ARCHIVES OF ACOUSTICS
32, 2, 405–415 (2007)

THE INFLUENCE OF A VIBRATING RECTANGULAR PISTON ON THE ACOUSTIC POWER RADIATED BY A RECTANGULAR PLATE

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(*received March 23, 2007; accepted April 18, 2007*)

The equation of motion of a flat simply supported rectangular plate has been solved. The plate has been excited by a surface force. The influence of the acoustic pressure radiated by the plate on its vibrations has been included. The corresponding sound pressure distributions have been presented as their backward Fourier transforms. The acoustic active and reactive sound power has been computed including the influence of the sound pressure radiated by the piston. The acoustic mutual sound power of both sources has also been presented.

Keywords: acoustic pressure, radiation, rectangular acoustic sources, radiation efficiency, sound power.

1. Introduction

The modal and intermodal radiation efficiency values are useful for some further theoretical analyzes of the sound power radiated. The modal radiation efficiency values have been considered purely analytically and reported in a few papers [1–8]. The mutual impedance of the two rectangular pistons located in a flat infinite baffle has been presented in [9, 10]. Arase and Wyrzykowska have considered the influence of the pistons on the sound field generated including the effect of the shift in phase of their vibrations. However, the piston-piston sound source idealization is not always enough for some real-life systems. For example the cover of the power transformer casings should be modeled by a plate rather than by a piston. Therefore, some purely theoretical considerations of the total sound power radiated by a rectangular plate-piston system would be essentially important. So far, there is no such an analysis presented in the literature and this paper presents a solutions to the problem.

2. Governing equations

The two flat rectangular acoustic sources are embedded into a flat rigid infinite baffle for $z = 0$. One of the sources is a thin simply supported plate of sizes $a_1 \times b_1$ and the other one is a flat piston of sizes $a_2 \times b_2$ (cf. Fig. 1). The distance between the central points of the two sources amounts to l_0. Their appropriate edges are in pairs parallel while their areas are $S_1 = a_1 b_1$ and $S_2 = a_2 b_2$, respectively. The plate is a component of a power transformer casing and is excited by the surface force $f(x, y, t) = f(x, y) \exp(-i\omega t)$. It is assumed that the distribution of the amplitude $f(x, y)$ and the excitation frequency ω are given or measured and that they are independent of any available control systems. As a result the plate become a source of noise.

The normal component of the piston vibration velocity is

$$v^{(2)}(x, y, t) = v^{(2)} e^{i(\delta - \omega t)} = -i\omega W^{(2)} e^{i(\delta - \omega t)}, \tag{1}$$

where $v^{(2)}$ is the vibration velocity amplitude, δ is the initial shift in phase at the vibration frequency ω, $W^{(2)}$ is the piston transverse deflection amplitude. Since Eq. (1) describes harmonic vibrations it can be expressed in its amplitude form $v^{(2)}(x, y) = v^{(2)} e^{i\delta} = -i\omega W^{(2)} e^{i\delta}$. It is assumed that the acoustic power control system can control the plate-piston interaction system via such quantities as $W^{(2)}$ and δ. Additionally, the vibration frequency of the piston should be identical as the excitation frequency ω. The control system should this way select the quantities $W^{(2)}$ and δ so that the total acoustic power radiated is minimal for steady state vibrations. It is the amplitude-phase control system of the acoustic power where the piston is acting as the antisource (cf. Fig. 2).

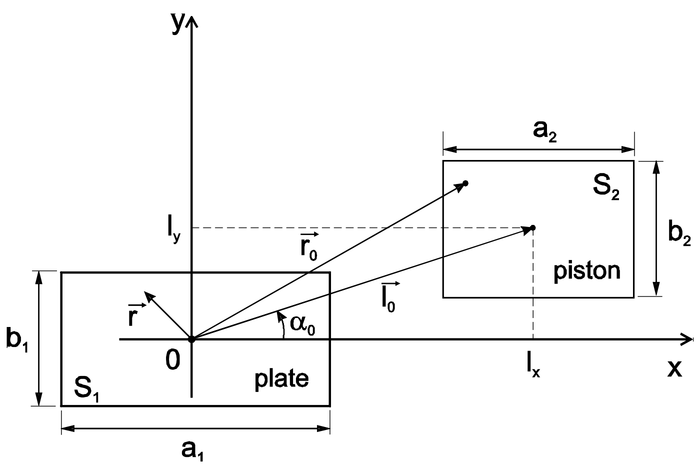

Fig. 1. Location of the flat plate and the flat piston both embedded into a flat infinite baffle for $z = 0$.

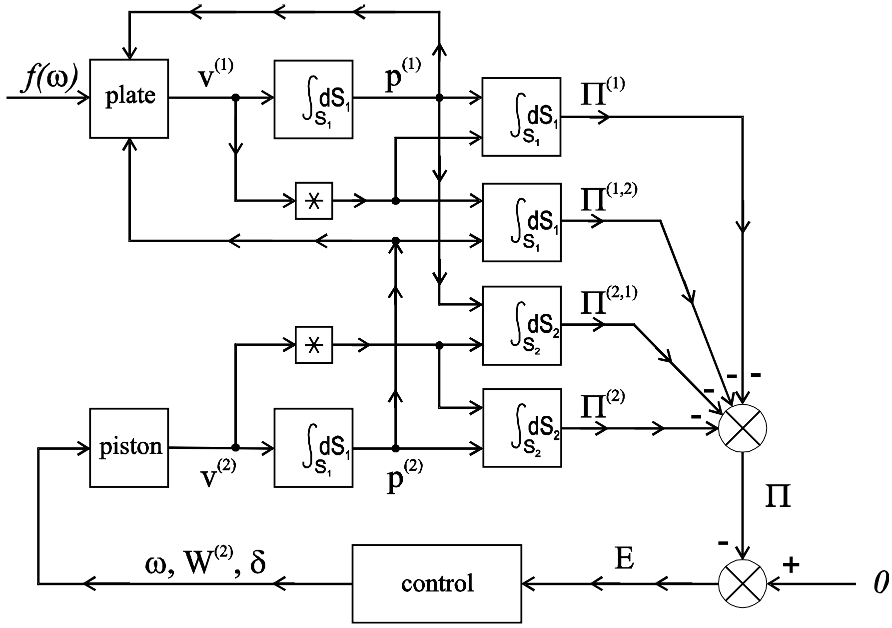

Fig. 2. The acoustic power computation and control scheme. Key: is the conjugate value, is the integration over the surface S_1 (time-averaged), is the integration over the surface S_1 (time-averaged), is the summation.

Analyzing the acoustic power control system scheme it is worth noticing that the acoustic pressure $p^{(1)}$ radiated by the vibrating plate does not influence the piston vibration velocity $v^{(2)}$. However both pressure $p^{(1)}$ and pressure $p^{(2)}$ influence the plate vibration velocity $v^{(1)}$. The influence is particularly essential for the excitation frequencies ω being close to the plate eigenfrequencies and represents the aeroacoustic damping. In the case of the excitation frequencies being far from the eigenfrequencies the aeroacoustic damping can be neglected. The computational scheme presented in Fig. 2 can turn out to be difficult or impossible for the practical accomplishment on account of a large number of data and six-times integrating over the surfaces of vibrating sources. For this reason farther theoretical and numerical analysis of these quantities has been carried out through calculating the acoustic power at set values of $f(\omega)$, ω, $W^{(2)}$ and δ. The very control algorithm has not been investigated, since in the state of the steady state vibrations, at established excitation $f(\omega)$, ω, once selected values $W^{(2)}$ and δ would not be changing, and the acoustic power radiated will remain minimal.

The governing equation of the excited plate presented in [11] can be rearranged and formulated in its amplitude form

$$\left\{k_D^{-4}\nabla^4 - (1 + i\beta/\omega)\right\}\ W^{(1)}(x,y) = \frac{1}{\omega^2 \rho h}\left\{ f(x,y) - p^{(1)}(x,y) - p^{(2)}(x,y)\right\}, \qquad (2)$$

where $k_D^4 = \omega^2 \rho h/D$, $D = D_E \{1 - i(\eta + \zeta\omega)\}$, $D_E = Eh^3/12\,(1-\nu^2)$, E, h, ρ, ν are the Young modulus, thickness, density and the Poisson ratio of the plate, respectively, β, η, ζ are the internal damping factors associated with the Maxwell model, the hysteretic curve and the deformation velocity of the plate, $f(x,y)$, $p^{(1)}(x,y)$, $p^{(2)}(x,y)$ are the amplitudes of the surface excitation force, the plate acoustic pressure and the piston acoustic pressure, respectively, $\nabla^4 = \nabla^2\nabla^2$, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, $W^{(1)}(x,y)$ is the plate's transverse deflection amplitude. The solution of Eq. (2) can be formulated as the eigenfunction series

$$W^{(1)}(x,y,t) = e^{-i\omega t} \sum_{m,n=1}^{\infty} c_{mn} W_{mn}(x,y), \tag{3}$$

where c_{mn} are the unknown coefficients,

$$W_{mn}(x,y) = A_{mn} \sin \frac{m\pi}{a_1}\left(x + \frac{a_1}{2}\right) \sin \frac{n\pi}{b_1}\left(y + \frac{b_1}{2}\right) \tag{4}$$

is the eigenfunction of the plate's vibration mode (m,n), the coefficient $A_{mn} = 2$ has been determined in virtue of the orthogonality condition

$$\frac{1}{S_1} \int\limits_{S_1} W_{mn}(x,y)\, W_{pq}(x,y)\, \mathrm{d}S_1 = \delta_{mp}\delta_{nq} \tag{5}$$

and $W^{(1)}(x,y,t) = W^{(1)}(x,y)\exp(-i\omega t)$.

The homogeneous governing equation of the plate with the real coefficients (for $\beta, \eta, \zeta = 0$) in its form of $(k_D^{-4}\nabla^4 - 1)\, W^{(1)}(x,y,t) = 0$, where $W^{(1)}(x,y,t) = \sum\limits_{m,n=1}^{\infty} \exp(-i\omega_{mn}t) c_{mn} W_{mn}(x,y)$, can be rearranged and expressed in its amplitude form by using Eq. (5)

$$(k_{mn}^{-4}\nabla^4 - 1)\, W_{mn}(x,y) = 0, \tag{6}$$

where $k_{mn}^4 = \omega_{mn}^2 \rho h/D_E = \{(m\pi/a_1)^2 + (n\pi/b_1)^2\}^2$ and ω_{mn} is the eigenfrequency of the mode (m,n).

The solution (3) has been multiplied by the term $\exp(i\omega t)$, inserted into Eq. (2) (Eq. (6) has been used), multiplied by the eigenfunction $W_{pq}(x,y)$, integrated over the plate's surface S_1 (Eq. (5) has been used), and resulted in the following equations system

$$c_{mn}\left\{k_D^{-4}k_{mn}^4 - (1 + i\frac{\beta}{\omega})\right\} = \frac{1}{\omega^2\rho h}\left\{f_{mn} - p_{mn}^{(1)} - p_{mn}^{(2)}\right\}, \tag{7}$$

where the following modal factors have been denoted as

$$
\begin{aligned}
f_{mn} &= \frac{1}{S_1}\int\limits_{S_1} f(x,y)W_{mn}(x,y)\mathrm{d}S_1,\\
p^{(1)}_{mn} &= \frac{1}{S_1}\int\limits_{S_1} p^{(1)}(x,y)W_{mn}(x,y)\mathrm{d}S_1,\\
p^{(2)}_{mn} &= \frac{1}{S_1}\int\limits_{S_1} p^{(2)}(x,y)W_{mn}(x,y)\mathrm{d}S_1.
\end{aligned}
\tag{8}
$$

Solving the equations system (7) and finding the coefficients c_{mn} first requires computing the acoustic pressure values, and then computing the modal coefficients f_{mn}, $p^{(1)}_{mn}$ and $p^{(2)}_{mn}$. The computations should be performed for a finite number of modes, i.e. for $m = 0, 1, 2, \ldots, M + 1$ and $n = 0, 1, 2, \ldots, N + 1$. The mode numbers M, N should be chosen in such a way so that the following condition is satisfied $k_{\max} \le k_{MN}$, where $k_{\max}$ is the upper bound of the acoustic wavenumber band $k \in (0, k_{\max})$, where the approximation is precise enough.

3. Acoustic pressure of the vibrating piston and plate

The distribution of the vibrating piston acoustic pressure exerted on a flat rectangular plate can be described by the Rayleigh formula [12]

$$
p^{(2)}(\mathbf{r}) = -\frac{i}{2\pi}\rho_0 ck \int\limits_{S_2} v^{(2)}(\mathbf{r}_0)\frac{e^{ik|\mathbf{r}-\mathbf{r}_0|}}{|\mathbf{r}-\mathbf{r}_0|}\mathrm{d}S_2,
\tag{9}
$$

where $\mathbf{r} = (x, y, z)$, $\mathbf{r}_0 = (x_0, y_0, z_0)$ are the leading vectors of the sound field point and the source point, respectively, ρ_0, c are the air density and the speed of sound in the air. The following formula is valid within the zone $z \ge 0$ for $z_0 = 0$

$$
\frac{e^{ik|\mathbf{r}-\mathbf{r}_0|}}{|\mathbf{r}-\mathbf{r}_0|} = \frac{i}{2\pi}\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} \exp\left\{i\left[k_x(x-x_0)+k_y(y-y_0)+k_z z\right]\right\}\frac{\mathrm{d}k_x\mathrm{d}k_y}{k_z},
\tag{10}
$$

where $\mathbf{k} = (k_x, k_y, k_z)$ is the wavevector and $k_z^2 = k^2 - k_x^2 - k_y^2$. Further applying the following wavevector coordinates transformations $k_x = k\sin\vartheta\cos\varphi$, $k_y = k\sin\vartheta\sin\varphi$ and $k_z = k\cos\vartheta$, where $\varphi \in [0,\, 2\pi]$, $\vartheta \in [0,\, \pi/2 - i\infty)$ and using the following Jacobian $\mathrm{d}k_x\mathrm{d}k_y = k^2\sin\vartheta\cos\vartheta\mathrm{d}\vartheta\,\mathrm{d}\varphi$ make it possible to formulate the vibrating piston acoustic pressure as (cf. Fig. 3)

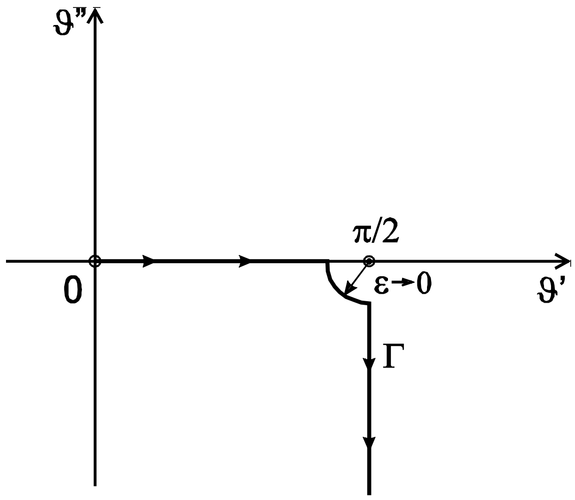

Fig. 3. The integration contour in Eq. (11) for $\vartheta = \vartheta' + i\vartheta''$.

$$p^{(2)}(x,y) = \rho_0 c \left(\frac{k}{2\pi}\right)^2 S_2 \int\limits_0^{2\pi} \int\limits_0^{\pi/2-i\infty} M^{(2)}(\vartheta,\varphi) \exp(i\mathbf{k}\cdot\mathbf{l}_0)\, \exp\left\{i(k_x x + k_y y)\right\} \sin\vartheta\, \mathrm{d}\vartheta \mathrm{d}\varphi, \qquad (11)$$

where the scalar product $\mathbf{k}\cdot\mathbf{l}_0 = k_x l_x + k_y l_y = k l_0 \sin\vartheta \cos(\varphi - \alpha_0)$, the following function has been introduced

$$M^{(2)}(\vartheta,\varphi) = \frac{1}{S_2} \int\limits_{S_2} v^{(2)}(x,y)\, \exp\left\{-i(k_x x + k_y y)\right\} \mathrm{d}S_2 = -i\omega W^{(2)} e^{i\delta} \cdot \frac{\sin(\alpha^{(2)}/2)}{\alpha^{(2)}/2} \cdot \frac{\sin(\beta^{(2)}/2)}{\beta^{(2)}/2} \qquad (12)$$

and it has been denoted $\alpha^{(2)} = k a_2 \sin\vartheta\cos\varphi$, $\beta^{(2)} = k b_2 \sin\vartheta \sin\varphi$. Further, the acoustic pressure (11) has been inserted into Eq. (8) which resulted in the modal coefficient

$$p_{mn}^{(2)} = -\frac{i}{\omega_{mn}} \rho_0 c \left(\frac{k}{2\pi}\right)^2 S_2 \int\limits_0^{2\pi} \int\limits_0^{\pi/2-i\infty} M^{(2)}(\vartheta,\varphi)\, M_{mn}^{(1)*}(\vartheta,\varphi) \exp(i\mathbf{k}\cdot\mathbf{l}_0) \sin\vartheta\, \mathrm{d}\vartheta\mathrm{d}\varphi, \qquad (13)$$

where $M_{mn}^{(1)*}(\vartheta,\varphi)$ is the conjugate value for

$$M_{mn}^{(1)}(\vartheta,\varphi) = \frac{1}{S_1}\int\limits_{S_1} v_{mn}(x,y)\ \exp\{-i(k_x x + k_y y)\}\,\mathrm{d}S_1$$

$$= \frac{-i\omega_{mn}}{S_1}\int\limits_{S_1} W_{mn}(x,y)\exp\{-i(k_x x + k_y y)\}\,\mathrm{d}S_1$$

$$= -i\frac{2\omega_{mn}}{\pi^2 mn}\cdot\frac{[1-(-1)^m]\cos(\alpha^{(1)}/2) + i\,[1+(-1)^m]\sin(\alpha^{(1)}/2)}{1-(\alpha^{(1)}/m\pi)^2}$$

$$\cdot\frac{[1-(-1)^n]\cos(\beta^{(1)}/2) + i\,[1+(-1)^n]\sin(\beta^{(1)}/2)}{1-(\beta^{(1)}/n\pi)^2} \qquad (14)$$

but the conjugate value does not concern the variable ϑ within the whole analysis, and it has been denoted $\alpha^{(1)} = ka_1\sin\vartheta\cos\varphi$, $\beta^{(1)} = kb_1\sin\vartheta\sin\varphi$.

The normal component of the plate's vibration velocity amplitude has been formulated using $v^{(1)}(x,y,t) = -i\omega W(x,y,t)$ and Eq. (3) as

$$v^{(1)}(x,y) = -i\omega\sum_{m,n=1}^{\infty} c_{mn}W_{mn}(x,y) \qquad (15)$$

and inserted into Eq. (9) instead of $v^{(2)}$, Eq. (10) has been used providing the acoustic pressure distribution on the surface $z = 0$ in the form of

$$p^{(1)}(x,y) = \rho_0 c\left(\frac{k}{2\pi}\right)^2 S_1\int\limits_0^{2\pi}\int\limits_0^{\pi/2-i\infty} M^{(1)}(\vartheta,\varphi)$$

$$\exp\{i(k_x x + k_y y)\}\sin\vartheta\,\mathrm{d}\vartheta\mathrm{d}\varphi, \qquad (16)$$

where it has been denoted (cf. Eq. (16))

$$M_{mn}^{(1)}(\vartheta,\varphi) = \frac{1}{S_1}\int\limits_{S_1} v^{(1)}(x,y)\ \exp\{-i(k_x x + k_y y)\}\,\mathrm{d}S_1$$

$$= \omega\sum_{m,n=1}^{\infty}\frac{c_{mn}}{\omega_{mn}}M_{mn}^{(1)}(\vartheta,\varphi). \qquad (17)$$

The acoustic pressure from Eq. (16) has been inserted into Eq. $(8)_2$ resulting in the modal coefficient

$$p_{mn}^{(1)} = -\frac{i}{\omega_{mn}}\rho_0 c\left(\frac{k}{2\pi}\right)^2 S_1\int\limits_0^{2\pi}\int\limits_0^{\pi/2-i\infty} M^{(1)}(\vartheta,\varphi)\,M_{mn}^{(1)*}(\vartheta,\varphi)\sin\vartheta\,\mathrm{d}\vartheta\,\mathrm{d}\varphi$$

$$= -i\frac{\omega}{\omega_{mn}}\rho_0 c\left(\frac{k}{2\pi}\right)^2 S_1\sum_{p,q=1}^{\infty}\frac{c_{pq}}{\omega_{pq}}F_{pq,mn}, \qquad (18)$$

where the intermodal coefficient of the pair of modes (p, q) and (m, n) has been denoted as

$$F_{pq,mn} = \int_0^{2\pi} \int_0^{\pi/2 - i\infty} M_{pq}^{(1)}(\vartheta, \varphi)\, M_{mn}^{(1)*}(\vartheta, \varphi) \sin\vartheta \,\mathrm{d}\vartheta \,\mathrm{d}\varphi. \tag{19}$$

Further, the determined coefficients $p_{mn}^{(1)}$ from Eq. (18), $p_{mn}^{(2)}$ from Eq. (13) and f_{mn} from Eq. (8) for a given distribution of the plate's surface excitation force $f(x, y)$ have been inserted into the equations system (7). Solving this equations system provides the coefficients c_{mn}.

4. Acoustic power

The time-averaged acoustic power of an excited and damped rectangular plate has been formulated using Eqs. (15), (16) and (17) as [13]

$$\begin{aligned} \varPi^{(1)} &= \frac{1}{2} \int_{S_1} p^{(1)}(x, y) v^{(1)*}(x, y) \mathrm{d}S_1 \\ &= \frac{1}{2} \rho_0 c \left(\frac{k}{2\pi}\right)^2 S_1^2 \int_0^{2\pi} \int_0^{\pi/2 - i\infty} M^{(1)}(\vartheta, \varphi)\, M^{(1)*}(\vartheta, \varphi) \sin\vartheta \,\mathrm{d}\vartheta\vartheta \,\mathrm{d}\varphi \\ &= \frac{1}{2} \rho_0 c \left(\frac{k}{2\pi}\right)^2 S_1^2 \sum_{pq,mn=1}^{\infty} c_{pq} c_{mn}^* \frac{\omega}{\omega_{pq}} \frac{\omega}{\omega_{mn}} F_{pq,mn}. \end{aligned} \tag{20}$$

The integration within the first line of Eq. (20) has been performed over the surface S_1 for $z = 0$ (the impedance approach) whereas within the integration over the variable ϑ within the second line of Eq. (20) has been performed over the contour $\varGamma$ (Fig. 3). The fourfold series in the third line of Eq. (20) can be substituted by a double series. For this purpose Eq. (7) has been multiplied by c_{mn}^*, summed up over the indices $m, n = 1, \ldots, \infty$ giving

$$\varPi^{(1)} = \frac{i\omega}{2} S_1 \sum_{m,n=1}^{\infty} c_{mn}^* \left\{ f_{mn} - p_{mn}^{(2)} - \omega^2 \rho h \, c_{mn} \left[k_{mn}^4 k_D^{-4} - \left(1 + i\frac{\beta}{\omega}\right)\right]\right\}. \tag{21}$$

The coefficients $p_{mn}^{(2)}$ have been given in Eq. (13).

The time-averaged acoustic power of a rectangular piston vibrating with velocity $v^{(2)}$ (cf. Eq. (1)) has been computed using Eqs. (11) and (12) (the impedance approach)

$$\Pi^{(2)} = \frac{1}{2} \int\limits_{S_2} p^{(2)}(x,y) v^{(2)*}(x,y) dS_2$$

$$= \frac{1}{2}\rho_0 c \left(\frac{k}{2\pi}\right)^2 S_2^2 k^2 \int\limits_0^{2\pi} \int\limits_0^{\pi/2-i\infty} M^{(2)}(\vartheta,\varphi)\, M^{(2)*}(\vartheta,\varphi) \sin\vartheta \mathrm{d}\vartheta\vartheta \mathrm{d}\varphi, \tag{22}$$

where from Eq. (12)

$$M^{(2)}(\vartheta,\varphi)\, M^{(2)*}(\vartheta,\varphi) = \left\{ v^{(2)} \frac{\sin(\alpha^{(2)}/2)}{\alpha^{(2)}/2} \; \frac{\sin(\beta^{(2)}/2)}{\beta^{(2)}/2} \right\}^2 . \tag{23}$$

A formula convenient for numerical computations has been presented earlier in [10] where the radiation resistance and reactance of a rectangular piston have been expressed as a fast convergent infinite series containing the integral Bessel and Neumann functions. On the other hand in the case of small argument values $ka_2,\, kb_2 \ll 1$ in the active acoustic power Eq. (22) (integration within the limits $0 \le \vartheta \le \pi/2$) the following approximation has been used

$$\left\{ \frac{\sin(\alpha^{(2)}/2)}{\alpha^{(2)}/2} \frac{\sin(\beta^{(2)}/2)}{\beta^{(2)}/2} \right\}^2 \cong 1 - \frac{\sin^2\vartheta}{12} \left\{ (ka_2)^2 \cos^2\varphi + (kb_2)^2 \sin^2\varphi \right\}$$

which leads to

$$\mathrm{Re}\Pi^{(2)} \cong \rho_0 c \frac{(k^2 S_2 v^{(2)})^2}{4\pi} \left\{ 1 - \frac{1}{3} \left[(ka_2)^2 + (kb_2)^2 \right] \right\} . \tag{24}$$

The time-averaged mutual acoustic power of a rectangular plate vibrating with amplitude $v^{(1)}(x,y)$ given in Eq. (15) under the acoustic pressure $p^{(2)}(x,y)$ (Eqs. (11) and (12)) generated by a vibrating piston has been formulated as

$$\Pi^{(1,2)} = \frac{1}{2} \int\limits_{S_2} p^{(2)}(x,y)\, v^{(1)*}(x,y)\, \mathrm{d}S_1 = \frac{1}{2}\rho_0 c \left(\frac{k}{2\pi}\right)^2 S_1 S_2 \int\limits_0^{2\pi} \int\limits_0^{\pi/2-i\infty}$$

$$M^{(2)}(\vartheta,\varphi)\; M^{(1)*}(\vartheta,\varphi)\; \exp(-i\,\mathbf{k}\cdot\mathbf{l}_0) \sin\vartheta\, \mathrm{d}\vartheta\vartheta\, \mathrm{d}\varphi, \tag{25}$$

where the scalar product $\mathbf{k}\cdot\mathbf{l}_0$ has been given after Eq. (11). The mutual acoustic power of a rectangular piston vibrating with amplitude $v^{(2)}(x,y) = v^{(2)} \exp(i\delta)$ Eq. (1) under the influence of the acoustic pressure $p^{(1)}(x,y)$ (Eqs. (16) and (17)) generated by a vibrating plate has been formulated as

$$\Pi^{(2,1)} = \frac{1}{2}\int\limits_{S_2} p^{(1)}(x,y)\, v^{(2)}(x,y)\,\mathrm{d}S_2 = \frac{1}{2}\rho_0 c\left(\frac{k}{2\pi}\right)^2 S_1 S_2 \int\limits_0^{2\pi}\int\limits_0^{\pi/2-i\infty} M^{(1)}(\vartheta,\varphi)\, M^{(2)*}(\vartheta,\varphi)\,\exp(-i\,\mathbf{k}\cdot\mathbf{l}_0)\sin\vartheta\,\mathrm{d}\vartheta\vartheta\,\mathrm{d}\varphi. \tag{26}$$

Considering Eqs. (25) and (26) it is obvious that $\Pi^{(1,2)} = \Pi^{(2,1)*}$ and $\Pi^{(2,1)} = \Pi^{(1,2)*}$.

The total acoustic power of the vibrating plate-piston system embedded into a flat rigid baffle has been expressed as a sum of the acoustic self-power of the plate Eqs. (20) or (21) and the piston Eqs. (22) or (24), and the mutual acoustic power of the vibrating sources Eqs. (25) and (26)

$$\Pi = \Pi^{(1)} + \Pi^{(2)} + \Pi^{(1,2)} + \Pi^{(2,1)}. \tag{27}$$

Considering equations used to formulate the total sound power radiated Eq. (27) leads to conclusions that this quantity depends on the surface force amplitude $f(x,y)$ exciting the rectangular plate, on the amplitude $v^{(2)} = -iW^{(2)}\exp(i\delta)$ and on the generated acoustic pressure levels. Since however it is not possible directly to influence none of mentioned quantities except for the piston vibration velocity, so controlling the acoustic power can take place only via the amplitude $W^{(2)}$ and phase δ Eq. (1) at a measured frequency ω.

5. Concluding remarks

It has been proved analytically that the sound power radiated by a rectangular plate can be minimized by a vibrating flat piston antisource. Formulas presented herein make it possible to perform some numerical simulations of some steady state harmonic processes to help further designing some acoustic cancellation systems for a power transformer casings.

References

[1] WALLACE C. E., *Radiation resistance of a rectangular panel*, J. Acoust. Soc. of Amer., **51**, 3, 946–952 (1972).

[2] GOMPERTS M. C., *Sound radiation from baffled thin, rectangular plates*, Acta Acustica/Acustica, **37**, 93–102 (1977).

[3] DAVIES H. G., *Acoustic radiation from fluid loaded rectangular plates*, MIT, Acoust. and Vibr. Labor., Tech. Rep. No. **71476–12** (1969).

[4] LEVINE, H., *On the short wave acoustic radiation from planar panels or beams of rectangular shape*, J. Acoust. Soc. of Amer., **76**, 2, 608–615 (1984).

[5] LEPPINGTON F. G., *The acoustic radiation efficiency of rectangular panels*, Proc. Royal Soc. London, **A382**, 245–271 (1982).

[6] BERRY A., GUYADER J. L., NICOLAS J., *A general formulation for the sound radiation from rectangular, baffled plates with arbitrary boundary conditions*, J. Acoust. Soc. of Amer., **86**, 6, 2792–2802 (1990).

[7] STEPANISHEN P. R., EBENEZER D. D., *A joint wavenumber-time domain technique to determine the transient acoustic radiation loading on planar vibrators*, J. Sound and Vibration, **157**, 3, 451–465 (1992).

[8] ZAWIESKA W. M., RDZANEK W. P., *Low frequency approximation of mutual modal radiation efficiency of a vibrating rectangular plate*, Archives of Acoustics, **31**, 4, 123–130 (2006).

[9] ARASE E. M., *Mutual impedance of square and rectangular pistons in a rigid infinite baffle*, J. Acoust. Soc. of Amer., **36**, 8, 1521–1525 (1964).

[10] WYRZYKOWSKA B., *The acoustic impedance of flat pistons of a uniform vibration velocity distribution*, University Press, Rzeszów 1979.

[11] MEIROVITCH L., *Analytical methods in vibration*, MacMillan, New York 1967.

[12] RAYLEIGH J. W. S., *Theory of sound*, MacMillan, London 1929.

[13] SKUDRZYK E., *Basic mathematics & Basic acoustics*, Springer-Verlag, Wien, New York 1971.