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A NEW METHOD FOR ANALYSIS OF THE MODAL CHARACTERISTICS OF SMALL RECTANGULAR ROOMS

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The main aim of the study was to compare acoustic comfort in eighteen rectangular classrooms, using a new method for modal analysis. Taking into regard the Schroeder frequency as the upper frequency limit, a statistical fluctuation in frequency spacing was calculated for all modes (parameters A and B) and axial modes separately (parameters C and D). Considering the range extended by about 30% towards higher frequencies, when compared to Bolt assumptions, it appeared that the rooms did not satisfy the Bolt criterion changed significantly their modal regularity. The enclosure with the ideal proportions, according to R. H. Bolt $(1 : 1.5 : 2.5)$ had the A value higher than a few rooms of other proportions and hence, was characterized by greater statistical fluctuation in frequency spacing.

Key words: classrooms, eigenmodes.

1. Introduction

In many earlier papers the acoustic comfort in classrooms was considered, depending on the reverberation time, background level of noise or speech intelligibility [1]. None of the above parameters was determined by the geometric proportion of the room. However, it seems that these proportions should affect the acoustic conditions in some way [2, 4]. The acoustic comfort in small and middle-size rectangular enclosures can decrease as a result of the elementary physical phenomenon of standing wave generation. This decrease can be caused by a significant strengthening of the perceived level of the sound corresponding to particular frequency components of the noise, generated by such devices as overhead projector or computer [2, 3]. Such an enhanced component can be a source of unpleasant acoustic effects disturbing concentration and causing fatigue of the hearing system.

In designing enclosures, one of the main objectives is to achieve ideal proportions ensuring a regular distribution of the lowest modes on the frequency scale. The Bolt curve defines the geometric size that would guarantee a regular distribution of these modes in a relatively limited band. Considering the range extended by about 30% towards higher frequencies, it appears that the rooms that do not satisfy the Bolt criterion after the extensions are characterised by greater regularity.

2. Numerical calculations

Modal analysis has been performed in the frequency range up to the Schroeder frequency, defined as the limiting value below which the subsequent eigenmodes are at distances great enough not to disturb the linearity of the frequency response of a given room. The Bolt curve treated hitherto as a criterion of the smoothest characteristics of the frequency response in a given room, was prepared on the basis of the 25 lowest modes. The mean square of the deviations of the actual distances between subsequent modes from the mean value in a certain frequency range was, according to BOLT [3], a measure of the regularity of distribution of the eigenmodes on the frequency scale:

$$\psi = \frac{1}{\mu_b - \mu_a} \sum_a^b \left(\frac{\delta^2}{\overline{\overline{\delta}}} \right), \tag{1}$$

where μ_a, μ_b – dimensionless parameters corresponding to the lower and upper frequency limits of the range studied, a – the lower bound of frequency range, b – the upper bound of frequency range, δ – the distance between subsequent modes on the frequency scale (dimensionless parameter), $\overline{\overline{\delta}}$ – the mean distance between the subsequent frequencies (dimensionless parameter).

Given the frequencies of subsequent eigenmodes (f_i) in any range, the value of ψ can be found alternatively on the basis of the parameters in Hz. Let us assume:

$$f_{i+1} - f_i = \delta_i = \overline{\overline{\overline{\delta}}} + \varepsilon_i, \tag{2}$$

where δ_i is the distance between subsequent eigenmodes in the room [Hz], $\overline{\overline{\overline{\delta}}}$ is the mean distance between subsequent eigenmodes in the room [Hz], ε_i is the deviation from the mean value [Hz] and i is the index denoting the pair of modes whose distance was considered. The dimensionless expression $\mu_b - \mu_a$ from Eq. (1) can be replaced by $(n-1)\overline{\overline{\overline{\delta}}}$, where n is the number of modes in the frequency range considered. In consequence, Eq. (1) can be written as:

$$\psi = \frac{1}{(n-1)\overline{\overline{\overline{\delta}}}} \sum_{i=1}^{n-1} \left(\frac{\delta_i^2}{\overline{\overline{\overline{\delta}}}} \right). \tag{3}$$

Since

$$\sum_{i=1}^{n-1} \varepsilon_i = 0.$$

Equation (3) can be finally presented in the form:

$$\psi = \frac{\sum_{i=1}^{n-1} \varepsilon_i^2}{(n-1)\overline{\overline{\delta}}^2} + 1. \tag{4}$$

As it was mentioned before, ψ is a measure of statistical fluctuation in frequency spacing [3]. However, ψ does not bring complete information on the character of this non-uniformity. To get this information we introduce an additional parameter. Let Ω be defined as follows:

$$\Omega = \frac{\sum_{i=1}^{n-1} \left(|\varepsilon_i| - \Gamma\right)^2}{(n-1)\Gamma^2}, \tag{5}$$

where:

$$\Gamma = \sqrt{\psi - 1}.$$

The higher the value of Ω, the larger the "gaps" in the characteristics (the irregularity increases).

Knowing the accurate frequencies of subsequent eigenmodes of a given room, it is possible to calculate easily the values of ψ and Ω. The question is: how many of the lowest eigenmode frequencies (n) should be taken into regard in the calculation? Bolt assumed n to be close to twenty-five; however, taking into regard the SCHROEDER frequency (6), this value does not seem to be sufficient. This frequency denotes approximately the boundary between the reverberant room behaviour and the discrete room modes [6]:

$$f_{\mathrm{Sch}} = 2000\sqrt{\frac{T_{60}}{V}}, \tag{6}$$

$$T_{60} = \frac{0.161\,V}{Ab}, \tag{7}$$

where T_{60} – reverberation time [s], V – room volume [m^3], Ab – absorption of the room [m^2]. The Schroeder frequency [Hz] versus acoustic absorption of the room is presented in Fig. 1.

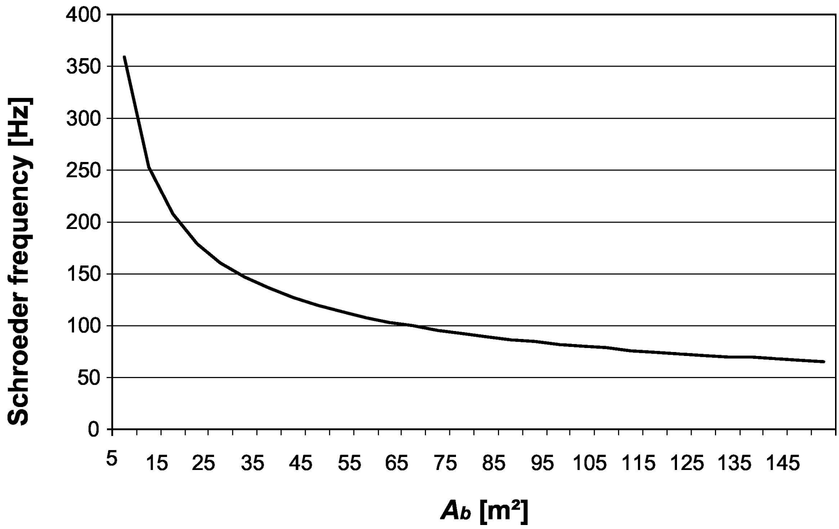

Fig. 1. Schroeder frequency [Hz] versus acoustic absorption [m^2].

3. Parameters *A*, *B*, *C* and *D*

The analysis was performed in two ways: for all modal frequencies (axial, tangential, and oblique) and for axial modes separately (as they have higher energy). Hence, for the enclosures analysed (Table 1), the following four parameters were calculated:

A – for axial, tangential and oblique modes (ψ, Eq. (4)),
B – for axial, tangential and oblique modes (Ω, Eq. (5)),
C – for axial modes separately (ψ, Eq. (4)),
D – for axial modes separately (Ω, Eq. (5)).

Determination of the parameters A, B, C and D was performed taking into regard the eigenmodes in the frequency range up to the Schroeder frequency and calculated for each enclosure individually. Fourteen different classrooms (p1–p14) were compared with the cube (p15), the cuboid on a square base (p16), and with the best Bolt's proportions (p17, p18).

Calculations were compared in the two ranges: for twenty-five lowest eigenmodes (first bar in the pair) and up to the Schroeder frequency (second bar in the pair), see Fig. 2. The Schroeder frequency was calculated by RoomModeCalculator013 [5]. The maximum frequency spacing of all modes varied from 7 Hz (p14) to 21 Hz (p2), while for axial modes from 16 Hz (p11) to 42 Hz (p15).

The lower the value of parameter A, the higher the regularity of frequency spacing. The value $\psi = 1$ gives the ideal distribution of the eigenmodes. The highest value of A was found for a cubic room $(\psi = 4.7)$.

Analysis of twenty-five lowest eigenmodes has shown that the most advantageous distribution was in room p3, p13, p14 and p18, what was consistent with the Bolt criteria. When the frequency range analysed was extended up to the Schroeder frequency,

Table 1. Geometric dimensions of the classrooms analysed.

room	geometric size (height, width, length) [m]	geometric proportion $(1 : x : y)$	volume [m^3]
p1	$3.3 \times 4.15 \times 7.3$	$1 : 1.26 : 2.21$	100
p2	$3.0 \times 6.07 \times 7.0$	$1 : 2.02 : 2.33$	127
p3	$3.34 \times 5.1 \times 7.05$	$1 : 1.53 : 2.11$	120
p4	$2.6 \times 4.75 \times 4.65$	$1 : 1.83 : 1.79$	57
p5	$2.6 \times 4.7 \times 9.6$	$1 : 1.81 : 3.69$	117
p6	$2.6 \times 3.13 \times 9.6$	$1 : 1.2 : 3.69$	78
p7	$3.3 \times 5.65 \times 6.4$	$1 : 1.71 : 1.94$	119
p8	$2.6 \times 4.6 \times 9.45$	$1 : 1.77 : 3.63$	114
p9	$3.05 \times 6.8 \times 9.2$	$1 : 2.23 : 3.02$	191
p10	$2.9 \times 6.3 \times 8.7$	$1 : 2.17 : 3$	159
p11	$4.1 \times 7.6 \times 8.7$	$1 : 1.85 : 2.12$	271
p12	$4.1 \times 5.85 \times 6.05$	$1 : 1.43 : 1.48$	145
p13	$4.1 \times 5.85 \times 9.0$	$1 : 1.43 : 2.2$	216
p14	$4.1 \times 6.6 \times 8.7$	$1 : 1.61 : 2.12$	235
p15	$4.1 \times 4.1 \times 4.1$	$1 : 1 : 1$	69
p16	$3 \times 6 \times 6$	$1 : 2 : 2$	108
p17	$4 \times 6 \times 10$	$1 : 1.5 : 2.5$	240
p18	$3 \times 3.78 \times 4.77$	$1 : 1.26 : 1.59$	54

the values changed. For example, parameter A for room p13 (consistent with the Bolt criterion) was higher than for rooms p10 and p11 (not satisfying the Bolt criteria). If the values of parameter A were equal for some rooms, parameter B was calculated (Fig. 4, Eq. (5)).

The values of parameter A were almost the same for rooms p3, p10 and p11, but the lowest parameter B was for room p3. It means that the frequency spacing was most regular in this enclosure (Fig. 5).

As the axial modes are characterised by the highest energy, their distribution on the frequency scale seems to be important. Parameters C and D (Fig. 6), analogous to A and B but referring only to the axial modes, can bring additional information on the frequency response.

For rooms p2, p8, p9 and p10, parameter C took the same value but the parameter D was the lowest for room p8. It means that among these rooms, p9 was characterised by the greatest regularity of the axial eigenmode distribution.

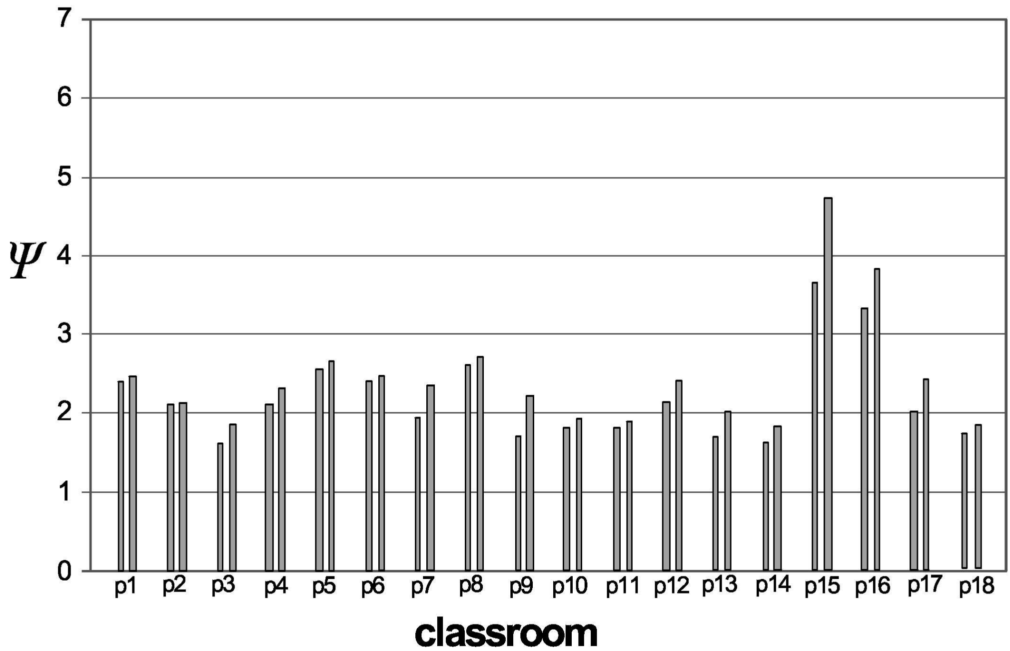

Fig. 2. Statistical fluctuation in frequency spacing for two frequency ranges: for the twenty-five lowest eigenmodes and up to the Schroeder frequency (parameter A).

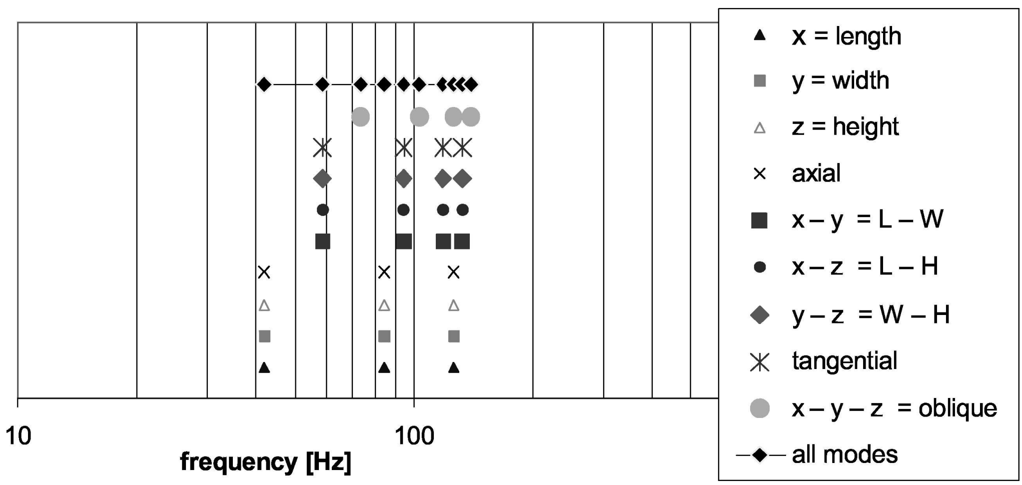

Fig. 3. Frequency spacing in a cubic enclosure (p15).

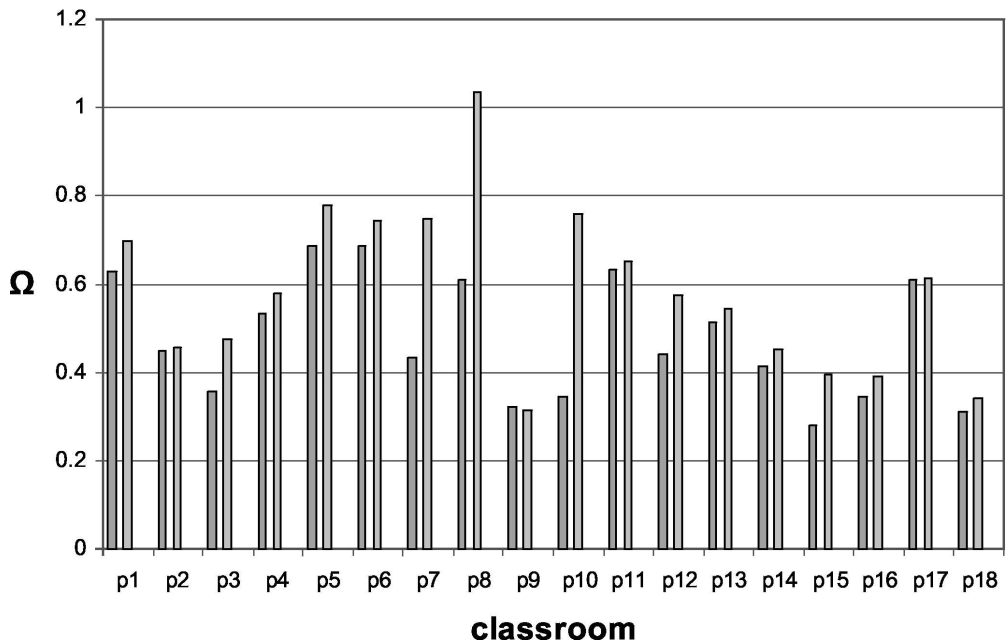

Fig. 4. Calculated values of Ω for the 25 lowest eigenmodes and for frequencies up to the Schroeder frequency (parameter B).

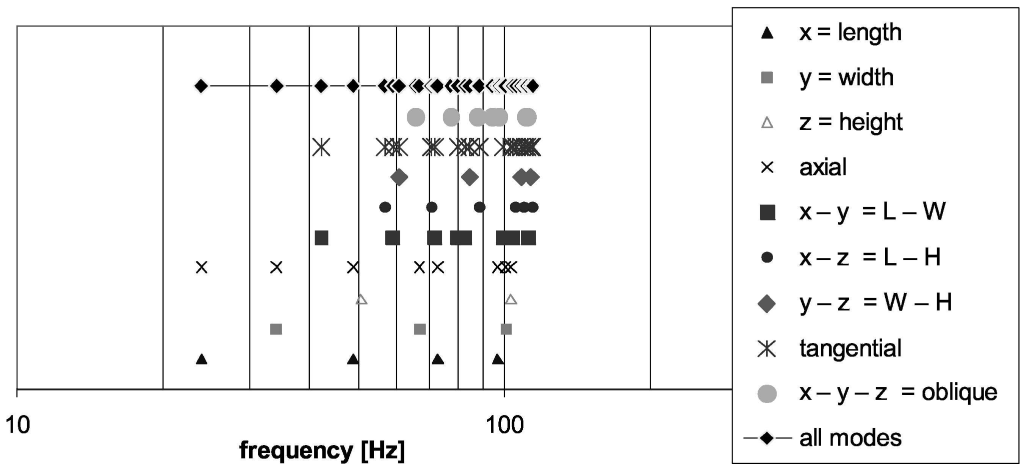

Fig. 5. Frequency spacing in room (p3).

a)

b)

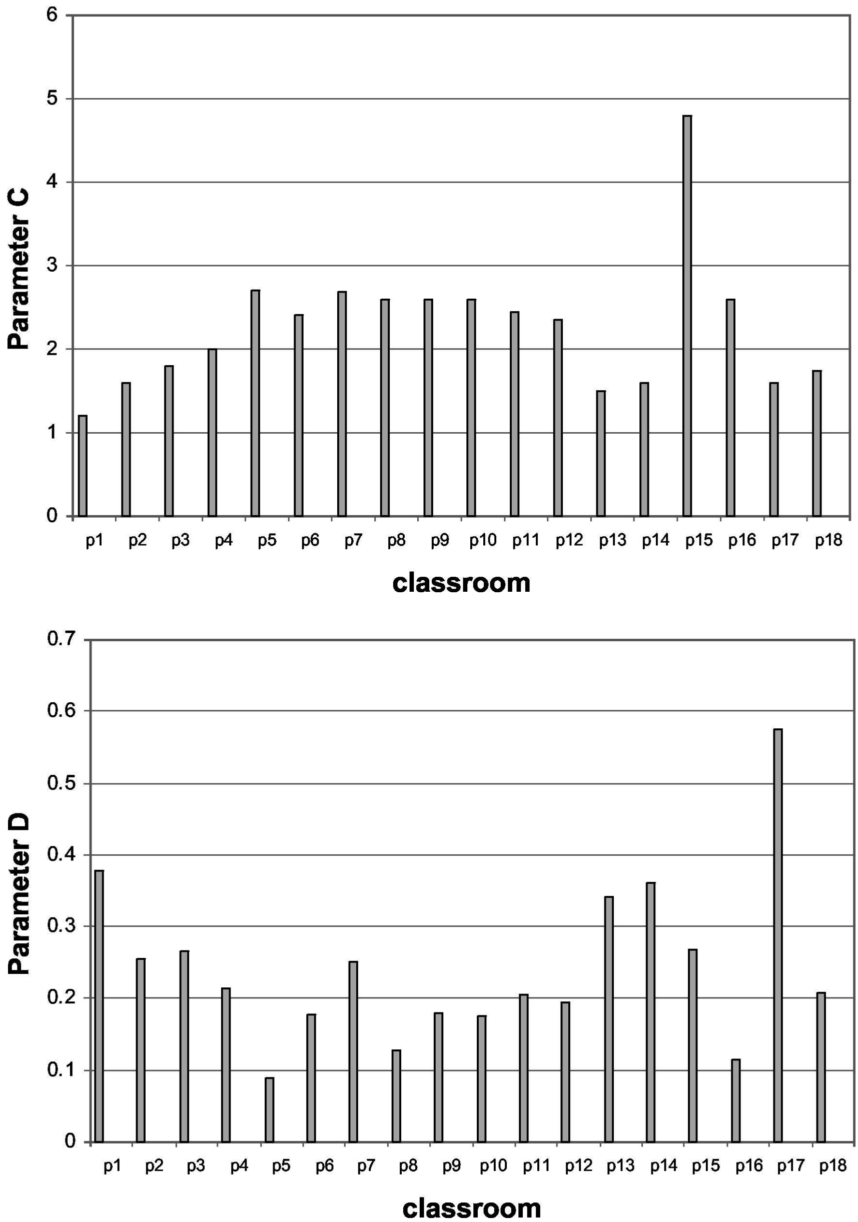

Fig. 6. Parameters C (a) and D (b) for analyzed enclosures.

4. Conclusion

The curve proposed by R. H. Bolt as the criterion of regularity of distribution of eigenmodes in a given room seems to be not precise enough because it takes into account only about twenty-five lowest modes. A new criterion based on the values of parameters A, B, C and D seems to be more accurate in determination of the regularity of eigenmodes distribution. For the enclosures with the same parameter A, the value of B determined the room which had the best acoustic properties.

As the axial modes have the greatest energy, their distribution on the frequency scale seems to be important. For this reason parameters C and D bring additional information on the response function. What was most important: parameters A, B, C and D depend on the Schroeder frequency. It is possible that their values may be different for two enclosures with the same geometrical size, because the Schroeder frequency depends on the total acoustic absorption of a room related to the time of reverberation T_{60}. Hence, a comparison of the geometrical sizes of rooms makes sense only if the rooms have the same absorbing properties of the surfaces. The greater the acoustic absorption, the lower the Schroeder frequency and the narrower the band of different frequency response. The curves analogous to the Bolt curve but based on the new procedure taking into account the acoustic absorption of a given room will be a subject of a separate paper.

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