ARCHIVES OF ACOUSTICS
32, 3, 737–748 (2007)

PERCEIVED ROUGHNESS OF TWO SIMULTANEOUS HARMONIC COMPLEX TONES

Andrzej MIŚKIEWICZ, Tomira ROGALA, Joanna SZCZEPAŃSKA–ANTOSIK

The Fryderyk Chopin Academy of Music
Department of Sound Engineering
Musical Acoustics Laboratory
Okólnik 2, 00-368 Warszawa, Poland
e-mail: misk@chopin.edu.pl

(*received May 29, 2007; accepted July 24, 2007*)

Two experiments were carried out to determine the dependence of perceived roughness on the frequency ratio of two simultaneous harmonic complex tones. In the first experiment, the frequency ratios of the tone pairs corresponded to 35 within-octave intervals of various musical tuning systems. In the second experiment 12 intervals were used; six of them ranged from 10 cents below to 10 cents above an equally-tempered fourth and the other six encompassed a similar range centred around the equally-tempered fifth. In both experiments the amount of roughness was assessed by absolute magnitude estimation. Results show that roughness considerably varies with the frequency ratio of a pair of harmonic complex tones, which is a well-known phenomenon. A new finding, that is in contrast to published theories of roughness, is that equally-tempered intervals produce less roughness than their counterparts based on integer frequency ratios. This effect is attributed to slow beats that arise between the harmonics of two complex tones when the frequency ratio of an equally-tempered interval slightly departs from integer ratio. Such beats, heard as fluctuations, impart a smooth character to the sound.

Keywords: timbre, roughness, dissonance.

1. Introduction

This article reports a study carried out to determine the dependence of perceived roughness on the frequency ratio of two simultaneous harmonic complex tones. Roughness is a characteristic auditory sensation elicited by rapid temporal variations of sound, such as beats, amplitude modulation, and frequency modulation. Roughness produced by two simultaneous harmonic complex tones is an effect of beats that occur between the harmonics of the tones.

The amount of roughness elicited by beats depends on their rate. For a pair of pure tones, roughness reaches a maximum at a certain beat rate, equal to the difference in frequency between the tones, and gradually decreases below and above that rate [5, 8]. Such a simple relation of roughness to the tone frequencies does not hold for a pair of harmonic complex tones in the case of which roughness is a compound effect of beats with different rates, produced by various combinations of the component tones of the two harmonic complexes. As the frequencies of two harmonic complex tones are moved apart, roughness markedly fluctuates so that its several local maxima and minima may be observed at certain values of frequency difference [3, 6, 10].

The relation of roughness to the frequencies of a pair of harmonic complex tones was first described by von HELMHOLTZ [2] as an acoustical explanation of musical consonance and dissonance. In his theory von Helmholtz postulated that musical dissonance is an effect of the sensation of roughness evoked by beats; a dyad or a chord is consonant when it does not produce roughness. Classification of intervals into consonant and dissonant has been a fundamental problem to music theory and musical acoustics since the times of ancient theorists. Traditionally, consonance has been associated with intervals represented by a ratio of small integers. According to von HELMHOLTZ's theory [2] the reason why the sound of two simultaneous harmonic complex tones with small integer frequency ratio is consonant is that the harmonics of the two tones either coincide in frequency and produce no beats, or are spaced far apart on the frequency scale, and the beats occur at rates exceeding the range that evokes the sensation of roughness.

TERHARDT [9] proposed the term "sensory dissonance" to distinguish the purely sensory component of dissonance related with the sensation of roughness from dissonance perceived in a musical context which is a compound effect, influenced by various musical factors. It has been demonstrated in a number of experiments [1, 6, 7] that auditory judgements of roughness are highly correlated with those of unpleasantness and sensory dissonance. The terms "roughness", "unpleasantness" and "sensory dissonance" have been therefore often used in the literature interchangeably, with reference to the sensation produced by beats [3, 6].

Published data on the sensation of roughness produced by a pair of harmonic complex-tones were either predicted from calculation models based on auditory judgements of roughness elicited by beats of pure tones [3, 7] or from relative judgements of consonance or dissonance made in a paired-comparison task [10]. The purpose of the present study was to map out the dependence of roughness on the frequency ratio of dyads corresponding to various within-octave musical intervals, by direct judgement of the sensation magnitude called explicitly "roughness". To do so, two experiments were carried out in which the judgements of roughness were obtained using the method of absolute magnitude estimation.

2. Experiment I

2.1. Method

The stimuli were dyads formed by combining two harmonic complex tones, each composed of the fundamental and nine harmonics, with a decreasing amplitude envelope of 6 dB/oct. All component tones of the harmonic complexes were gated on with a 0 phase. Each dyad was 1 s in duration, including a 25-ms rise and fall. Fundamental frequency of the lower complex tone in a dyad was 261.6 Hz and corresponded to the note C4 on the equally-tempered musical scale; the upper tone's fundamental frequency depended on the frequency ratio of the two tones.

The set of stimuli comprised 35 dyads; twenty four of them were within the span of an octave in equally-tempered, quarter-tone steps and the remaining nine dyads formed various within-octave intervals based on integer frequency ratios. Table 1 lists the frequency ratios of the intervals, their size in cents, and – where applicable – the musical name of the interval. The interval size, n, in cents, was calculated using the following formula:

$$n = 1200 \frac{\log\left(\dfrac{f_2}{f_1}\right)}{\log 2}, \tag{1}$$

where f_1 and f_2 are respectively the frequencies of the lower and the upper tone in Hz.

The dyads were generated using a PC-compatible computer with a signal processor (TDT AP2) and a 16-bit digital-to-analogue converter (TDT DD1) with a 50-kHz sampling rate. The signal at the converter's output was low-pass filtered (TDT FT5, $fc = 20$ kHz), attenuated (TDT PA4), and led to a headphone amplifier (TDT HB6) which fed one earphone of a Beyerdynamic 911 headset. All dyads were presented at a loudness level of 50 phons. The signal level was determined for each dyad individually, by measuring the loudness of sound reproduced through the earphone and adjusting the setting of the attenuator to obtain a 50-phon loudness level. The loudness level was measured with the use of an artificial ear (B&K type 4153), a 1/4-inch microphone (B&K type 4134), and a spectrum analyzer (B&K type 4144) equipped with software for the measurement of loudness, according to Zwicker's procedure (ISO, 1966). Earphone calibration was 103.8 dB SPL for a 1-V input.

The judgements of roughness were made using the method of absolute magnitude estimation [11]. The listeners were tested individually in a sound-proof booth. A series of judgements comprised 35 dyads presented in random order. The listener's task was to assign a number to the amount of roughness produced by each dyad. The listener activated a single presentation of a dyad by pressing a button on the response box and could repeat the presentation at will before reporting the number through an intercom to the experimenter. The experimenter entered the number to the computer and a visual signal was displayed on the response box in the booth to indicate a next judgement. In accordance with the procedure of absolute magnitude estimation described by ZWISLOCKI and GOODMAN [11], the listeners were instructed to use only positive numbers in their judgements. They also were told to judge the roughness of each dyad separately, that is not to think about numbers assigned to preceding dyads in a series, while making a judgement.

Table 1. Frequency ratios, size in cents, and names of musical intervals used in Experiment I. The integer frequency ratios are shown in parentheses.

	Frequency ratio	Interval size (ct)	Interval name
1	1.0000	0	unison
2	1.0125	22	syntonic comma (81:80)
3	1.0293	50	equally-tempered quarter tone
4	1.0595	100	equally-tempered semitone
5	1.0905	150	
6	1.1225	200	equally-tempered major second
7	1.1554	250	
8	1.1892	300	equally-tempered minor third
9	1.2000	316	just minor third (6:5)
10	1.2241	350	
11	1.2500	386	just major third (5:4)
12	1.2599	400	equally-tempered major third
13	1.2660	408	Pythagorean major third (81:64)
14	1.2968	450	
15	1.3348	500	equally-tempered quarter
16	1.3740	550	
17	1.4142	600	tritone
18	1.4557	650	
19	1.4983	700	equally-tempered fifth
20	1.5000	702	just fifth (3:2)
21	1.5422	750	
22	1.5874	800	equally-tempered minor sixth
23	1.6000	814	just minor sixth (8:5)
24	1.6339	850	
25	1.6670	885	just major sixth (5:3)
26	1.6818	900	equally-tempered major sixth
27	1.7311	950	
28	1.7500	969	harmonic minor seventh (7:4)
29	1.7818	1000	equally-tempered minor seventh
30	1.8000	1018	just minor seventh (9:5)
31	1.8340	1050	
32	1.8750	1088	just major seventh (15:8)
33	1.8877	1100	equally-tempered major seventh
34	1.9431	1150	
35	2.0000	1200	octave (2:1)

Sixteen students, 19–23 years old, with normal hearing (10 dB HL or less, at audiometric frequencies from 0.25 to 8 kHz), served as listeners. All of them were sound engineering majors at the Fryderyk Chopin Academy of Music and had previous experience in absolute magnitude estimation of auditory sensations in classroom demonstrations. Each listener completed five series of judgements, so that a total of 80 judgements was obtained for each dyad (16 listeners $\times 5$ judgements).

2.2. Results and discussion

Results of roughness scaling are shown in Fig. 1. The main abscissa is the size in cents of the interval formed by a dyad and the secondary abscissa is the frequency ratio corresponding to that interval. The data are geometric means of 80 estimates multiplied by a constant such that the maximum of roughness obtained in the experiment equals 1.

As seen in Fig. 1, roughness markedly changes as the interval between the two tones is increased from a unison (0 ct) to an octave (1200 ct). Maximum roughness is produced by an interval of 150 ct (frequency ratio 1.0905) and the lowest values of roughness are obtained for the unison and the octave.

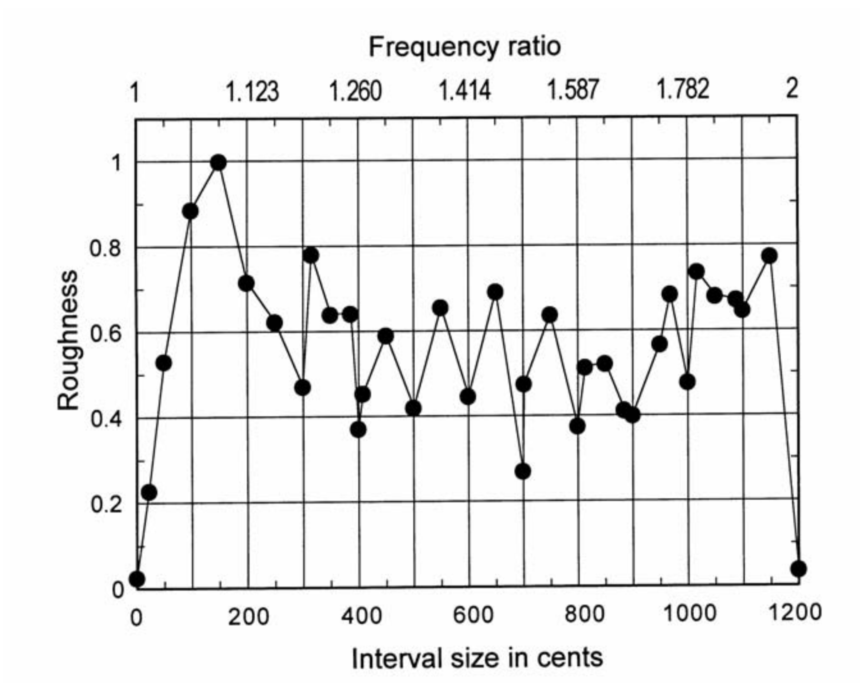

Fig. 1. Roughness of musical intervals composed of two simultaneous harmonic complex tones. The primary abscissa is the interval size in cents and the secondary abscissa is the interval's frequency ratio. Plotted are geometric means of 80 judgements (16 listeners $\times$ 5 series of judgements) multiplied by a constant such that the roughness maximum equals 1.

In Fig. 2 compared are roughness values obtained for intervals based on integer frequency ratios and for their equally-tempered counterparts. The data are geometric means replotted from Fig. 1. Figure 2 shows that equally-tempered intervals produce less roughness than intervals with integer frequency ratios. This difference is largest for the perfect fifth, the major third, and the minor third, in the case of which the roughness values obtained for the equally-tempered intervals are only 57–60% of the roughness produced by the respective integer-ratio intervals.

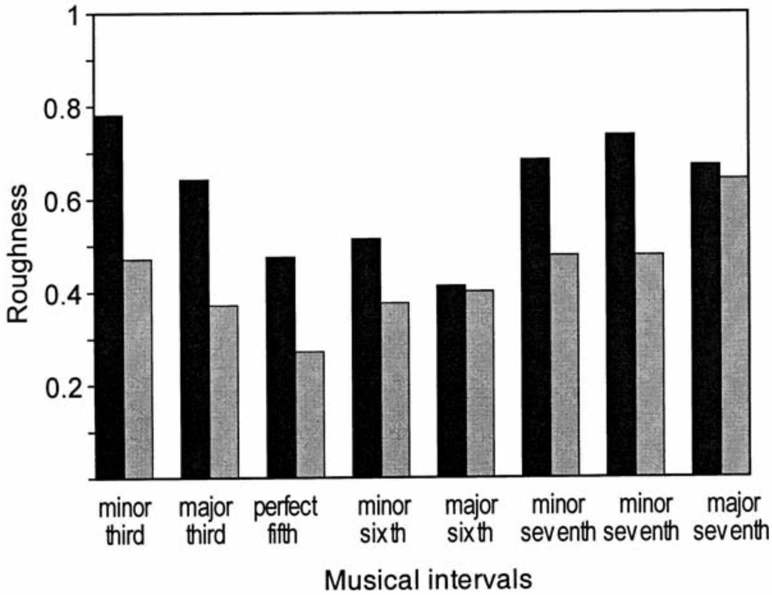

Fig. 2. A comparison of roughness judgements obtained for intervals based on integer frequency ratios (left bar in each pair) and for intervals of the equally-tempered scale, ETS (right bar). The data are group geometric means replotted from Fig. 1.

To examine whether the differences in roughness observed between integer-ratio and equally-tempered intervals were statistically significant, a t-test for grouped data was performed for the pairs of intervals compared in Fig. 2. The calculations were made for individual data transformed on a logarithmic scale. The values of probability distribution, p, were less than 0.001 for the minor third and the major third, less than 0.01 for the perfect fifth and for the two versions of the minor seventh, and less than 0.03 for the minor sixth. The differences in roughness were therefore statistically significant for the above pairs of integer-ratio and equally-tempered intervals. For the major sixth and the major seventh the p values exceeded 0.05 so the differences in roughness could not be considered significant.

In order to find an explanation for the difference in roughness observed for equally-tempered intervals and their integer-ratio counterparts we calculated the rates and amplitudes of beats produced by combinations of component tones of the two harmonic complex tones composing a dyad. The calculations were made for six out of eight pairs of dyads shown in Fig. 2, for which the differences in roughness were statistically significant. The rate of beats, f_{jk}, elicited by component j of one complex tone and component k of the other one is:

$$f_{jk} = |f_j - f_k| \,, \tag{2}$$

where f_j and f_k are the frequencies of components j and k. The amplitude of beats, A_{jk}, is:

$$A_{jk} = A_j + A_k - |A_j - A_k| \,, \tag{3}$$

where A_j and A_k denote the amplitudes of component tones j and k. The amplitudes of beats, calculated using equation (3), were then converted to decibels. The calculated beat rates and amplitudes are shown for each dyad in separate panels in Fig. 3, for integer-ratio (left column) and equally-tempered (right column) intervals. The beat rates are represented by the location of the bars on the abscissa; the dots on a bar indicate beat amplitudes produced at a given rate by different pairs of component tones. The value of 0 dB on the ordinate is the maximum beat amplitude produced by the fundamental tones of the two harmonic complexes.

An apparent difference in the distribution of beat rates in integer-ratio and equally-tempered intervals is that the range of beat rates produced by equally-tempered intervals is extended down to lower values (Fig. 3). This difference is most readily seen in the case of the fifth: the lowest beat rate produced by a just perfect fifth (3:2) is 130.8 Hz whereas the equally-tempered fifth produces beats at as low rates as 0.9, 1.8, and 2.7 Hz. Such very slow beats do not produce roughness but are heard as fluctuations of sound [5]. During an informal session, after completion of the judgements of roughness, we played back the two versions of the fifth to some of the listeners and asked them to describe the difference in the sound character of those intervals. The listeners all agreed that the equally-tempered fifth produced a smoother sound that to some extent resembled vibrato.

The above explanation for the lesser roughness of equally-tempered intervals may also hold for the minor third, the major third, and for the major sixth. As seen in Fig. 3, the equally-tempered variants of those intervals produce beats at rates below 20 Hz, a range that does not produce a pronounced sensation of roughness [5, 7]. In the case of the minor seventh, the reason for the lesser roughness produced by the equally-tempered interval cannot be ascribed to the effect of slow beats and remains unclear. The lowest beat rate produced by the component tones of the equally-tempered seventh is about 30 Hz (Fig. 3) and falls in the range of beat rates that produce an intense sensation of roughness [5, 7].

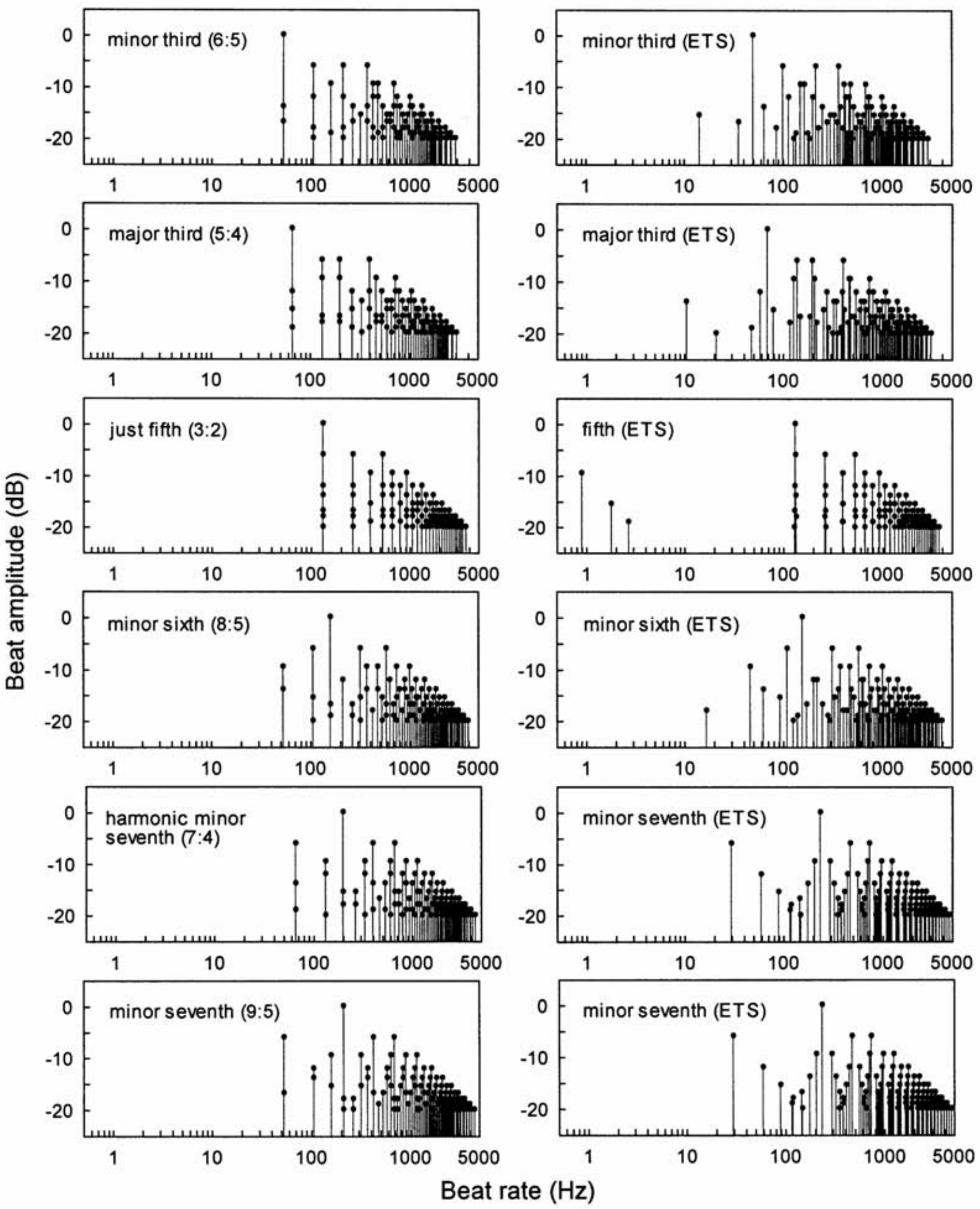

Fig. 3. Rates and amplitudes of beats produced by combinations of the components of two harmonic complex tones constituting various musical intervals based on integer frequency ratios (left column of panels) and intervals of the equally-tempered scale, ETS (right column of panels). The dots on the bars indicate the amplitudes of beats produced at a given rate by different pairs of component tones. The value of 0 dB on the ordinate is the maximum beat amplitude produced by the fundamental tones of the two complexes.

3. Experiment II

3.1. Rationale and method

To verify the findings obtained in Experiment I and examine in more detail the effect of small departures from integer frequency ratio of two harmonic complex tones on the amount of roughness, a supplementary experiment, called Experiment II, was carried out. The set of dyads used in Experiment II comprised 12 intervals; six of them were within a range from 10 cents below to 10 cents above an equally-tempered fourth and the other six encompassed a similar range around the equally-tempered fifth. Similarly as in Experiment I, fundamental frequency of the lower complex tone was 261.6 Hz in all dyads. The frequency ratios of intervals and their size in cents are shown in Table 2. The stimuli included a perfect fourth (4:3), omitted in Experiment I due to a programming error. The apparatus, the tone spectra and levels, the procedure of stimulus presentation and the procedure of roughness judgement were same as in Experiment I. The data were collected from a different group of 13 students, 20–23 years old, majors in sound engineering. All of them had normal hearing, 10 dB HL or less, at audiometric frequencies from 0.25 to 8 kHz. Each listener completed five series of judgements so that a total of 65 judgements was obtained for each dyad (13 listeners $\times$ 5 judgements).

Table 2. Frequency ratios, size in cents, and names of musical intervals used in Experiment II. The integer frequency ratios are shown in parentheses.

	Frequency ratio	Interval size (ct)	Interval name
1	1.3272	490	
2	1.3310	495	
3	1.3333	498	just fourth (4:3)
4	1.3348	500	equally-tempered fourth
5	1.3387	505	
6	1.3426	510	
7	1.4897	690	
8	1.4940	695	
9	1.4983	700	equally-tempered fifth
10	1.5000	702	just fifth (3:2)
11	1.5026	705	
12	1.5070	710	

3.2. Results and discussion

Results of roughness scaling are plotted in Fig. 4. Each point on the graph represents the geometric mean of 65 estimates, multiplied by a constant such that the maximum roughness value equals 1. Circles show the data for intervals within a range of $\pm$ 10 cents around an equally-tempered fourth and squares indicate the results for intervals within $\pm$ 10 cents around an equally-tempered fifth.

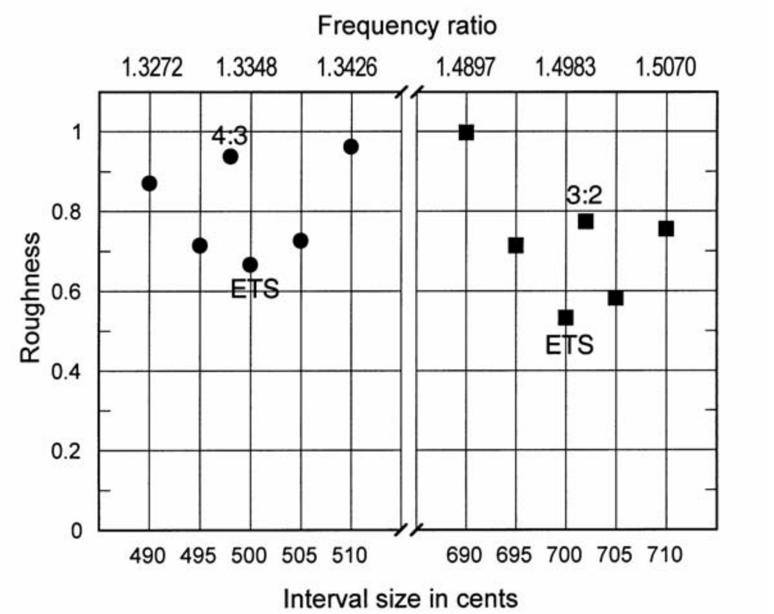

Fig. 4. Roughness of musical intervals composed of two simultaneous harmonic complex tones. The intervals encompass a range of ± 10 ct around the equally-tempered fourth (circles) and the equally-tempered fifth (squares). The primary abscissa is the interval size in cents and the secondary abscissa is the interval's frequency ratio. Plotted are geometric means of 65 judgements (13 listeners $\times$ 5 series of judgements) multiplied by a constant such that the roughness maximum equals 1.

Figure 5 shows the rates and amplitudes of beats produced by combinations of the component tones of two harmonic complex tones constituting each of the dyads used in Experiment II. The data have been calculated and plotted in the same manner as in Fig. 3. For clarity of presentation, the range of beat rates shown on the graphs was limited to 300 Hz.

The data plotted in Fig. 4 show that the least amounts of roughness are produced by intervals with smallest departures from exact, integer frequency ratio. Among the various versions of the fourth (circles), the lowest roughness values have been obtained for intervals of 495 ct (an interval smaller by 3 cents than the just fourth, 4:3) and 500 ct (equally-tempered fourth, larger by 2 cents than the just fourth). As the departure of the interval size from the just fourth (4:3) becomes larger, the amount of roughness considerably increases. A similar pattern of data is also seen for the fifths (Fig. 4, squares). The lowest roughness values have been obtained for intervals of 700 ct (equally-tempered fifth, smaller by 2 ct than the just fifth) and 705 ct (an interval by 3 cents larger than the just fifth).

A comparison of data plotted in Figs. 4 and 5 shows that the amount of roughness is related to the presence of very slow beats. The dyads that were judged least rough (495, 500, 700 and 705 ct) included a pair of component tones that produced beats with a rate of about 1 Hz. In the case of dyads for which high roughness values were found the lowest beat rate was about 5 Hz (intervals of 490 and 690 ct), 7 Hz (510 ct) or 87 Hz (Just fourth, 4:3).

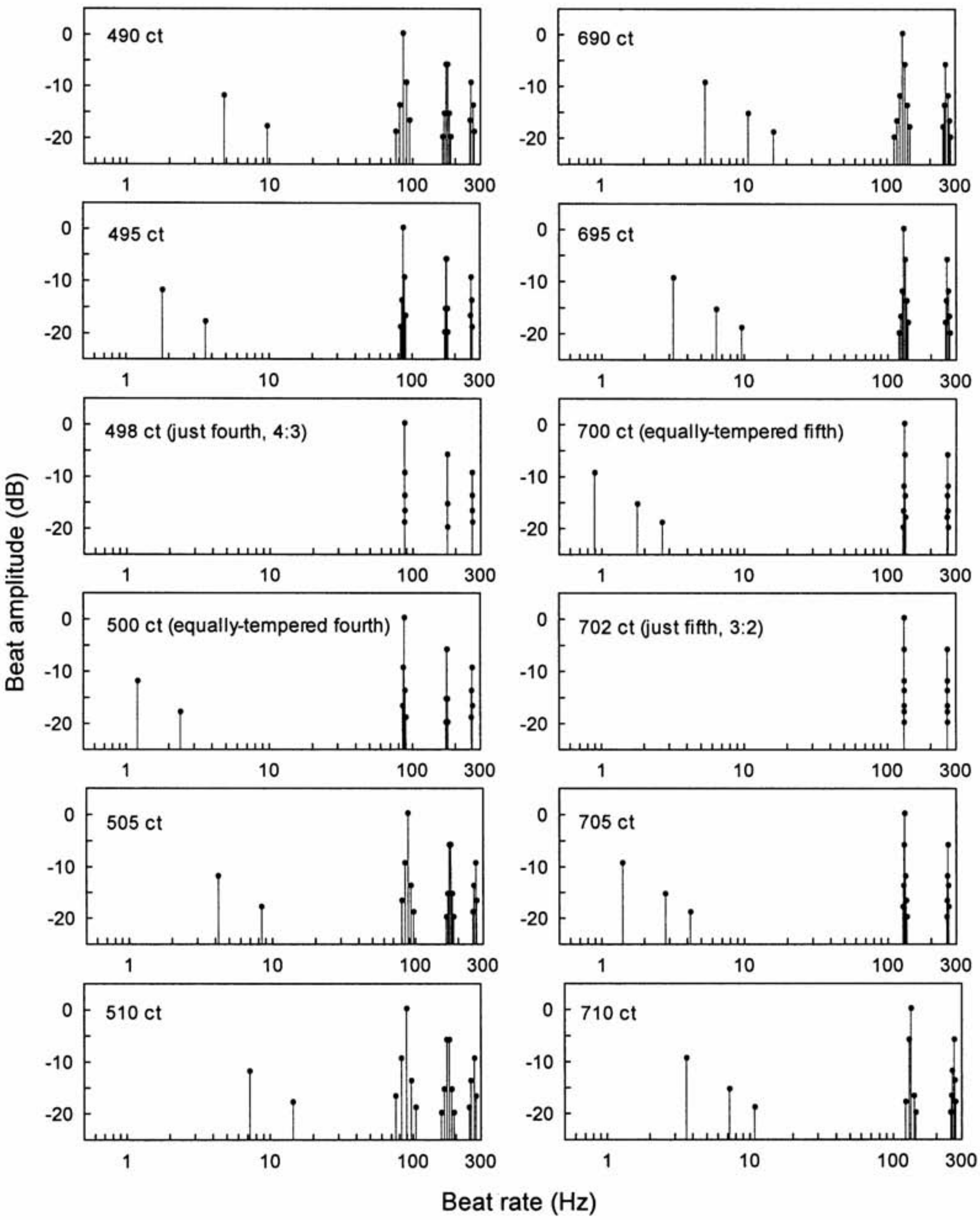

Fig. 5. Rates and amplitudes of beats produced by combinations of the components of two harmonic complex tones constituting various musical intervals. The intervals encompass a range of ± 10 ct around the equally-tempered fourth (left colums of panels) and the equally-tempered fifth (right column of panels). The dots on the bars indicate the amplitudes of beats produced at a given rate by different pairs of component tones. The value of 0 dB on the ordinate is the maximum beat amplitude produced by the fundamental tones of the two complexes.

4. Conclusions

The present study has demonstrated that dyads composed of harmonic complex tones, with frequency ratios corresponding to equally-tempered intervals are generally perceived less rough than dyads with integer frequency ratios. This finding is in contrast to the classical acoustical theories of roughness and sensory dissonance [2, 3, 7] as well as to musical theories of dissonance which assume that the least amount of roughness and the least amount of dissonance are both produced when an interval exactly corresponds to a small integer frequency ratio. The decrease of roughness below the value obtained for an integer-ratio interval, observed in the experiments reported here, is an effect of slow beats that occur when an interval slightly departs from integer frequency ratio.

In all intervals explored in this study the lower tone was C4 (261.6 Hz) therefore the findings reported here are limited to the middle register of the pitch scale used in music. Further investigations are needed to determine the effect of the interval's frequency ratio on the sensation of roughness in other pitch registers.

References

[1] GUTHRIE E. R., MORRILL H., *The fusion of non-musical intervals*, American Journal of Psychology, **40**, 624–625 (1926).

[2] HELMHOLTZ H. VON, *Die Lehre von den Tonempfindungen als psychologische Grundlage für die Theorie der Musik*, F. Vieweg und Sohn, Braunschweig, 1863, English translation: A. Ellis, *On the Sensations of Tone*, Dover, New York 1954.

[3] KAMEOKA A., KURIYAGAWA M., *Consonance theory part II: Consonance of complex tones and its calculation method*, Journal of the Acoustical Society of America, **45**, 1460–1469 (1969).

[4] INTERNATIONAL ORGANISATION FOR STANDARDISATION, *Method for calculating loudness level*, R-532B, ISO, Geneva 1966.

[5] MIŚKIEWICZ A., RAKOWSKI A., ROŚCISZEWSKA T., *Perceived roughness of two simultaneous pure tones*, Acta Acustica united with Acustica, **92**, 331–336 (2006).

[6] PLOMP R., *Timbre as a multidimensional attribute of sound*, [in:] Frequency Analysis and Periodicity Detection in Hearing, R. Plomp, G. F. Smoorenburg [Eds.], Sijthoff, Leiden, 397–414, 1970.

[7] PLOMP R., LEVELT W. J. M., *Tonal consonance and critical bandwidth*, Journal of the Acoustical Society of America, **38**, 548–560 (1965).

[8] PLOMP R., STEENEKEN H., *Interference between two simple tones*, Journal of the Acoustical Society of America, **43**, 883–884 (1968).

[9] TERHARDT E., *Ein psychoakustisch begründetes Konzept der Musikalischen Konsonanz*, Acustica, **36**, 121–137 (1976).

[10] TUFTS J. B., MOLIS M. R., LEEK M. R., *Perception of dissonance by people with normal hearing and sensorineural hearing loss*, Journal of the Acoustical Society of America, **118**, 955–967 (2005).

[11] ZWISLOCKI J. J., GOODMAN D. A., *Absolute scaling and sensory magnitudes: A validation*, Perception and Psychophysics, **28**, 28–38 (1980).