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A FAST METHOD FOR THE DETERMINATION OF PSYCHOPHYSICAL TUNING CURVES: FURTHER REFINING

Aleksander SĘK, Andrzej WICHER,
Szymon DRGAS

Adam Mickiewicz University
Institute of Acoustics
Umultowska 85, 61-614 Poznań, Poland
e-mail: oleksek@amu.edu.pl, awaku@amu.edu.pl

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Psychophysical tuning curves (PTCs) are usually measured by determining the level of a narrowband noise required just to mask a fixed, low-level tone, for several masker centre frequencies. PTCs are often used to assess the frequency selectivity of the auditory system and they have also been used to detect "dead regions" in the cochlea, especially to define the frequency boundaries of the dead regions. However, the traditional method of PTC determination is too-time consuming for use in clinical practice. This paper is concerned with further evaluation and refining of a fast method for determining PTCs, based on the use of a sweeping band of noise. The fixed sinusoidal signal is turned on and off at regular time intervals and is masked by a band of noise, whose centre frequency sweeps over a range of two octaves during four minutes. A Békésy method is used to determine the masker level required for threshold; the subject presses a button to indicate that the signal is inaudible, and releases it when the signal is audible, and the masker level is adjusted accordingly by a computer. The fast method was evaluated using normally hearing subjects and showed good agreement with the results obtained with the use of the traditional method. The shapes of the PTCs, the slopes of the low- and high-frequency skirts, and the positions of the minima were very similar when the fast and the traditional methods were used. However from the point of view of clinical usage the determination of the PTC tip, that is the masker centre frequency at which the masker level is lowest is the most important issue. The position of the PTC minimum with reference to the tone frequency indicates the presence of a dead region. Therefore, in this study several methods of the PTC minimum estimation were evaluated and compared. It has turned out that a fitting method of a single PTC by means of a square function yielded the best results. The method gave the smallest standard deviation, the highest kurtosis and the narrowest range of the PTC minima.

Keywords: psychophysical tuning curve, dead regions.

1. Introduction

1.1. Frequency selectivity in the auditory system

Frequency selectivity refers to the ability of the auditory system to separate or resolve, to some extent, the spectral components in complex sounds. This ability is also known as frequency resolution or frequency analysis. Frequency selectivity plays an important role in nearly all aspects of auditory perception and is usually demonstrated and measured in masking experiments, because masking is regarded as reflecting the limits of the frequency resolution of the auditory system. If a sound at a given frequency is masked (inaudible) by another sound at a different frequency, then the auditory system has failed to resolve (separate out) the two sounds. Therefore, by measuring the masking of one sound by another one it is possible to describe the frequency resolution of the auditory system.

It is widely believed that the frequency selectivity measured in masking experiments is largely determined by the filtering properties at the basilar membrane in the cochlea [10, 18]. The tuning of any point of the basilar membrane is determined by two components. The first one is a passive component reflecting mechanical properties of the basilar membrane and the surrounding fluids. The second one is an active component, reflecting influence of the electromotility of the outer hair cells [19]. While the passive component seems to be linear and does not depend on the sound level, the active component shows high nonlinearity [20, 21], especially at medium sound levels.

Frequency selectivity was first shown by FLETCHER [1] in a band-widening experiment that was repeated several times [3, 22]. The masked threshold of a tone increases as the bandwidth of the masking noise (centered at the frequency of the tone and having constant spectrum level) increases, but at some bandwidth value it flattens off, so that further increase in the bandwidth does not change the signal threshold significantly. To account for these results, Fletcher suggested that the auditory system contains an array of bandpass filters and while detecting a tone in the presence of noise the subject tends to use the filter, with a center frequency equal or close to that of the tone. The effect of the initial increase in noise bandwidth is that a larger amount of noise passes through the filter centered at the tone frequency and the threshold of the tone increases. However, once the noise bandwidth exceeds the filter bandwidth, further increase in the noise bandwidth does not change the amount of the noise passing through the filter. Therefore the tone threshold remains approximately constant.

The filters suggested by Fletcher are nowadays called the auditory filters and they provide a satisfactory measure of the frequency selectivity of the auditory system. Therefore while discussing the frequency resolution of the auditory system, different aspects of the auditory filter parameters are mentioned (the bandwidth as a function of frequency, the slopes of the upper and lower skirts of the filters, the dynamic range, etc.).

There are several methods of the auditory filter shape (frequency characteristics) determination, for example, the notched noise method [14, 15], the rippled-noise method [5] and the measurement of the psychophysical tuning curve (PTC).

Although the measurement of the frequency characteristics of the auditory filter provides very useful information concerned with the auditory system functioning, the above mentioned methods are not used as diagnostic tools in clinical practice. The main reason for this is that these methods are very time-consuming and they have not been implemented in standard clinical equipment. However, one of them, namely the PTC measurement method, turned out to be very useful, precise and effective in so-called dead region detection [23–25]. Therefore the following paragraphs present details of the PTC measurement method as well as a brief description of the dead regions.

1.2. Psychophysical tuning curves measurement

The measurement of psychophysical tuning curves (PTCs) is usually done in a simple masking experiment. The sinusoidal signal is fixed in level, usually at a very low level (e.g., 10 dB SL). The masker can be either an other sinusoidal signal or, preferably, a band of noise. Masking noise is preferred because this type of masker has inherent fluctuations, therefore beats arising from superposition of the signal and the masker usually do not provide effective cues to the subject. However, it has to be assumed that the masker bandwidth must not be too narrow, because temporal interaction between the signal and masker's low-frequency fluctuations may be still detectable [7].

The level of the masker required just to mask the signal is determined for several masker centre frequencies. Because the signal level is low, it is assumed that the signal will produce activity at the output of a single auditory filter centered at its frequency. It is also assumed that at the signal threshold, the masker produces a constant output from that filter, in order to mask the fixed signal. Thus, the PTC indicates the masker level needed to produce a fixed output from the auditory filter centered at the signal frequency, as a function of the centre frequency of the masker. Filter characteristics is usually determined by plotting the level of the output signal from the filter for an input varying in frequency and fixed in level. However, if the filter is linear the same result can be obtained by plotting the level of the input signal that gives a fixed output. Thus, if linearity is assumed, the frequency characteristics of the auditory filter can be obtained simply by inverting the PTC.

It has been assumed that only one auditory filter is involved in the PTC determination. However there a substantial evidence that off-frequency listening [17] can influence the PTC shape. When the masker frequency is above the signal frequency, the highest signal-to-masker ratio occurs at the filter output centered below the signal frequency. Conversely, when the masker centre frequency is below that of the signal the highest signal-to-masker ratio occurs at the filter output centered above the signal frequency. In both of these cases the masker level necessary for threshold is higher than would be in the case if off-frequency listening did not occur. This means that the sharpness of the PTC is slightly higher than would be expected if off-frequency listening did not exist and the minimum of the PTC shows exactly the centre frequency of the filter that was actually used, while detecting the signal. In normally hearing subjects the slopes of the low-frequency part of the PTC is shallower than the high-frequency part.

Moreover, the position of the PTC minimum is very close or exactly equal to the signal frequency.

As results from the above description, the measurement of PTC is not the best way of the auditory filter frequency characteristics determination. The notched noise method proposed by PATTERSON [14, 15] is the best method in this case, as it reduces the off-frequency listening and gives the most reliable estimates of the auditory filter shape. However in some cases the determination of the centre frequency of the auditory filter that was used in certain detection task, is more crucial, than precise determination of its frequency characteristics. This happens while doing diagnose of a dead region (see below for details) existence. In this case the most important is the determination of the PTC minimum that shows the frequency limit of the dead region, while the slopes of low- and high-frequency skirts of the PTC are not the most important. Therefore, even if the PTC is not the best way of the auditory filter frequency characteristics determination it is still considered as a useful tool in measuring other aspects of the auditory perception.

1.3. Dead region

Cochlear hearing loss is often associated with damage of the hair cells within the cochlea. This damage can give rise to hearing impairment in two main ways. Damage of the outer hair cells (OHCs) impairs the active mechanism in the cochlea, and results in reduced basilar membrane vibration for a low sound level [12]. Inner hair cells (IHCs) damage, however, can result in reduced efficiency of transduction of basilar membrane vibration into action potentials (activity) in the auditory nerve. However, the IHCs at certain places along the basilar membrane may be completely non-functioning (or even missing) and/or neurones innervating those cells may be non-functioning or degenerated to some extent. Such areas are referred to as "dead regions" in the cochlea [11]. A dead region is defined in terms of the characteristic frequencies (CFs) of the IHCs and/or neurones immediately adjacent to the dead region [9]. It should be noted, that in the dead region only biological features of the basilar membrane are degraded while its mechanical features (i.e. tonotopic organization) are believed to be preserved. If a dead region is present at some area of the basilar membrane (BM), the audiogram will give a misleading estimation of the amount of hearing loss, for a tone whose frequency falls in the dead region [9]. Effectively, the "true" hearing loss in the dead region is infinite. However, the audibility of a tone falling in the dead region is possible and an absolute threshold at its frequency can be easily measured. It is possible when off-frequency listening is taken into account. Off-frequency listening enables detection of a tone of a particular frequency by means of places on the basilar membrane (as well as IHCs and neurons) with different characteristic frequencies from that of the tone [6, 16]. Since the mechanical properties of the basilar membrane are seem to be preserved in the dead region it has to be assumed that in response to a sound whose frequency falls into the dead region, some vibrations can be observed both in the dead region as well as in the areas adjacent to this region. In a response to a tone certain areas of the basilar membrane are active instead of its single point having characteristic frequency equals to that of the tone. Thus, it is quite possible that the basilar membrane vibration in the dead region can be detected via neurones innervating different areas of the basilar membrane, therefore the results of audiometric measurements are not reliable in this case.

Proper diagnosis of the dead regions is an important issue in hearing aid fitting. As shown by MOORE *et al.*, [12], subjects with a high-frequency dead region do not benefit from amplification of sounds falling in that region. Such amplification generally leads to a poorer vowel recognition. Thus, it seems that the dead region diagnostic should be done before the final decision about the hearing aid gain in different frequency bands is made.

One of the most commonly used method for detecting dead region is the Threshold Equalising Noise (TEN) method [12]. This method is based on the measurement of the detection threshold of a tone masked by specially prepared noise, that produces equal or no more than 10 dB higher masked thresholds than nominal masker level in one-ERB band for normally hearing or for hearing-impaired subjects without dead regions. If the frequency of the tone falls into the dead region, its detection threshold in the presence of the TEN masker is much higher (more than 10 dB) than the TEN level in one-ERB wide band. The tone level must be high enough to evoke an excitation perceived in the presence of TEN on the threshold level that indicates the presence of the dead region. The TEN method is very useful and leads to very fast diagnose. However, it does not give precise frequency boundaries of the dead region.

Measurement of the PTC provides an alternative diagnostic method for determining the frequency boundaries of dead regions [7, 9, 24]. When a PTC is measured for a hearing-impaired listener with a dead region, and when the frequency of the signal falls within the dead region, the tip of the PTC is shifted away from the signal frequency. When the tip of the PTC is shifted downwards in frequency, this indicates a high-frequency dead region beginning at the frequency of the shifted tip. When the tip is shifted upwards in frequency, this indicates a low-frequency dead region, whose upper boundary lies at the tip frequency of the PTC. In principle, the measurement of PTCs provides a more precise way of estimating the edge frequency of a dead region than the TEN test. However, the diagnosis of dead regions based on PTCs is time-consuming and the method has usually been applied only in laboratories. To determine a single PTC it is necessary to use at least five masker centre frequencies, and defining the frequency of a shifted tip may require many more masker centre frequencies. This typically takes at least two hours, and often takes longer, for a single PTC. However, a proper diagnose of the dead region requires the measurement of several PTCs that makes the overall time much longer. Thus, PTCs measured in the traditional way are not suitable for use in routine clinical practice.

Recently SĘK *et al.*, [24] proposed a method for PTCs determination that is very promising as it is much faster than the traditional one and can be used in clinics. However, this method is still under development and needs further evaluation and should be extensively refined before it is accepted as a diagnostic tool.

2. The aim of the study

The main aim of the present study was further evaluation of a fast method for determining PTCs, based on the use of a masker whose centre frequency is not fixed but sweeps from below to above the signal frequency or vice versa. This method called here the FAST-PTC method is presented below in details. Initially, for several subjects, the PTC were measured using the classical approach described above, i.e. the measurements were carried out using bands of noise with arbitrary chosen centre frequencies. This was done in order to compare the results of classical measurements with the results obtained using the FAST-PTC method.

At the next stage the PTCs were measured using the FAST-PTC method for noise of different bandwidth. The purpose of this experiment was to check the influence of the masker bandwidth on the position of the PTC minimum in the frequency domain. The purpose of this research was also to find out the optimal bandwidth of the masking noise leading to the most stable an repeatable estimates of the PTCs.

For a potential use of the FAST-PTC method in clinics the estimation of the PTC minimum position is the most important issue. A shift of the PTC minimum with respect to the signal frequency indicates the occurrence of a dead region. Therefore, several numerical procedures for estimation of the tip frequency of the PTC were designed, applied, evaluated and compared.

The classical method of determining PTCs consisted in measuring the masked threshold using a two-alternative forced-choice (2AFC) method, with a two-down one-up staircase procedure [8]. On each trial, the masker was presented in two intervals. However, one of them, chosen at random, contained the signal. The subject's task was to indicate the interval that contained the signal. The level of the noise masker was increased after two successive correct responses and decreased after one incorrect response. The step size was 4 dB until four "turnpoints" (changes from increasing to decreasing masker level or vice versa) had occurred, and was 2 dB thereafter. Twelve turnpoints were obtained and the level of the masker required for threshold was calculated from the mean masker level at the last eight turnpoints. The data included in this paper are averages of at least three separate measurements.

Eleven subjects with otologically normal hearing took part in the measurements of the PTCs carried out by means of the FAST-PTC and only four of them took part in the same measurements using classical method. Most of them were paid for their services. Subjects AW and SD are the co-authors of the paper. During experiment subjects were seated in a double-walled acoustically isolated booth and they used TDT-RBOX as a response keyboard.

3. The FAST-PTC method

The FAST-PTC method is actually based on the classical PTC measurement technique and takes advantages of both the classical method and the TEN method while avoiding their disadvantages. A series of 300-ms bursts of a sinusoidal tone (including a 20-ms rise and decay of Hanning shape) at a frequency of f_t are presented to a subject. The tone level is constant, and usually equals 10 dB SL (an analogy to classical measurements of the PTCs). The time intervals between the bursts are 200 ms in duration. The tone is masked by a continuous (in the time domain) band of noise at an arbitrary chosen bandwidth. However, unlike in the classical technique, the centre frequency of the masking band is not constant. The centre frequency gradually changes from the lowest value, usually set an octave below the tone frequency (i.e. from $f_{N\,\min} = 0.5f_t$), to one octave above the frequency of the tone (i.e. to $f_{N\,\max} = 2f_t$). The changes of the noise band centre frequency covers usually two-octave interval, but it may be easily adjusted to be narrower or broader. The rate of the noise band centre frequency change is constant not on a linear frequency scale, but on a logarithmic one, to reflect the tonotopic structure of the basilar membrane in the cochlea. The noise duration is initially set to 4 min (240 s) but it may be easily changed as it is just a parameter of the noise which is usually generated immediately before the measurement.

The measurement method was similar to that used in Békésy audiometry technique [4, 26]. However, here the noise level required just to mask the signal was determined as a function of the masker centre frequency. The measurement of a single PTC usually started with five pulses of the sinusoidal signal without the masker, to show the subject what to listen for. After those initial pulses, the masker was turned on with a centre frequency of $f_{N\,\min}$ and a level of 50 dB SPL. Subjects were requested to press a control button when the signal was inaudible and release the button when they heard the signal. While the subject kept the control button pressed, the level of the noise decreased at a rate set within a range from 0.1 to 8 dB per second. Otherwise, while the control button was released, the level of the noise increased at the same rate. Therefore, one can say that the subjects could have controlled the direction of the noise level change only. The level was changed in 0.1-dB steps. Initial measurements, however, showed that for normally hearing subjects the rate of 2 dB/s was optimal [24] and this rate was used in the present study.

4. Apparatus

The measurements were carried out using a Tucker-Davis Technologies (TDT) System II, controlled by a PC host. The masker and the signal were generated using separate channels of the digital-to-analogue converter (TDT-DD1) at a sampling rate of 32 kHz, with 16-bit resolution. The signals were fed to lowpass filters with cut-off frequency of 8 kHz, TDT-PF1. Then, the signals were delivered to separate programmable attenuators (TDT-PA4), a summer (TDT-SM1), and a headphone buffer (TDT-HB5). The signals were presented monaurally via Sennheiser HD 580 headphones.

For the fast method, the entire masker waveform was pre-calculated, using MATLAB, and stored on the hard drive of the PC. The calculation involved the following steps. The masker spectrum was specified in a series of time frames, and within each frame, the amplitude was specified for 2048 discrete frequency components, extending from 0 to 16 000 Hz, but only those corresponding to the chosen bandwidth within each frame were none-zero ones. The number of time frames, L_k, including 50% overlapping in time domain, can be specified by the following formula:

$$L_k = \frac{2tF_s}{N_0}, \tag{1}$$

where F_s denotes sampling rate, N_0 is the number of points of a single frame and t is the overall duration of the noise. For noise duration of 240 s it gives 7500 time frames. Within the successive frames, centre frequency of the noise band, f_n, was a function of the frame number and for the n-th time frame it could be specified by the following formula:

$$\begin{aligned} f_n &= 2^{\xi}, \\ \xi &= \log_2 f_{N\,\min} + n\frac{N_0 \log_2 \dfrac{f_{N\,\max}}{f_{N\,\min}}}{2tF_s} \end{aligned} \tag{2}$$

which means, that while playing the noise its centre frequency was changed linearly on a logarithmic frequency scale as a function of time. Indeed, the centre frequency ratio of the successive frames was constant:

$$\frac{f_{n+1}}{f_n} = \frac{N_0 \log_2 \dfrac{f_{N\,\max}}{f_{N\,\min}}}{2tF_s} = \text{const.} \tag{3}$$

Amplitude values were set to unity for each component within the desired passband for that frame, and were set to zero for all other components. The phase of each frequency component was randomly chosen from a uniform distribution within a range form 0 to 360 degrees. The spectrum within each frame was converted to a time waveform using the inverse Fast Fourier Transform (iFFT). Each frame resulted in a 64-ms waveform segment. Each of the segments was normalized to the same root-mean-square value and windowed using a Hanning window and successive segments were added together with overlapping of 50% (32 ms) between successive frames. The noise waveform was saved on a hard drive in a *.wav* file format. Prior to the start of a run, the masker waveform was retrieved from the hard drive and transferred to the TDT buffer, to allow continuous replay of the waveform. The noise file was usually generated immediately before the experimental run. For the longest noise duration (240 ms) it took about 20 s. The same method was also implemented on the TDT system III equipment.

5. Results

In the experiment the frequency of the tone was fixed at 2000 Hz and the tone level was 10 dB SL. Also the direction of the centre frequency change of the masking noise was upward only, i.e. the noise always started from the frequency of 1000 Hz and ended up at 4000 Hz. Nine different bandwidths of the masking noise were chosen, i.e. 0% (that stands for a sinusoidal masking signal), 1%, 2%, 3.5%, 5%, 6.5%, 8%, 10% and 15% of the centre frequency of the noise. This means that the relative bandwidth of the masker was kept constant, to mimic the increase in bandwidth of the auditory filter with its centre frequency. These values were chosen to cover a range from the narrowest bandwidth to that approximately equal to the width of the corresponding auditory filter. The rate of the noise band level change was fixed at 2 dB/s. In general the increase in the level rate brings about an increase in the scatter of the results [24]. However, the rate of 2 dB/s was proved to be an optimal one in the case of normally hearing subjects. While measuring the PTCs by means of the classical method five different centre frequencies of the masking noise were used, i.e. 1.2, 1.6, 2, 2.4 and 2.8 kHz and two different bandwidths, namely 10 and 15% of the centre frequency of the masking band.

As in the experimental sessions 11 subjects were tested, individual results obtained for each of them will not be presented. The data gathered for all subjects were very similar. In each case they were characterized by a very prominent minimum at a frequency very close to the tone frequency. Moreover, in each case the low-frequency skirt of the PTC was much shallower (being a function of the making bandwidth) than the high-frequency skirt. However, a typical example of the results of a single experimental run for subject MR in conditions with a 10-% relative bandwidth of the masking noise (left panel) and 15-% bandwidth (right panel) are presented in Fig. 1. The thin, jagged lines in the figure show the data gathered by means of the FAST-PTC method while unfilled circles connected with a continuous straight line show the results gathered for the same subject, using the classical PTC measurement method. Asterisks denote the frequency and the level of the signal. As can be seen in Fig. 1 the data obtained by means of the FAST-PTC method resemble the results of Békésy audiometry. The local maxima and minima (described by means of the level of the masking noise for its centre frequency) show the masking noise parameters at which the subject pressed or released the control button respectively. It is worth to add the scatter of the result is not too large as it does not exceed 10 dB. Based on the presented results it can be stated that the general pattern of the results is very similar to that obtained by means of the classical method. However, a direct comparison of the jagged line with the classical data is not straightforward. Therefore, the data gathered by means of the FAST-PTC were subjected to two-point moving averaging (TPMA). In this transformation, which is a sort of lowpass filtering with the filter impulse response of $\{0.5, 0.5\}$, all two successive data points were averaged. The result of this simplest averaging method is presented in Fig. 1 by a thicker continuous line. The TPMA may be treated as one of useful (usually an initial one) ways of the PTC tip frequency estimation.

As seen in Fig. 1 the results obtained by means of the TPMA transformation are in very good agreement with the data gathered with the use of the classical measurement method. This agreement was observed for a vast majority of the data and was concerned both the position of the PTC minima and the general shape of the PTCs. This agreement was observed for all subjects in the experiment.

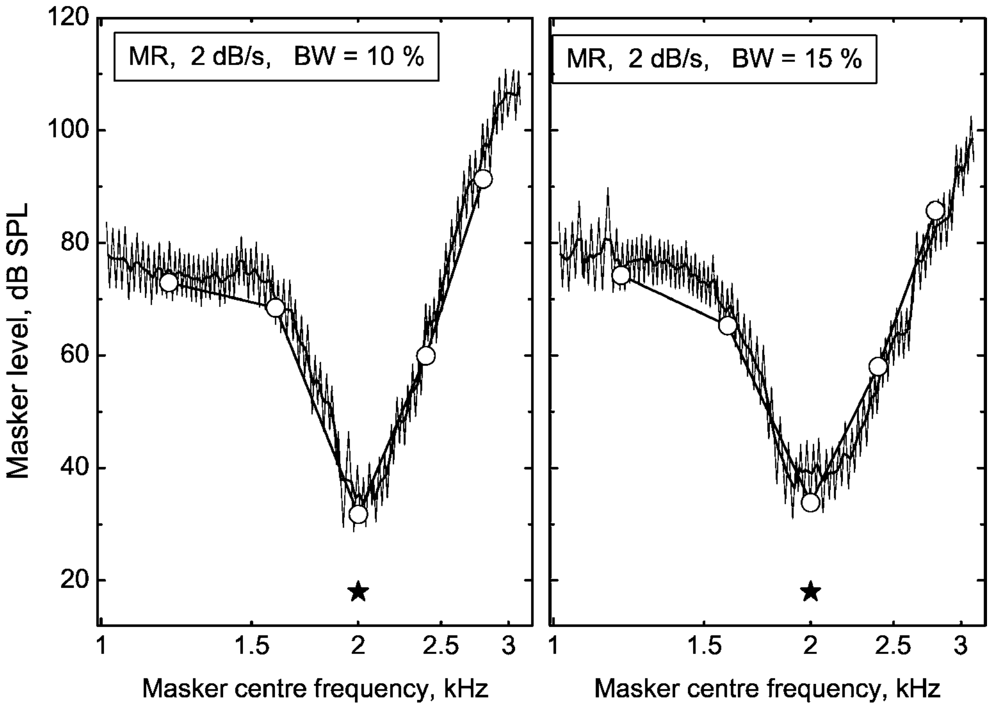

Fig. 1. Examples of the results of single measurements obtained for subject MR, relative bandwidth of 10% (left panel) and 15% (right panel) of the noise centre frequency and noise level rate of 2 dB/s. The results gathered by means of the FAST-PTC method are depicted as a tin, jagged, continuous line while the data gathered by means of the classical method are depicted by means of open circles. The filled stars indicate the level and the frequency of the tone.

6. Estimation of the PTC tip frequency

The above mentioned procedure of the two-point moving average (TPMA) may serve as one of the simplest methods of the PTC minimum estimation. This procedure is very useful especially when the noise band is narrow (i.e. less than 10% of the masker centre frequency, see left panel in Fig. 1). However, for broader bands, for which the minimum in the PC is not that prominent (see right panel in Fig. 1) this procedure is strongly biased by the bandwidth. Therefore, to find the best estimate of the PTC, apart of the TPMA method, several procedures presented below were considered.

6.1. Two linear regression method (TLR)

When the output from the FAST-PTC procedure is plotted on a linear frequency scale broad ranges of the data on the low- and high-frequency side of the curve are linearly correlated with the frequency. These data can be then used to estimate the slopes of the respective parts of the PTC. Based on the slopes the minimum of the PTC can be easily determined as a crossing points of the best fitting straight lines to these parts of the data. This idea was initially presented by SĘK *et al.*, [24] and was also used in this work.

Figure 2 presents a next example of a single measurement of the PTC (subject AW, 2 dB/s) and the method described is based on these data. In the first step of the procedure, the raw data from a single measurement (the thin jagged line) are smoothed using a two-point moving average (the thicker continuous line in the middle of the jagged line). The next step is to determine the frequency at the absolute minimum of the smoothed curve. This frequency is denoted as $f_{L\,\mathrm{min}}$. The segments of the PTC just below and above the absolute minimum can be approximated reasonably well by straight lines, although this approximation breaks down for masker frequencies well below the tip and for masker frequencies very close to the tip, especially for broader bands of noise. Therefore, in the third step, two separate linear regression analyses are performed, one using masker frequencies in a range from $0.75 f_{L\,\mathrm{min}}$ to $0.96 f_{L\,\mathrm{min}}$ and the other one using masker frequencies in a range from $1.05 f_{L\,\mathrm{min}}$ to $1.4 f_{L\,\mathrm{min}}$. However, prior to the regression analysis, the data in the above determined ranges were subjected to an r-Pearson coefficient calculation. For all data gathered in the experiments this coeffi-

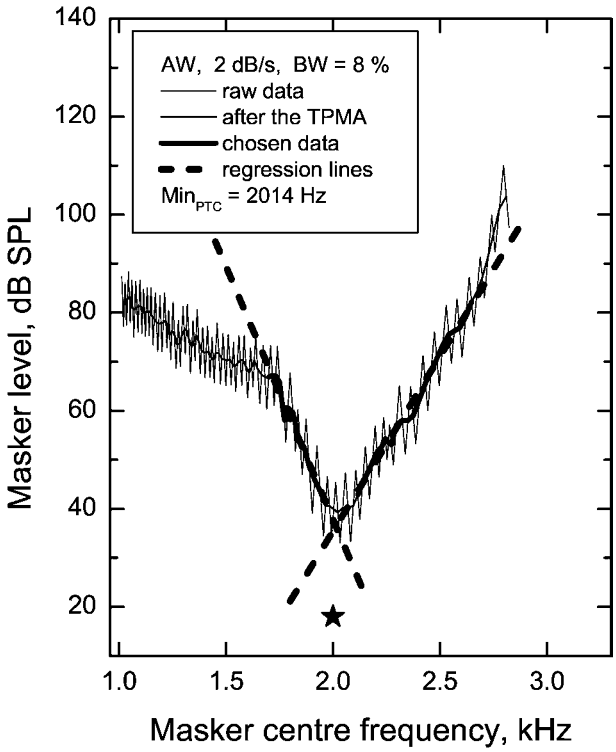

Fig. 2. An example of the TLR procedure for finding the PTC minimum position proposed by SĘK *et al.* [24]. Raw data were gathered for subject AW, noise level rate of 2 dB/s and masker bandwidth of 8%. The filled star indicates the level and the frequency of the tone.

cient was typically close to 0.92 (average value) and its value was never less than 0.81. The data subjected to the linear regression analyses are indicated in Fig. 2 by the thickest parts of the smoothed curve. The fitted regression lines are shown in Fig. 2 as the dashed lines. In the final step, the intersection point of the two fitted regression lines is determined. This intersection point may be considered as an objective estimate of the tip frequency of the PTC.

The TLR method was also used to estimate the PTC minimum based on the classical measurements of the PTC. The low- and high-frequency part of the PTC were subjected to the analysis of regression and the intercept of the two straight lines was taken as the PTC tip position.

6.2. Parametric approximation methods (ROEX and SQ)

As stated in Sec. 1.2, when linearity of the auditory system is assumed, the frequency characteristics of the auditory filter can be obtained simply by inverting the PTC. The squared frequency response of the auditory filters can be approximated by the rounded exponential function $(\mathrm{roex}(f_c, a, b, c, d, e))$, originally applied to the description of the auditory filter shape [2, 15], i.e.:

$$\mathrm{roex}(f, f_c, a, b, c, d, e) = \begin{cases} a(1 + b|f - f_c|)\exp(-b|f - f_c|) + e(c - a) & f < f_c\,, \\ c(1 + d|f - f_c|)\exp(-d|f - f_c|) + e & f \geq f_c\,, \end{cases} \tag{4}$$

where b and d define the left and right slope of the filter respectively, f_c denotes its center frequency while a, c, e are coefficients that determine the filter gain. The data gathered in the experiment can be modeled as a sum of the model curve (roex) and an error function, ε:

$$d(f) = \mathrm{roex}(f, f_c, a, b, c, d, e) + \varepsilon(f, f_c, a, b, c, d, e), \tag{5}$$

where $d(f)$ denotes the data. An iterative approach is required in order to find the values of parameters that give the smallest error function ε. The iterative procedure may be summarized in following steps. First, initial values of f_c, a, b, c, d, and e are chosen. In the second step fitted curve for current coefficients (f_c, a, b, c, d, e) is determined and squared error is calculated. In the next step the coefficients are adjusted with the *trust-region* algorithm [13]. Function E which describes the sum of squared error for a given set of parameters f_c, a, b, c, d, e.

$$E(f_c, a, b, c, d, e) = \sum_f \varepsilon^2(f, f_c, a, b, c, d, e) \tag{6}$$

is approximated with a simpler function which reasonably well reflects the behavior of function E in the neighborhood N around point (f_c, a, b, c, d, e). The above steps are repeated until the convergence criteria are reached by the fit. The estimate of the PTC tip is the value of the f_c parameter of the fitted curve. An example of calculation by means of this procedure is shown in Fig. 3.

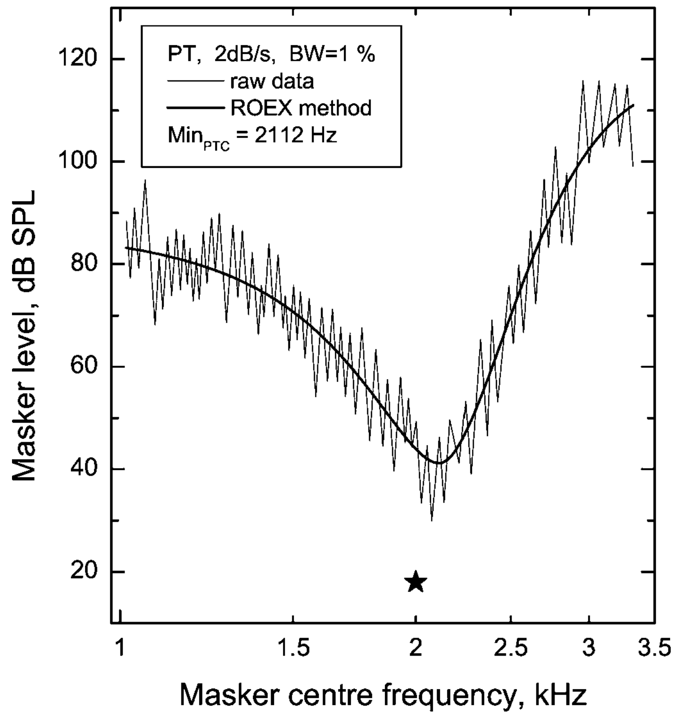

Fig. 3. An example of the PTC tip position estimation by means of the ROEX procedure (continuous line). Raw a data were gathered for subject PT, noise level rate of 2 dB/s and masker bandwidth of 1%. The filled star indicates the level and the frequency of the tone.

Another parametric method that was applied to the gathered data was based on the simple square function i.e.:

$$\mathrm{PTC} = af^2 + bf + c, \tag{7}$$

where a, b and c parameters are to be determined.

As in the TLR method, initially a two-point moving average was calculated based on raw data and the argument of the minimum of the average $(f_{\mathrm{AV\_MIN}})$ was found. Next, the averaged data were limited to the following frequency region, allowing to choose the data situated very close to the PTC tip only:

$$0.85 f_{\mathrm{AV\_MIN}} \leq f \leq 1.15 f_{\mathrm{AV\_MIN}}. \tag{8}$$

In the next step an iterative procedure was searching for the a, b and c parameters of the square function that give the smallest sum of squares of differences between raw data and fitted square function. The PTC tip position was then calculated as a ratio of respective parameters of the square function. An example of PTC minimum betermined by means of this procedure is shown in Fig. 4.

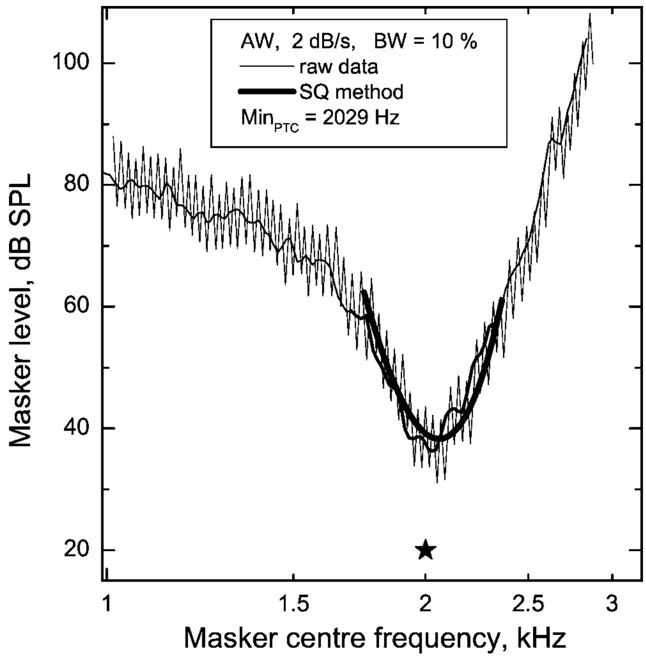

Fig. 4. An example of the PTC tip position calculation by means of the SQ method (continuous line). Raw data were gathered for subject AW for masker bandwidth of 10%. The filled star indicates the level and the frequency of the tone.

6.3. Double low-pass filtering (DLF) method

The data gathered in the PTC measurements using the FAST-PTC method can be considered as samples of a signal in the time domain: successive data points are separated by certain time lags that roughly correspond to a certain shift of the noise band centre frequency. The signal contains two basic components: a low- and high-frequency one. The low-frequency component corresponds to the PTC to be determined while the high-frequency one reflects an unwanted noise reflecting the reaction time and concentration of subjects and depends on the rate of the noise level change [24]. Thus it seems that a reasonable method aiming at the determination of the frequency position of the PTC should be a low-pass filtering of the collected data points. Therefore a simple Butterworth low-pass filter can be used to filter out the high frequency component included in the raw data. However the filtering by means of a low-pass filter gives an unwanted linear phase shift, that may influence the final result. To compensate for this shift, after low-pass filtering the resulting signal was reversed and filtered again by means of the same filter. An example of results obtained by means of this procedure is shown in Fig. 5. An important problem that must be considered before the method is used concerns the cutoff frequency, f_{cutoff}, of the filter. It is reasonable to assume that a given PTC constitutes a single period of a signal to be determined while the noise-like component has much higher frequency. A single measurement run gave at least 100 data points to be averaged, therefore it can be assumed that the sampling frequency was at least 100 times higher than the signal frequency to be determined. Thus, the cutoff frequency of the low-pass filter can be very close to the Nyquist frequency. Initial calculation showed, however, that if the cutoff frequency was expressed in terms of the Nyquist frequency, the best fit to the experimental results was obtained when $f_{\text{cutoff}} = 0.5$ (see Fig. 5). In the initial testing of the DLF method it was also found that the order of the filter did not influence markedly the quality of the fit, therefore it was assumed that a fourth order of the low-pass filter would be used. An example of the fitted line for cutoff frequency of 0.5 (expressed in terms of the Nyquist frequency) is shown in Fig. 5.

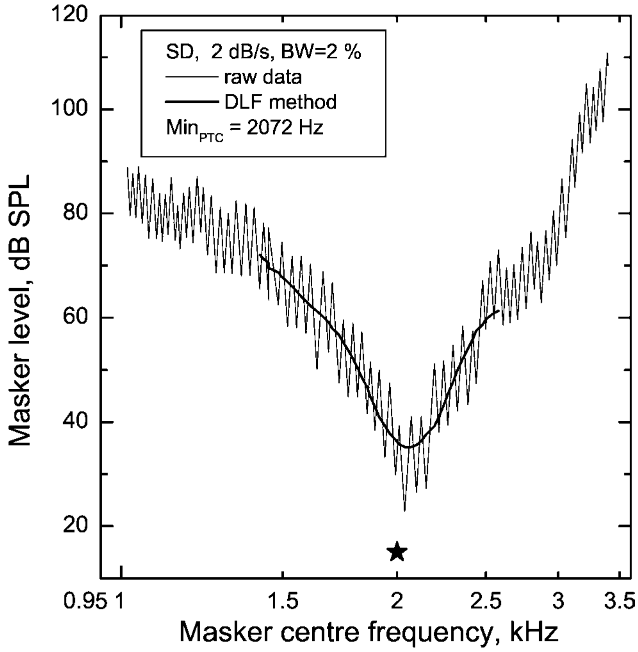

Fig. 5. An example of the PTC tip position determination by means of the DLF method (continuous line). Raw data were gathered for subject SD for masker bandwidth of 2%. The filled star indicates the level and the frequency of the tone.

6.4. Savitzky–Golay Filtering (SGF) method

Another smoothing method that has been applied to the gathered data was the Savitzky–Golay filtering (SGF) method. The method is also known as a digital smoothing polynomial filtering or least-squares filtering method. The method is usually used to smooth out a noisy signal whose frequency span is large, what actually happens in the case of the FAST-PTC measurements. It is well known that the performance of the SGF method in this type of application is much better than standard averaging by means of FIR filters, as the filters tend to filter out a significant portion of high-frequency content of the signal along with the noise. On the other hand it is necessary to add that the SGF method is an optimal one as it minimizes the least-squares error in curve fitting of a polynomial to each frame of data.

In this method the signal is divided into n-point frames of successive data points. In each frame the signal is approximated with a polynomial of r-th order. Therefore two parameters must be arbitrary chosen, i.e. the polynomial order r and the frame length n. An example of using the SGF method is shown in the left panel in Fig. 6 for $r = 2$ and $n = 11$. As can be seen in this figure, the SGF method preserves high-frequency components of the signal and therefore is less efficient at rejecting noise than the double low-pass filtering method.

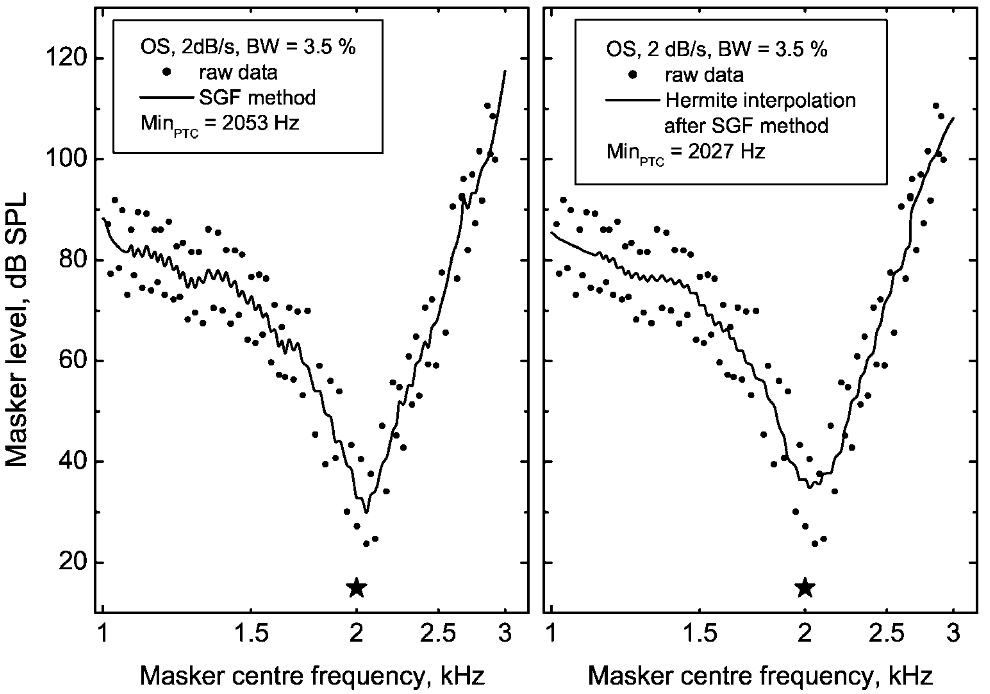

Fig. 6. An example of the PTC minimum determination by means of SGF method for subject OS and masker bandwidth of 3.5%. Left panel shows raw (filled circlers) and smoothed raw data (solid line) while the right one raw data as well as smoothed and interpolated data. The filled stars indicate the level and the frequency of the tone.

After initial smoothing the data have to be interpolated. To preserve a curve shape cubic spline, or Hermite interpolation can be used. The right panel of Fig. 6 shows smoothed and interpolated data by means of the Hermite interpolation method. The last step is to find a global minimum of the smoothed and interpolated data.

7. Statistical analysis of the data

In the first step of statistical analysis a comparison of the PTC minima estimated with the use of the classical and the fast measurement methods was carried out. With respect to the results of the classical measurements, in which four out of eleven subjects were tested only, the TLR method was used. To confirm an agreement between the data gathered by means of the FAST-PTC and classical measurement method the data were subjected to the within-subject analysis of variance (ANOVA) with the following factors: bandwidth (10 and 15% of the noise centre frequency), measurement method (the classical and the FAST-PTC) and four repetitions of each measurement. The results of the analysis proved that neither the bandwidth nor measurement method were significant. Also the interaction between the factors was not significant. This result is entirely consistent with the data presented by SĘK *et al.*, [24]. It suggests that the FAST-PTC method yields similar results to those obtained by means of the classical measurements. However the FAST-PTC method is markedly faster and therefore may be used instead of the classical method and provide the same results.

As described above the data gathered by means of the FAST-PTC measurement method enabled estimation of the PTC minima by means of five different methods. The estimates were subjected to a two-way within-subject analysis of variance (ANOVA) by means of the GenStat package with the following factors: bandwidth (9 values) and estimation method (5 values). For 10 out of 11 subjects four repetitions of a single measurement were taken into account. However, for one subject only three repetitions were available only, therefore the 45 missing values were approximated by means of the software package. This type of ANOVA analysis is equivalent to a two-way analysis of the mean PTC tip estimations if mean values across repetitions are taken into account. The grand mean of the all estimated PTC tips was equal to 2054 Hz, which means that frequency coinciding with the PTC minimum was slightly higher than that of the tone. The factor of method was not statistically significant $[F(4,10) = 1.30,\ p = 0.285]$. Indeed the range of the mean PTC tip position was very small and equal to 13 Hz only, i.e. from 2058 Hz (the TLR method) to 2045 Hz (the ROEX method). This means that all the PTC estimation methods yielded very similar results as far as the position of the PTC minimum is concerned. The factor of bandwidth turned out to be statistically insignificant: $[F(8,80) = 1.87,\ p = 0.095]$. The range of the mean PTC location in the frequency domain was also very narrow, equal to 16 Hz, i.e. from 2039 Hz for 15-% bandwidth to 2062 Hz for 1-% bandwidth. Therefore one can say that as far as the bandwidth of the masking noise does not exceed the width of the appropriate auditory filter, it does not influence the PTC tip position. However, it is also worth to add that it does not mean that the bandwidth does not influence the shape of the PTC (i.e. slopes of the low- and high-frequency side of the PTC). The interaction between the analyzed factors was also statistically insignificant $[F(32,320) = 1.24,\ p = 0.176]$. This means that a simultaneous change of the masker bandwidth and the estimation method does not markedly change the position of the PTC minimum.

8. Discussion

Similarly to what was reported by SĘK *et al.*, [24] the FAST-PTC measurement method yielded estimates of the PTC minima very close to those obtained in classical measurements. As the time needed for a measurement with the FAST-PTC method is markedly shorter the proposed method may be very useful for the auditory system diagnosis.

However, the main purpose of the present paper was to test several methods of the PTC minimum estimation. Therefore Fig. 7 presents histograms of the estimated PTC minima obtained by means of five different methods as well as the means, the ranges and the standard deviations. As the bandwidth of the masking noise came out to be statistically insignificant the data gathered for all bandwidths and all subjects were combined.

One of the most striking conclusions that can be easily drawn from the present data is that in nearly each case the mean of the PTC tip is higher than the frequency of the masked signal used in the experiment (grand mean of 2054 Hz). This tendency is observed for each of the method of the PTC tip calculation. The highest efficiency of

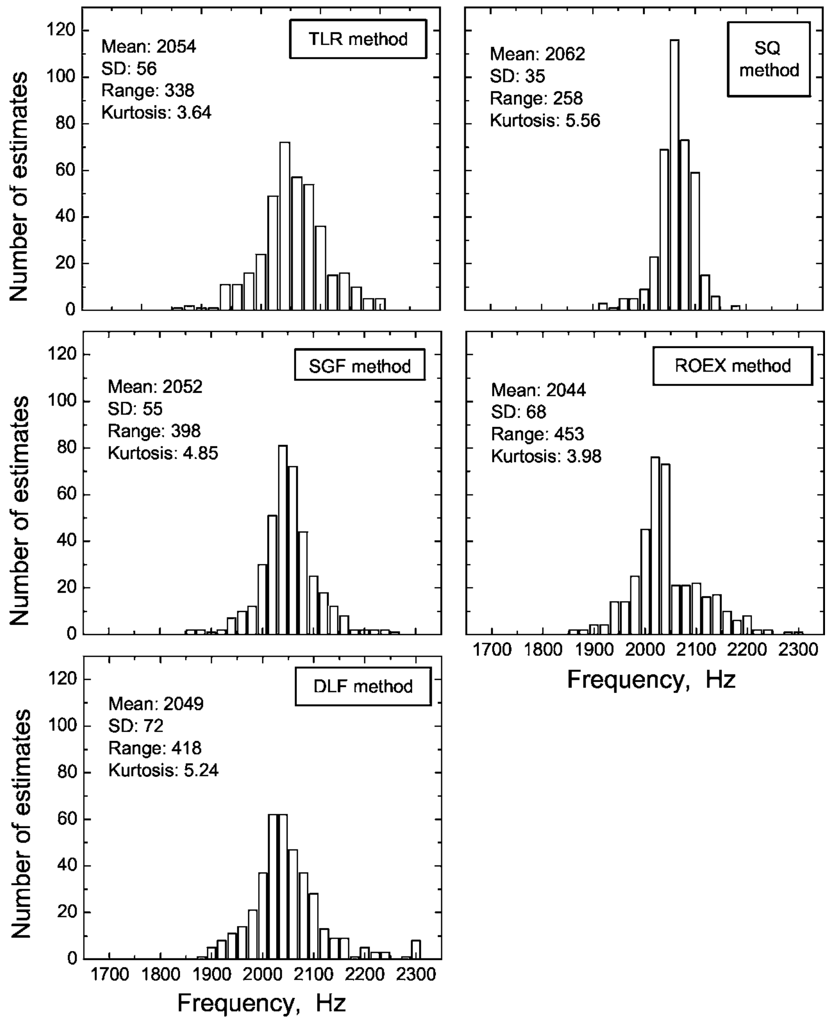

Fig. 7. Histograms of the estimated PTC minima obtained for five different methods.

masking, as well as PTC tips positions were rather expected to coincide with the frequency of the masked tone. However, the results obtained for each method and each bandwidth of the masking noise were always slightly higher than the frequency of the tone. A similar effect was also observed by SĘK *et al.*, [24]. One of potential problems related with the application of the FAST-PTC method is the fact that the subject's response (pressing or releasing the button) is slightly delayed with respect to the change in percept. For example, when the subject presses the button to indicate that the signal is inaudible, the masker level and its centre frequency increase, and the subject then releases the button *after* (i.e. with a certain delay) the signal becomes audible. Because of this delay, the tip frequency of the PTC may fall above the signal frequency when the masker frequency is sweeping upwards. An opposite effect can be also expected, i.e. the PTC minimum is below the signal frequency, when the band of noise is sweeping downwards. Indeed, for each subject and each signal frequency used by SĘK *et al.* [24], the mean of estimated PTC minima for an upward sweep was above that obtained for downward sweep. Therefore this effect was called by SĘK *et al.*, [24] the hysteresis effect. However the hysteresis effect is rather week and the shift of the PTC minimum has never exceeded 3% of the signal frequency.

The present data clearly show an upward shift of the PTC minimum, as only upward sweeps of the centre frequency of the masking noise were used. However, although the effect is clearly visible its magnitude does not exceed 3% of the tone frequency, which is exactly the same as reported by SĘK *et al.*, [24].

In general, the five methods of PTC minimum estimation provided very similar results falling in a very narrow frequency range and the differences between the data obtained by those methods are not statistically significant. However, as can be seen in Fig. 7, the estimates of the PTC minimum are characterized by a certain scatter which strictly depends on the method. The smallest scatter, measured by means of standard deviation, was obtained for one of the simplest methods, namely the SQ method (35 Hz) while the highest scatter was found for the DLF method (72 Hz). The largest number of estimates being close to the mean was obtained for the SQ method and the smallest number for DLF method. Also the range of the results is markedly different for the methods used. The smallest range was obtained for the SQ method (258 Hz) while the largest for the ROEX method (453 Hz). However, in the case of the SQ method the difference between the mean of PTC tip positions and the signal frequency was the largest.

One of the best parameters that describe the concentration of the distribution of results is the kurtosis. Kurtosis is defined as the ratio of the fourth cumulant and the square of the variance. A high-kurtosis distribution has a sharper peak, while the peak of a low-kurtosis distribution is more rounded. High-kurtosis distributions are also characterized by a large scatter of the results, but most of the data are concentrated around average value. Low-kurtosis distributions may be characterized by means of slightly smaller data scatter, but the data are more evenly distributed in the available range of values. As can be seen in Fig. 7 the highest kurtosis, reaching 5.56 was obtained for the SQ method for which the standard deviation was the smallest. The lowest kurtosis, on the other hand, was obtained for the TLR method (3.64).

To summarize the above it should be stressed that all five methods of the PTC minimum assessment gave very similar results if the mean value is considered. This finding was confirmed by the analysis of variance, as the factor of method was not significant. However, the histograms plotted for the five methods (see Fig. 7) revealed significant differences between the distributions of results. If the kurtosis and standard deviation are taken into account, one may say that the best method for the PTC minimum estimation seems to be the approximation of raw data by means of a square function (see Eq. (7)). If this method is applied to the central part of the raw experimental data selected based on the minimum of the TPMA transform (see Eq. (8)), then the estimates of the PTC minima are characterized by the smallest SD and highest kurtosis. This means that most of the estimates are situated in a narrow frequency range close to the mean value. It is worth to add that the mean PTC minimum for this method was not the closest to the frequency of the tone. However both, kurtosis, standard deviation as well as the range of the obtained assessments suggest that this method was the best among the methods used.

The data presented in this paper were gathered for nine different bandwidth of the masking noise and for one centre frequency of the signal. However, the bandwidth of the masking noise, as intended, was usually narrower than the auditory filter bandwidth whose centre frequency corresponded to the instantaneous centre frequency of the noise band and hardly exceeded its width. As came out from the analysis of variance the bandwidth was proved to be statistically insignificant. The highest estimate was obtained for 1-% bandwidth (2062 Hz) while the lowest for the 15-% bandwidth (2040 Hz). The change in the bandwidth brought about a merely 1-% change in the position of the PTC minimum if averages across the subjects and the estimation methods are considered. However, a variation of the PTC tip as a function of the estimation method for each bandwidth is slightly larger and reaches about 50 Hz.

It is worth to add that the variation of the PTC minimum estimates was markedly lower for the SQ method which was chosen as the best one. In the case of this method the highest (2069 Hz) and the lowest (2058 Hz) estimates were determined for the 1-% and the 15-% bandwidth respectively. This confirms that the PTC tip position does not depend on the bandwidth of the noise band.

However, even though a clear dependency between the bandwidth of the masking noise and the PTC position has not been found it does not mean that the shape of the PTC remains unchanged as the bandwidth changes. For the narrowest masking noise bands the beats coming from the superposition of the signal and the masker may be used as additional cues by the subjects and may influence the shape of the PTC [7]. It is best to use a possibly widest masker band to avoid problems associated with beats as an additional detection cue. However, if the masker bandwidth is too wide (i.e. much wider that the width of the auditory filter centered at the signal frequency), this may make it difficult to estimate the position of the PTC tip in the frequency domain. A reasonable compromise for signal frequencies below 2000 Hz, would be to use a masker bandwidth close to the width of the auditory filter centered at the signal frequency. This is in a good agreement with the recommendations of Kluk and Moore [7] for traditional PTCs.

9. Conclusions

The data gathered in this work as well as the use of five different methods of the PTC minimum estimation enable to formulate the following conclusions:

• The FAST-PTC measurement method yields nearly the same estimates of the PTC tip positions as the classical measurement method.

• For an upward sweep in the centre frequency of the noise band the PTC minimum occurs at a slightly higher frequency than the tone frequency. The frequency shift is less than 3% of the tone frequency.

• Among the five different methods of the PTC minimum determination the method based on a simple square function (SQ method) yielded the best results. Using this particular method the highest kurtosis, the lowest standard deviation and the narrowest range of the determined PTC minima were obtained. However for this method the difference between the mean of the PTC tip positions and the signal frequency was the largest.

• The position of the PTC minimum does not depend on the bandwidth of the masking noise. This is true for bands of noise no wider than the width of the auditory filter whose centre frequency is equal to the centre frequency of the masking noise band.

• The FAST-PTC method proved to be as precise as the classical measurement of the PTC. However the FAST-PTC method is much less time consuming and therefore should be considered as a method for frequency selectivity determination, fully equivalent to other methods. The method can be also used to determine the frequency boundaries of dead regions.

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