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LOCALIZATION OF AREAS OF INCREASED VIBROACTIVITY BY MEANS OF THE INVERSE METHOD

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Increased possibilities of parallel acquiring data and transforming them effectively together with the improvement of calculation tools made the inverse methods more important. Some examples of research, carried out by the author, during, which the possibilities of localization of increased vibroactivity areas were studied using the multi-microphone method, are presented in the paper.

The theoretical basis of the inversion method of sound sources localization and certain laboratory test results are also given.

Keywords: proszę podać słowa kluczowe.

1. Introduction

Increasing possibilities of parallel acquiring data and their effective processing as well as the improved calculating tools are the reasons of growing importance of inverse methods. These methods find application in various fields of acoustics: oceanic acoustics, vibroacoustics, aero acoustics, structural acoustics and as well as in geophysics.

Inverse methods allow for an acoustic assessment of machines on the bases of analysis of acoustic field parameters. By modelling the process of vibroacoustic energy radiation from the sound source to the receiver and knowing the actual value of an sound pressure in measuring points the propagation path can be reversed and the parameters of the sound source determined.

2. Method's sensitivity to the source localization

To be able to assess properly the parameters of the substitute sources those sources should be located exactly in the places of actual sources. The inverse method (especially when measurements are done with the phase shift angle) is highly sensitive to the localization of the substitute sources. This feature can be utilized for the determination of the position of actual sources. The accuracy of the method is also dependent on the arrangement of observation points. Some examples are presented below. Equations as well as figures were developed by the author.

2.1. Observation points arranged on the spherical surface

In the first case, the influence of the substitute source localization on the value of its sound power was investigated. Observation points were arranged in a continuous way on the surface of the semi-sphere. The omni-directional sound source of 20dB power (imitating the actual source) was situated in the centre of the coordinate system.

The conversion function between a source A moment [Pa m] and an sound pressure p in the observation point for the omni-directional source is given by the equation:

$$p = A\frac{\exp(ikr)}{r} \quad \text{[Pa]}. \tag{1}$$

Source sound power can be determined as:

$$N = \frac{2\pi A^2}{\rho_0 c} \quad \text{[W]}. \tag{2}$$

where $\rho_0 c$ – acoustic impedance of a medium [kg m^{-2}s^{-1}].

During the tests the substitute source was being shifted inside a square of a side equal to the radius of the sphere (Fig. 1). Then parameters of the substitute source (sound power) were determined by the inverse method [4].

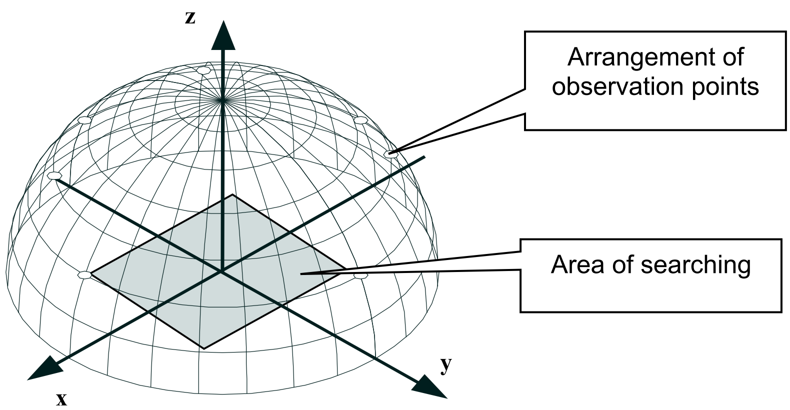

Fig. 1. Arrangement of sound sources and observation points on the surface of the semi-sphere.

The sound power value of the substitute source determined by the inverse method (with phase) can be, in this case, calculated from the formula:

$$N_z = N\frac{\sin(e\,k\,R)^2}{(e\,k\,R)^2} \quad \text{[W]}, \tag{3}$$

where N – actual source power, N_z – substitute source power [W], R – radius of the semi-sphere [m], k – wave number [m^{-1}], eR – distance between the substitute and the actual source [m], e – relative displacement of the substitute source versus the actual source being in the centre of the coordinate system.

Not knowing the phase shift angles we can estimate the substitute source power from the equation [4]:

$$N_z = N\frac{(1-e^2)^2}{2e}\,\log\left(\frac{1-e}{1+e}\right)\quad [\mathrm{W}]. \tag{4}$$

Examples of the calculation results – assuming that $R = 2$ m – are presented graphically in Fig. 2.

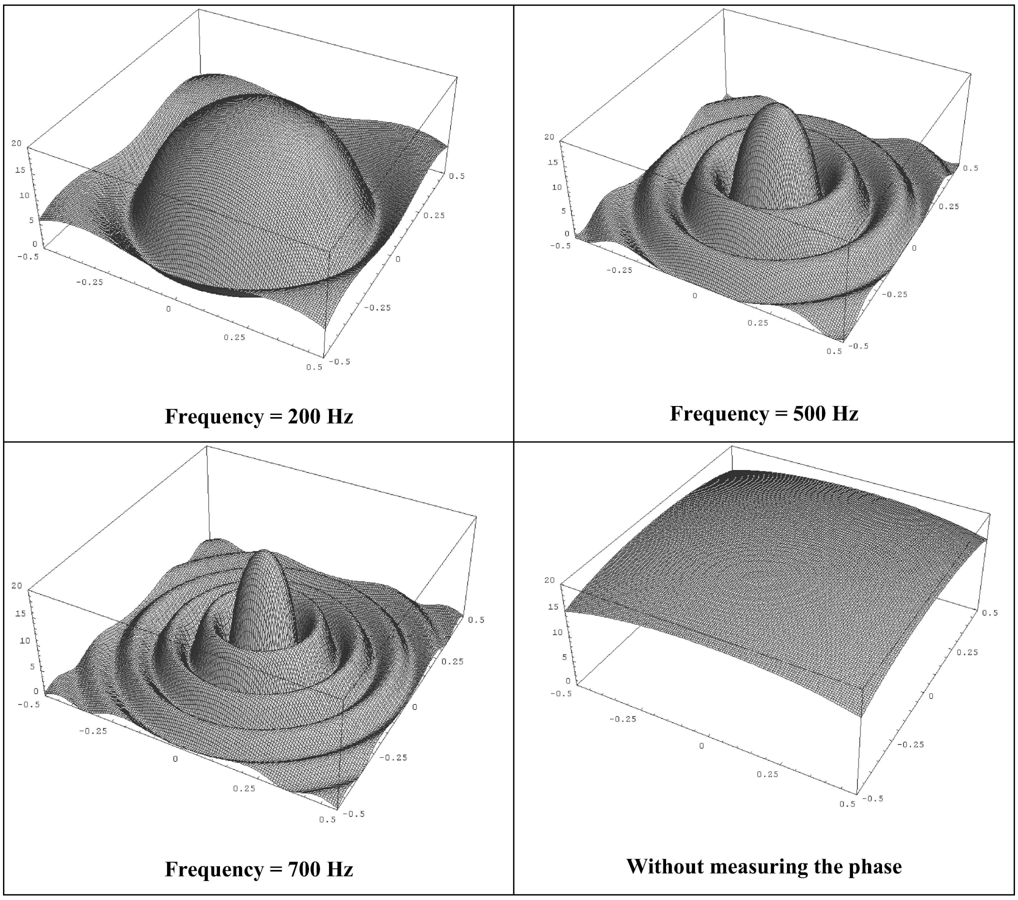

Fig. 2. Sound power level of the substitute source in dependence of its position – observation points arranged on the semi-sphere surface.

2.2. Observation points arranged in the nodes of the rectangular grid

The problem of the sound source localization becomes complicated in the case when the observation points do not surround the area of searching. The observation points arranged in the nodes of the small dimension grid, significantly remote from the area of searching, can serve as the example (Fig. 3).

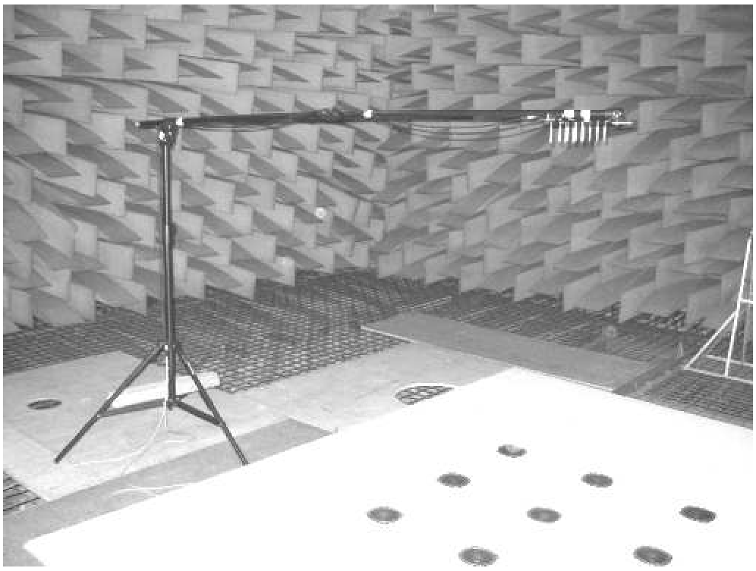

Fig. 3. Arrangement of sound sources (loudspeakers – in the lower part of the photo) and observation points (microphones – in the upper part of the photo).

An example of theoretical calculations in the case of 12 observation points (6×2 microphones) arranged in the nodes of a grid of a side length 5 cm located at the distance of 1.4 m from the area of searching is presented in Fig. 4.

The localization of sound sources is not so straightforward in this case (a sound source is not always in the point of the maximum sound power of the substitute sound source). However, the notion of a similarity criterion between the actual distribution of the sound pressure and such distribution around the model can be introduced in a form:

$$K = -10 \log 10 \left(\sum_{i=1}^{n} (p_{ri} - p_{zi}) \overline{(p_{ri} - p_{zi})} \right), \tag{5}$$

where p_{ri} – sound pressure measured by the i-th microphone; p_{zi} – theoretical sound pressure in the i-th observation point.

The distribution of the similarity criterion K on the searched surface – for the value from the previous example – is presented in Fig. 5.

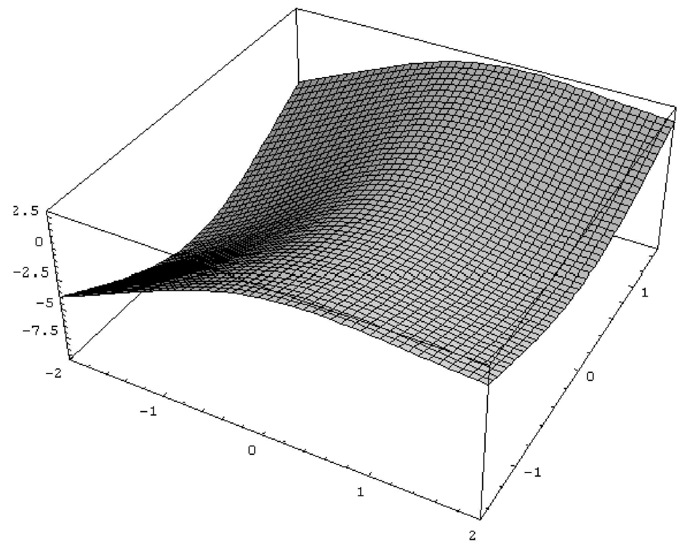

Fig. 4. Power level of a substitute source in dependence of its localization – observation points are located in the grid nods (frequency 1000 Hz).

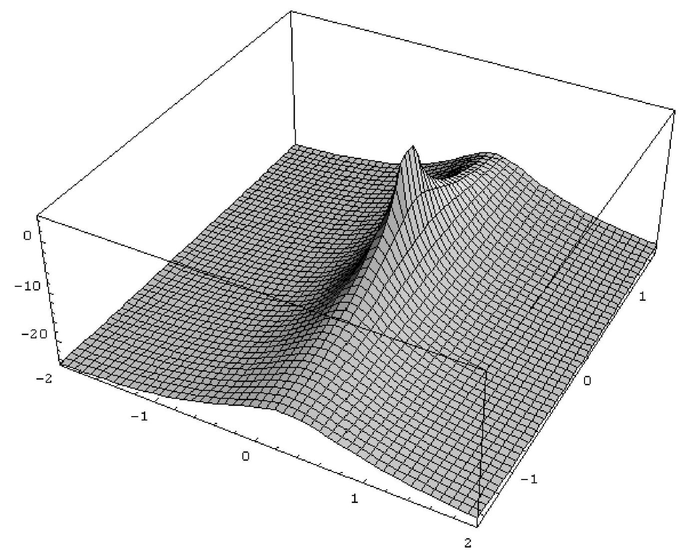

Fig. 5. Distribution of the similarity criterion on the searched surface (frequency 1000 Hz).

Analysing the distribution of the similarity criterion we are able to localise – with a high accuracy – the sound source and to determine its sound power. Figure 6 illustrates – as the example – the similarity distribution obtained for acoustic measurements of the setup presented in Fig. 3.

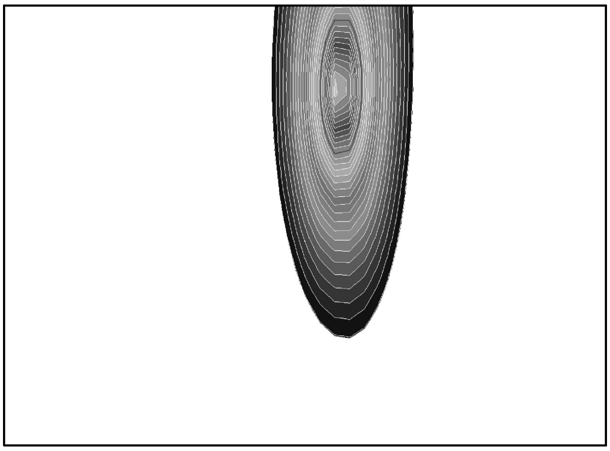

Fig. 6. Distribution of the similarity criterion during the operation of loudspeaker no 3 at the frequency of 1000 Hz.

3. Sound propagation by a vibrating plate

Apart from the model of sound propagation from the omni-directional source (monopole), models of sound propagation from the dipole and from the system consisting of the dipole and monopole, were applied for the localization of areas of an increased vibroactivity. Each time the results were very similar. An application of the sound propagation models related to radiation characteristics corresponding to vibration modes of the rectangular plate supported at the periphery was another approach to searching for areas of an increased vibroactivity. The spatial solution of equation of the rectangular plate vibrations simple-supported at the periphery constitute eigenfunctions in the following form [6]:

$$W_{m,n}(x,y) = A_{m,n} \sin\left(\frac{m\pi(x-0.5a)}{a}\right) \sin\left(\frac{n\pi(y-0.5b)}{b}\right), \tag{6}$$

where a, b – plate dimensions [m]; m, n – modes of plate vibrations; $A_{m,n}$ – plate vibrations amplitude, determined by the vibration mode m, n.

Knowing the vibration velocity distribution on the plate surface we are able to estimate the distribution of the sound pressure around the plate on the basis of the following formula [5]:

$$p(r,\theta,\varphi) = \frac{ik\rho_0 c}{2\pi}\frac{e^{-ikr}}{r}\int\limits_{S_0} v(\mathbf{r}_0)\exp\left[ikr_0\sin\theta\,\cos(\varphi-\varphi_0)\right]\,\mathrm{d}S_0 \quad [\mathrm{Pa}], \tag{7}$$

where r, θ, φ – polar coordinates of the observation point, r_0, φ_0 – polar coordinates of the point on the vibrating surface, k – wave number [m^{-1}], S_0 – vibrating surface [m^2], $v(\mathbf{r}_0)$ – distribution of amplitudes of vibration velocity on the vibrating surface [ms^{-1}].

After integration (7) of Eq. (6) we obtain functions determining the distribution of sound pressure around the vibrating plate of m, n modes of vibration as:

$$g_{m,n}(r,\theta,\varphi) = \frac{ik\rho_0 c}{2\pi}\,\frac{e^{-ikr}}{r}\,F_{x,m}(\theta,\varphi)\,F_{y,n}(\theta,\varphi) \quad [\mathrm{kg\,m^{-2}s^{-1}}], \tag{8}$$

where:

$$\begin{aligned} F_{x,m} &= -\frac{2a}{m\pi}\,\frac{i\pi^2 m^2 \sin\big(0.5a\,k\cos(\varphi)\sin(\theta)\big)}{m^2\pi^2 - a^2k^2\cos^2(\varphi)\sin^2(\theta)} &&\text{for odd } n;\\ F_{x,m} &= \frac{2a}{m\pi}\,\frac{\pi^2 m^2 \cos\big(0.5a\,k\cos(\varphi)\sin(\theta)\big)}{m^2\pi^2 - a^2k^2\cos^2(\varphi)\sin^2(\theta)} &&\text{for even } n \end{aligned}$$

and

$$\begin{aligned} F_{y,n} &= -\frac{2b}{n\pi}\,\frac{i\pi^2 n^2 \sin\big(0.5b\,k\sin(\varphi)\sin(\theta)\big)}{n^2\pi^2 - b^2k^2\sin^2(\varphi)\sin^2(\theta)} &&\text{for odd } m;\\ F_{y,n} &= \frac{2b}{n\pi}\,\frac{\pi^2 n^2 \cos\big(0.5b\,k\sin(\varphi)\sin(\theta)\big)}{n^2\pi^2 - b^2k^2\sin^2(\varphi)\sin^2(\theta)} &&\text{for even } m. \end{aligned}$$

Exemplary directional characteristics of the radiation $F_{x,m}(\theta,\varphi)\,F_{y,n}(\theta,\varphi)$ for the frequency 1000 Hz are presented in Fig. 7.

The total distribution of the sound pressure at the observation point of coordinates r_i, φ_i, θ_i can be described as:

$$p(r_i,\theta_i,\varphi_i) = \sum_{m,n} A_{m,n}\,g_{m,n}(r_i,\theta_i,\varphi_i) \quad [\mathrm{Pa}] \tag{9}$$

Performing simultaneous measurements of the sound pressure in L observation points $(L \geq m\cdot n)$ we obtain the set of linear equations written in the matrix form:

$$\mathbf{P} = \mathbf{G}\cdot\mathbf{A} \quad [\mathrm{Pa}]. \tag{10}$$

This set of equations can be solved on the basis of the inverse method [3]. However, the solution might not be stable enough due to the fact that the directivity radiation response characteristics, measured in the rectangular grid nodes at further distance from the sound source, are quite similar. The solution stability can be – in such case – increased by the Tichonow's regularisation.

The final solution of such presented problem is the distribution of vibration velocity on the plate surface described by the relation:

$$v(x,y) = \sum_{m,n} A_{m,n}\sin\left(\frac{m\pi(x-0.5a)}{a}\right)\sin\left(\frac{n\pi(y-0.5b)}{b}\right) \quad [\mathrm{m/s}]. \tag{11}$$

Exemplary solution achieved with computer simulations is presented in the Fig. 8.

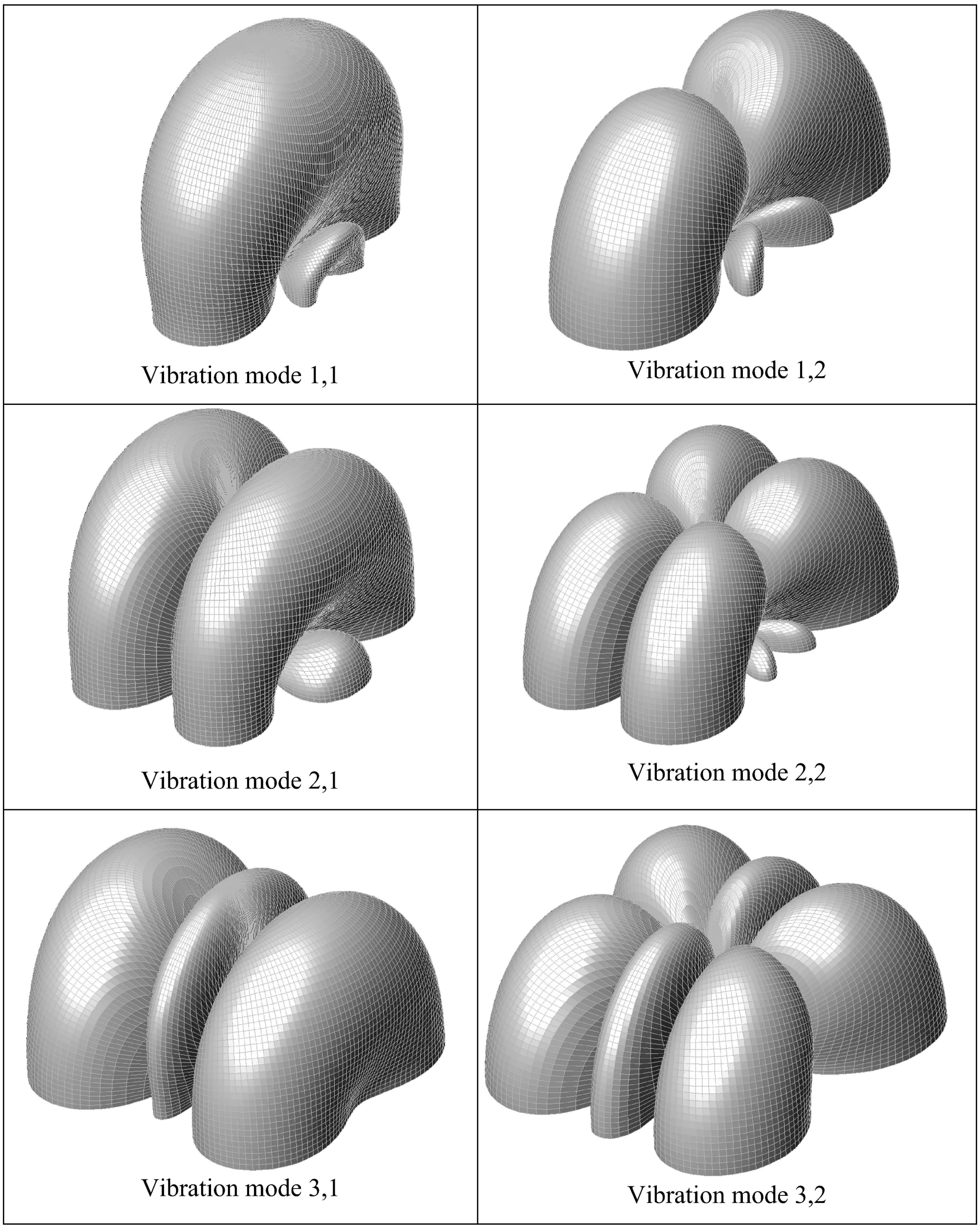

Fig. 7. Directional characteristics of the rectangular plate radiation.

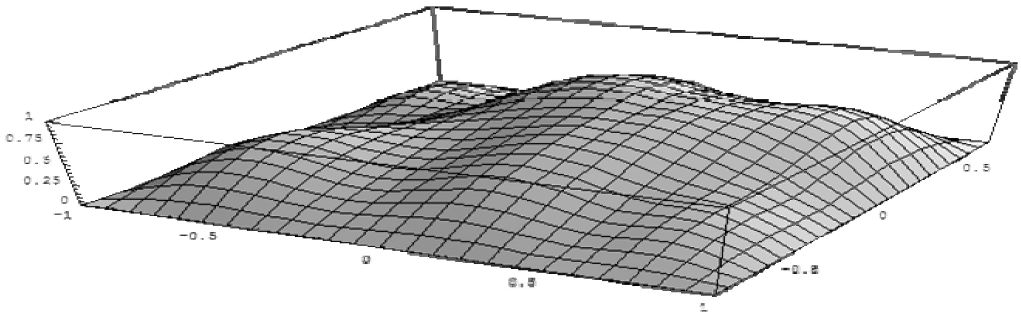

Fig. 8. Distribution of vibration velocity on the plate surface achieved with the inverse method.

Accuracy of presented calculations strongly depends on the presence of the disturbances during measurement and on the degree of reality projection by assumed generation model and on sound propagation. Solution stability is currently the subject of author's research.

4. Conclusions

Inverse methods can find application in problems of an identification of vibroacoustic energy sources and in the estimation of sound radiation.

The examples of the performed investigations illustrate how the acoustic field can be tested and how the models of the vibroacoustic emission of sound sources, based on the set of substitute sources, can be presented.

The knowledge of actual distributions of the sound pressure around sound sources is indispensable for the calculation of model parameters. However, to obtain this purpose it is necessary to determine the distribution of sound pressures amplitude as well as the distribution of phase shift angles in the selected measuring points. In the case when we do not know the shift angles we can also apply the inverse method, but we must realize that the calculation accuracy will decrease and that we will have to select the observation points more carefully.

The inverse method (especially when the measurements are done with the phase shift angle) is very sensitive to the localization of substitute sources. This property can be also used for the determination of the localization of actual sources. The sensitivity of the method depends also on the localization of the observation points.

Acknowledgments

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