2.3	3.1	2.7	2.8	2.9	2.0	1.6
a	2.0	3.0	2.5	1.9	2.0	1.5	0.9

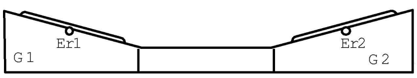

Fig. 4. SPL reduction [dB] of the broadband noise.

the control system to significant plant changes (the models were obtained prior to experiments and kept invariant) has also been verified in this way.

It is seen from the figures that the zones of quiet are correctly located, i.e. the highest attenuation is obtained at the nominal position of the head, for which path models have been obtained. Both lateral and forward practical head movements are allowed without leaving zones of significant attenuation for the given noise. For the tonal noise the attenuation and its gradient are significantly larger than for the broadband noise.

4. Conclusions

In this paper a two-stage multi-channel virtual-microphone active noise control system has been designed and analysed. The polynomial approach for designing Wiener filter matrix has been adopted. It requires spectral factorization of the matrix PSD of the disturbance, inner-outer factorization of a polynomial matrix and extraction of a casual part. This system has been experimentally verified. It successfully reduces the tonal and real noises for the active headrest system.

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