

ON CONVERGENCE AND STABILITY OF ADAPTIVE ACTIVE NOISE CONTROL SYSTEMS

M. PAWEŁCZYK

Silesian University of Technology
Institute of Automatic Control
Akademicka 16, 44-101 Gliwice, Poland
e-mail: Marek.Pawelczyk@polsl.pl

(received June 15, 2006; accepted September 30, 2006)

The problems of convergence and stability of adaptive active noise control systems with the Filtered-Reference LMS algorithm are discussed. First, the assumptions required to derive convergence conditions are collected and addressed. Then, it is demonstrated that feedback systems are subject to convergence conditions different than those used for feedforward systems. Moreover, the problems of convergence of the adaptation algorithm and stability of the structural feedback loop are coupled. Theoretical considerations are supported by simulations experiments.

Key words: active noise control, adaptive control, feedforward, feedback, stability, convergence.

1. Introduction

In active noise control secondary sound sources are used to reduce acoustic noise from the original primary noise. If a reference signal is measure-available and causality condition is satisfied, feedforward control is preferred – see Fig. 1. Then, a feedback loop is avoided and the control action can be undertaken in advance allowing successfully controlling broadband noise. However, if such signal is not available or the control system is mobile, feedback control is the interim solution – see Fig. 2. It is commonly used by many practitioners. Adaptive algorithms, usually analogous to those used for feedforward control are applied. Similar properties of the overall systems are then expected. However, adaptive feedback systems need to be differently analysed compared to their feedforward counterparts. This paper presents a review of the current knowledge in this field supported by additional analysis and experiments. The most popular in active control Filtered-Reference LMS (FXLMS) algorithm is used as the adaptation algorithm. It updates control filter parameters in the direction of the negative gradient of the squared output signal sample (the instantaneous value is taken instead of the

ensemble averaging) [2]:

$$\begin{aligned}\mathbf{w}(i+1) &= \gamma \mathbf{w}(i) - \mu \hat{\mathbf{r}}(i)y(i), \\ \hat{\mathbf{r}}(i) &= \hat{\mathbf{S}}^T \mathbf{x}(i).\end{aligned}\quad (1)$$

In this equation μ is greater than zero “step size”, and $\mathbf{w}(i)$ and $\hat{\mathbf{s}}$ are vectors of the control filter, W , and the plant model, \hat{S} , respectively, assumed to be of FIR structure. The γ parameter is equal to one for the FXLMS algorithm and is less than one for its robust version known as the Leaky FXLMS algorithm.

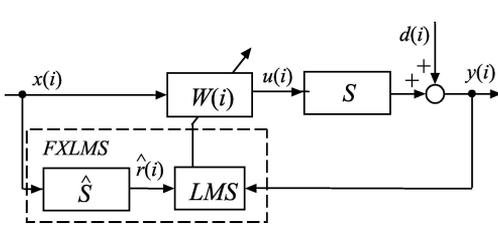


Fig. 1. Adaptive feedforward system with the FXLMS algorithm.

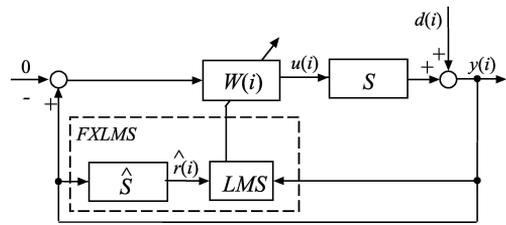


Fig. 2. Adaptive classical feedback system with the FXLMS algorithm.

2. Theoretical analysis

The problem of convergence of the FXLMS algorithm has been addressed in many publications. Below, all the convergence types and important assumptions used to derive convergence conditions are arranged in a coherent way. In stochastic systems convergence of two random variables can be defined in various ways [1, 5]:

- Convergence “with probability one” or “almost surely – a.s.”.
- Convergence “in probability”.
- “Weak” convergence.
- Convergence “in the mean-square sense”.
- Convergence “in the mean”.
- Convergence “of the mean”.

Among the above definitions, convergence “with probability one” is the strongest, whereas convergence “of the mean” is the weakest. The assumptions are:

- A.1. The step size is very small or tends to zero (frozen trajectories).
- A.2. The control filter structure is known.
- A.3. The control filter parameters are bounded.
- A.4. The control filter input is exciting.
- A.5. The control filter input and variations of the control filter parameters are statistically independent.

The most frequently used upper bound for the step size in (1) is:

$$\mu < \frac{2}{\lambda_{\max}}, \quad (2)$$

where λ_{\max} is the maximum eigenvalue of the correlation matrix $E\{\hat{\mathbf{r}}(i)\hat{\mathbf{r}}^T(i)\}$. However, this condition has been derived for feedforward systems only and is valid provided the assumptions A.1–A.5 are satisfied and the plant is perfectly modelled. Moreover, it is exclusively a sufficient condition for convergence “of the mean” [5]. This guarantees only that the control filter parameters are unbiased estimates of the optimal filter parameters, and it does not refer to their variance. Hence, in practice, the step size should be chosen significantly lower. Imperfect plant modelling sets further limitations to this condition.

A reliable convergence analysis of the FXLMS algorithm has been performed in [9] using the Ljung’s Ordinary Differential Equations method. Under assumptions A.1–A.5, the sufficient but not necessary condition for convergence “with probability one” of the FXLMS algorithm is that the so-called “phase error” between the control-to-output path and its estimate (the plant and the model, respectively, for feedforward control in Fig. 1) must not exceed $\pm\pi/2$ at all frequencies [9], i.e.:

$$\forall_{\omega T_S} \left| \angle\{S(e^{-j\omega T_S})\} - \angle\{\hat{S}(e^{-j\omega T_S})\} \right| < \frac{\pi}{2}, \quad (3)$$

where $\angle\{.\}$ stands for the phase of $\{.\}$. If the assumption concerning the step size tending to zero is not satisfied the control filter parameters converge “in probability” [9]. The above condition is usually considered as a rule of thumb for any control structure. However, it is extremely important to bear all the made assumptions in mind. For the classical feedback structure as presented in Fig. 2 two assumptions required for (3) to be valid can be violated.

Assumption A.4 requires the reference signal being the control filter input to be exciting. Fortunately, it is a much milder assumption than persistent excitation of respective degree required for the RLS algorithm. Nevertheless, particularly for narrowband signals, high suppression of the system output may make the reference signal insufficiently rich for the FXLMS algorithm to correctly update the controller parameters. This, in turn, may result in temporary divergence of the algorithm and may generate local bursts of the output signal as illustrated in Fig. 3. There are different ways, not considered in the paper, to mitigate this problem, including injecting low-level exciting noise or the adaptive dual control strategy.

Assumption A.5 concerning independence of the control filter input on the control filter parameters is also not satisfied for the classical feedback structure. The convergence analysis can be then approximately tackled after performing linearization of the path from the control filter input to the system output. Then, the sufficient condition for convergence “with probability one” is [7]:

$$\left| \angle\{S\} - \angle\{\hat{S}\} - \angle\{1 - SW\} \right| < \frac{\pi}{2}. \quad (4)$$

It differs from (3) by presence of the phase of the characteristic polynomial $1 - SW$, which sets also the condition for stability of the structural feedback loop. Thus, condition (4) binds the problem of stability of the feedback system and convergence of the

adaptation algorithm. This condition is usually more severe than that for feedforward system. However, it can be shown that due to presence of the adaptive filter such condition can be self-correcting during adaptation and decrease the phase error [7]. Moreover, it can even be less conservative than in case of feedforward systems (3).

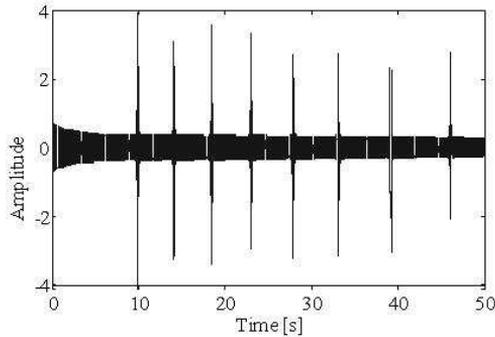


Fig. 3. Time sequence of the output signal for the adaptive classical feedback system.

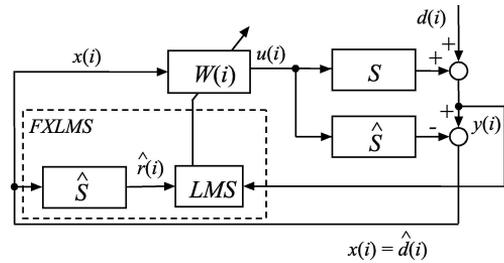


Fig. 4. Adaptive Internal Model Control system with the FXLMS algorithm.

The problem of signal excitation can be mitigated in the Internal Model Control (IMC) system structure presented in Fig. 4. In this structure the reference signal is an estimate of the output disturbance, the better the more exact the plant model is. Moreover, for perfect modelling the feedback system degrades to a feedforward system, which is stable provided the control filter is stable since the acousto-electric plant is stable. However, assumption A.5 is also violated for this structure. Using the linearization technique as mentioned above the following sufficient condition for convergence “with probability one” has been derived [8]:

$$\left| \angle \{S\} - \angle \{\hat{S}\} - \angle \left\{ 1 + W (\hat{S} - S) \right\} \right| < \frac{\pi}{2}. \quad (5)$$

As for the classical feedback structure limitation of the control filter gain may also reduce the phase error and protect against divergence of the FXLMS algorithm [6]. Such limitation is also advantageous for stability of the feedback loop (the characteristic polynomial is $1 + W(\hat{S} - S)$, and additionally $1 + \hat{S}W$ should be stable for internal stability).

The problem of convergence of the adaptive IMC system has also been addressed in the literature using different approaches. However, strong and unfeasible assumptions, like perfect plant modelling or its minimum phase character, are made there. It is believed that more reliable analysis can be approached using, e.g. the tool presented in [1] based on conditional expectations and particularly on the martingale method.

Another methodology to overcome the problems due to imperfect plant modelling is to update the model simultaneously with adaptation of the control filter. Several algorithms have been proposed for the on-line adaptation of the plant model. They usually

require injecting to the system additional wideband noise uncorrelated with the disturbance (see e.g., [4]). There are also some trials to identify the plant model without the additional noise but the result is generally not unique. Usefulness of such algorithms is usually comparable to the Leaky FXLMS in terms of the convergence itself. They can, however, improve the convergence rate. Some of them are particularly appreciated in feedback internal model-based structures. Then, in addition to updating the control filter parameters the plant model constitutes a part of the overall feedback controller. If changes of the plant are large enough stability of the feedback loop may be impaired.

It has been assumed in A.1 for the convergence analysis that the step size is very small or tends to zero. On the other hand, the adaptive system is also responsible for responding to time-variations of the plant or non-stationarity of the disturbance. However, since for the IMC structure the model of the plant is required to adapt the controller as well as estimate the reference signal variations of the plant are more crucial. In case of too small step size the so-called “lag effect” may appear, which can degrade the performance, even if the system remains stable [5]. In turn, too large step size increases the excess mean-square error or can even lead to divergence as the upper bounds demonstrate. It has been shown in [5] that the optimal step size is related to so-called “non-stationarity degree” introduced to describe time variations, and the LMS-based algorithm behaves much better in terms of the tracking capability when facing random time-variations than a deterministic trend.

3. Simulation analysis

A further insight into the stability related problems can be gained and behaviour of the adaptive algorithm can be examined with simulation analysis. However, it should be emphasised at the very beginning that stability, similarly to convergence, is an asymptotic property of a system. Therefore, any analysis performed over a short time horizon can rather refer to a tendency towards instability (or divergence) and the conclusions cannot be straightforwardly generalised.

The step size, has great impact on the convergence itself and the convergence time (rate). This dependence has been verified by means of simulations for the IMC system. The data come from a real-world active headrest system aimed at attenuating noise at ears of a person occupying the chair (label “original” in Fig. 5). Additionally, in the second experiment, the plant gain has been multiplied by two (“doubled”), and in the third experiment the original plant impulse response has been delayed by one sample (“delayed”). In these investigations the convergence time has been defined as the number of samples required for an examined tone of 250 Hz to be attenuated by 20 dB. It has been observed that for a small step size increase of its value decreases convergence time and the dependence is reciprocal,

$$t_C \sim \mu^{-1}, \quad (6)$$

regardless of plant modelling error, what coincides with the observations made in [8].

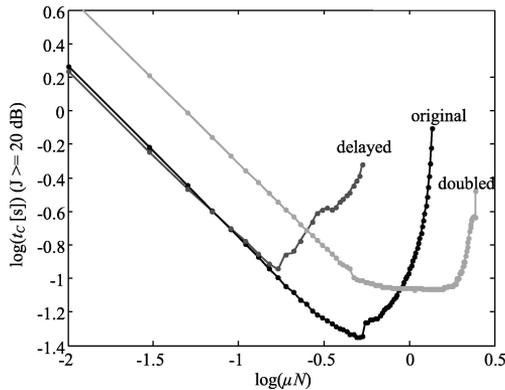


Fig. 5. Convergence time vs. normalised step size for the nominal plant, with additional one-sample delay, and with doubled gain; N – filter length.

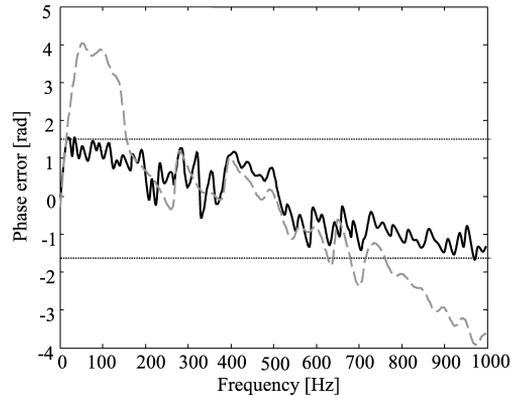


Fig. 6. Phase error for feedforward (dashed, grey), and IMC (solid) systems; the dotted lines represent phase error of $+\pi/2$ and $-\pi/2$.

For small values of μ the plant dynamics has little effect and only the plant gain and control filter length, N , influence the scaling in (6). Then, there exists an optimal value of μ in terms of the smallest convergence time (highest convergence rate). Minimum value of t_C depends both on the plant gain and delay for a given μ . However, the optimum value of μ and the range of its values are mainly limited by the delay. This has been confirmed in analysis performed in [3] for a simplified case of a feedforward system with the plant being a pure delay and a tonal reference signal. It has been experimentally found there that the optimal μ is inversely proportional to the overall path delay expressed in samples. Continuing the simulation analysis, further increase of the step size increases the convergence time due to fluctuations of the residual signal (or excess mean-square error). Finally, after crossing the critical value, the adaptive system suddenly diverges.

Figure 6 presents the phase error evaluated according to the condition for feedforward system (dashed, see (3)) and according to the condition for the IMC system (solid, see (5)). They have been obtained after convergence of the adaptive filter controlling a broadband noise for a plant significantly different in phase and gain compared to the assumed model. It is seen that the first condition is violated at low and high frequencies and adaptive feedforward system would diverge. However, the second condition is satisfied and the adaptive IMC system works stably provided it is updated slowly.

4. Conclusions

In this paper the problems of convergence and stability of adaptive active noise control systems have been discussed. Adaptive feedback systems, in contrary to feedforward systems, are very difficult to be fairly analysed. They operate with non-stationary signals due to the adaptation loop, have structural feedback loop and are generally employed to face time variations of the plant and non-stationarity of the disturbance. There-

fore, their analysis is usually subject to a variety of constraints that often significantly limit practical application of found conditions. The theoretical considerations have been supported by simulation experiments. Summarising the entire analysis it is suggested to support additionally the adaptive algorithm with a heuristic supervisory loop. This loop can be used to monitor the convergence and performance, and reset the algorithm or change some settings, if necessary. This is particularly important for the acousto-electric applications to avoid any unpleasant and annoying sound effects.

Acknowledgment

Financial support from the state budget for science is gratefully acknowledged.

References

- [1] CHEN H. F., GUO L., *Identification and stochastic adaptive control*, Birkhäuser, Boston 1991.
- [2] ELLIOTT S. J., *Signal processing for active control*, Academic Press, London 2001.
- [3] ELLIOTT S. J., STOTHERS I. M., NELSON P. A., *A multiple error LMS algorithm and its application to the active control of sound and vibration*, IEEE Trans. Acoustics Speech and Signal Processing, **35**, 1423–1434 (1987).
- [4] FIGWER J., *A new method of on-line model identification and update for multichannel active noise control systems*, Archives of Control Sciences, **13**, 2, 11–155 (2003).
- [5] MACCHI O., *Adaptive processing. The least mean squares approach with applications in transmission*, J. Wiley & Sons. Chichester 1995.
- [6] PAWEŁCZYK M., *Feedback control of acoustic noise at desired locations*, Wydawnictwo Politechniki Śląskiej, No. 1684, z. 141, Gliwice 2005.
- [7] PAWEŁCZYK M., *An adaptive feedback control system with the Filtered-Reference LMS algorithm*, Proceedings of 12th IEEE International Conference MMAR 2006.
- [8] VAUDREY M. A., BAUMANN W. T., SAUNDERS R., *Stability and operating constraints of adaptive LMS – feedback control*, Automatica, **39**, 595–605 (2003).
- [9] WANG A. K., REN W., *Convergence analysis of the multi-variable filtered-x algorithm with application to active noise control*, IEEE Trans. Signal Processing, **47**, 1166–1169 (1999).