

## ANALYSIS AND MODELLING OF ULTRASONIC PULSES IN A BIOLOGICAL MEDIUM

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In this work, formulas were presented, which allow modelling of linear and nonlinear propagation of ultrasonic wave in a biological medium, by simulating the shape of pulses which propagate in this medium. Analysed and measured in chosen biological media using transmission method, the linear and nonlinear ultrasonic pulses were modelled by means of developed formulas. Very good compatibility of the representation was achieved both in the area of time and frequency. In case of biological media causing high attenuation of ultrasonic wave, reduction of mid frequency in the receiving pulse caused by attenuation dispersion was observed.

**Keywords:** ultrasonic wave propagation, biological medium, pulse modelling.

### 1. Introduction

Modelling of propagation of ultrasonic wave in a biological medium on the basis of wave equation results requires the use of complicated numerical models and specialised calculation tools [2, 4]. In this work, the author presented simple formulas he previously developed, which allow modelling of linear and nonlinear propagation of ultrasonic wave in a biological medium, by simulating the shape of pulses which propagate in this medium. Several different equations based on detailed analysis of parameters of real ultrasonic pulses registered in biological media were elaborated. The formulas presented in this paper enable simulating pulses, which are generated in biological medium using ultrasonic piezoelectric transducers excited by sequence of *burst* type pulses.

These formulas are used in the process of optimization of measurement algorithms of the ultrasonic transmission tomography method (UTT) [3].

## 2. Linear propagation

In case of low intensity of ultrasonic waves generated in a biological medium, the shape of the wave does not differ much from sinusoidal waveform. In transmission ultrasonic measurements of biological media using UTT method, *burst* type pulses sequence is usually utilized to power the sending transducers [3]. Based on envelope function in the form of  $f(x) = 1 - |x|^m$ , the author developed a formula which allows determination of linear ultrasonic signal of the *burst* type in a biological medium. The signal takes the form of  $n$  number of sinusoidal pulses:

$$p = p_o \sum_{n=0}^{\infty} (1 - |2 \tau_n f_o / m_c - 1|^m) \cdot \sin(\omega_o \tau_n - \varphi_o), \quad (1)$$

where  $p_o$  – acoustic pressure amplitude,  $\tau_n = t - z/c_{av} - n/f_r$  indicates retardant time (with allowance made for a delay resulting from transit time and repetition frequency),  $z$  – the path of wave front,  $c_{av}$  – average sound velocity in the medium,  $m_c$  – the number of pulse width cycles, in which  $p = 0$  outside range  $\tau_n \in (0; m_c/f_o)$ . Electric *burst* type pulse sequence that powers the sending transducer can also be determined using formula (1), if acoustic pressure is substituted with voltage and transit time  $z/c_{av}$  with delay  $\Delta t$  (usually  $\Delta t = 0$ ) where exponent value is assumed to be  $m \rightarrow \infty$ .

Figure 1a shows an example of a receiving pulse measured by the author by means of a hydrophone (HPM05/2 Precision Acoustics: diameter – 0.5 mm, sensitivity – 250 nV/Pa) in a transmission system in water (distance between transducers  $d = 100.00$  mm), utilizing a rectangular elementary transducer ( $0.5 \times 18$  mm) of a multielement ultrasonic ring probe [1], after it was transmitted through a hard boiled hen's egg with removed shell. The sending transducer was powered by *burst* type pulses, which were characterised by frequency of  $f_o = 1.700$  MHz, number of cycles  $m_c = 15$ , phase  $\varphi_o = 0$ , amplitude  $U_o = 10$  V (20 Vpp) and repetition frequency  $f_r = 2$  kHz. The measured receiving pulse (Fig. 1a) is delayed in relation to the sending pulse by the time of  $t_p = 67.000$   $\mu$ s ( $c_{av} = 1492.54$  m/s). The receiving pulse envelope in comparison to the sending pulse changed – pulse attack and decay time significantly increased and the number of periods increased, which is caused by considerable sending transducer inertia. The amplitude of the measured receiving pulse was  $p_o = 4.667$  kPa (peak-to-peak), in which  $p_o^+ = 2.344$  kPa,  $p_o^- = -2.322$  kPa, which gives an average value of pressure level  $L_{p/pr} = 187.36$  dB in relation to reference pressure, which in water is assumed to be  $p_r = 1$   $\mu$ Pa. With the receiving pulse pressure values that low (resulting from high ultrasound attenuation in egg), it is impossible to observe distortions caused by the medium's nonlinearity; harmonics are also not observable in spectrum (Fig. 2a).

In order to reconstruct the pulse from Fig. 1a, numerical calculations were performed using a universal formula (1), with utilisation of a suitable composition of envelopes for attack part of the pulse ( $m = 2$ ) and decay part ( $m = 0.5$ ) for parameters  $f_o = 1.660$  MHz,  $m_c = 39$ ,  $z = 100.00$  mm,  $c_{av} = 1492.54$  m/s,  $\varphi_o = 0$ ,  $f_r = 2$  kHz. The result was a pulse shown in Fig. 1b. Figure 2a shows the comparison of a *burst* type sending pulse spectrum with a measured receiving pulse spectrum from Fig. 1a.

In case of receiving pulse, resonance band in the area of sending transducer operation frequency (1.7 MHz) is much thinner, and signal components of frequencies around the resonance are attenuated, which can be explained by significant  $Q$  factor of the sending transducer and a very long time of receiving pulse attack and decay (Fig. 1).

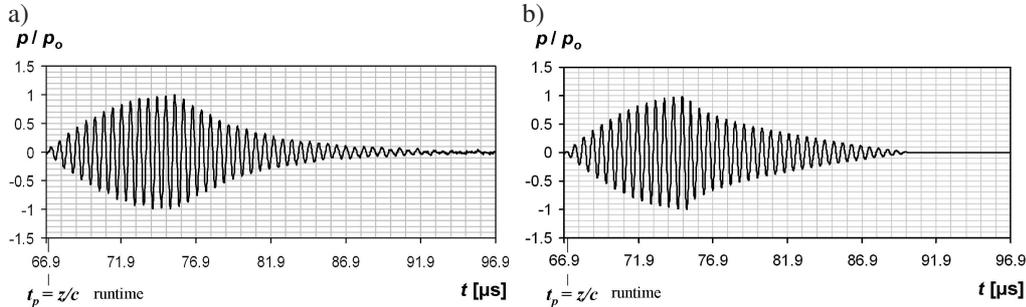


Fig. 1. Linear *burst* type pulse transmitted through a biological medium: a) measured, b) calculated.

Figure 2b shows a comparison of three pulses' spectra in a narrow band around resonance frequency: sending, measured (receiving) and modelled. Determined by means of FFT, the mid frequency of receiving signal 2,  $f_o = 1.675$  MHz is approximately 25 kHz lower than sending signal frequency 1; this difference is caused by attenuation dispersion in the biological medium [3].

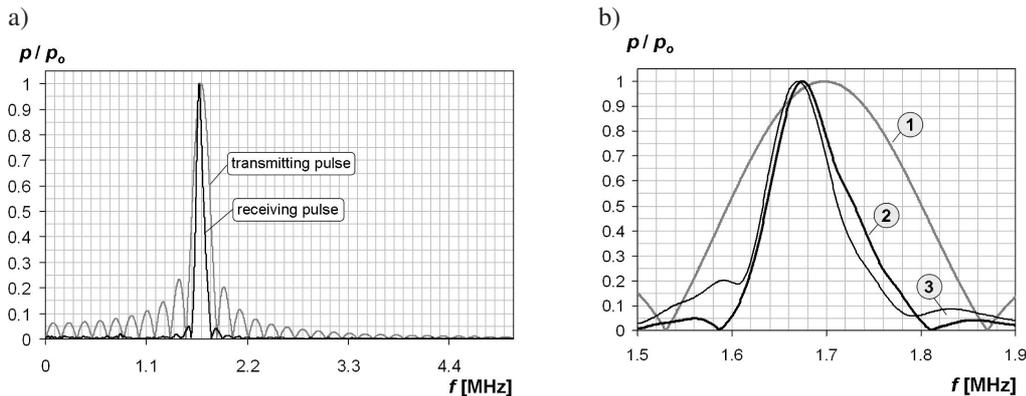


Fig. 2. Spectrum of: a) *burst* type sending pulse and measured receiving pulse from Fig. 1a, b) pulses: 1 – *burst* type sending pulse, 2 – measured receiving pulse from Fig. 1a, 3 – modelled receiving pulse from Fig. 1b.

### 3. Nonlinear propagation

On the basis of the analysis of calculations and measurements presented by KUJAWSKA [2] it can be claimed that in biological medium the relation of amplitudes  $p_o^+/p_o^-$  of an ultrasonic pulse begins to increase linearly, starting at the distance from

source, for which medium nonlinearity related evolution of wave shape begins. Analysis of individual harmonics (index  $h$ ) self-generated in the spectrum of ultrasonic wave propagated nonlinearly in a biological medium shows that their amplitudes decrease linearly in a logarithmic scale, which is related to attenuation coefficient exponential relation to frequency. The performed analysis also proves linear decrease in phase for successive harmonics. The above conclusions made it possible for the author to construct an expression in the form of an infinite sum of sinusoidal functions with suitably regulated amplitude and phase. Each function corresponds to the next harmonic component in the spectrum of a nonlinear pulse ( $h = 1, 2, \dots, \infty$ ):

$$S = \sum_{h=1}^{\infty} A_h \cdot \sin [h \cdot \omega_o \tau - \varphi_o - \varphi_h], \quad (2)$$

where the harmonic component amplitude  $A_h = 10^{-a_c \cdot f_o \cdot (h-1)/20}$ , decrease coefficient of amplitude of harmonic components  $a_c[\text{dB/MHz}] = (A_h [\text{dB}] - A_{h-1}[\text{dB}]) / f_o [\text{MHz}]$  and the harmonic component phase change  $\varphi_h = \varphi_c \cdot (h - 1)$ , where  $\varphi_c$  means a decrease coefficient of phase of harmonic components. In order to normalize the amplitude of function  $S$  to value 1 it is necessary to rescale  $\bar{S} = S/\text{Max}(|S|)$ . The number of harmonic components in signal spectrum related to the degree of its distortion, which results from medium nonlinearity, can be adjusted using coefficient  $a_c$ , which was determined on the basis amplitude difference of two successive harmonics  $A_h$  and  $A_{h-1}$ . The  $p_o^+/p_o^-$  relation can be adjusted using coefficient  $\varphi_c$  value. These coefficients depend on intensity and on attenuation of ultrasonic wave propagated in a medium. Analysis of measurements indicates that some relationship exists among  $a_c$  and  $\varphi_c$  coefficients, what will be researched in the next work. The expression (2) enables simulation of nonlinear continuous ultrasonic wave in biological medium (transmission method). Sequence of ultrasonic wave pulses, which propagate in a biological medium and are distorted as a result of medium nonlinearity, can therefore be modelled using formula (1), where *sinus* function is substituted with  $\bar{S}_n = S_n/\text{Max}(|S_n|)$  function:

$$S_n = \sum_{h=1}^{\infty} A_h \cdot \sin [h \cdot \omega_o \tau_n - \varphi_o - \varphi_h], \quad (3)$$

where  $n$  is a number of pulse in the sequence. Figure 3a shows an example of a nonlinear receiving pulse measured by the author by means of a hydrophone HPM05/2 in a transmission system ( $d = 126.56$  mm) utilizing a ring-shaped ultrasonic probe ( $r = 2.5$  mm), after it was transmitted through an agar gel cylinder ( $r = 28$  mm) submerged in water. The transducer was powered by *burst* type pulses, which were characterised by frequency of  $f_o = 5.000$  MHz, number of cycles  $m_c = 15$ , phase  $\varphi_o = 0$ , amplitude  $U_o = 10$  V (20 Vpp) and repetition frequency  $f_r = 2$  kHz. In order to reconstruct measured nonlinear receiving pulse from Fig. 3a using Eq. (1) with *sinus* function substituted with  $\bar{S}_n$  function, coefficient  $a_c = 4.25$  was chosen on the basis of measurement of amplitudes of harmonics in pulse spectrum, and coefficient  $\varphi_c = 0.82$  was chosen empirically, so that relation  $p_o^+/p_o^- = 1.125$  was obtained. The other parameters

were given as follows:  $f_o = 5.000$  MHz,  $m_c = 18$ ,  $z = 126.56$  mm,  $c_{av} = 1512.84$  m/s,  $\varphi_o = 0$ ,  $f_r = 2$  kHz,  $m = 6$ . Calculation results can be seen in Fig. 3b. The amplitude of the measured receiving pulse 2 was  $p_o = 20.010$  kPa (peak-to-peak), in which  $p_o^+ = 10.593$  kPa,  $p_o^- = -9.416$  kPa, which gives a maximum value of pressure level  $L_{p/pr} = 200.50$  dB in relation to reference pressure, which in water is assumed to be  $p_r = 1$   $\mu$ Pa. The significant difference of amplitudes of acoustic pressure in compression and decompression phase is a prove, that the phenomenon of medium nonlinearity influences the shape of a studied pulse.

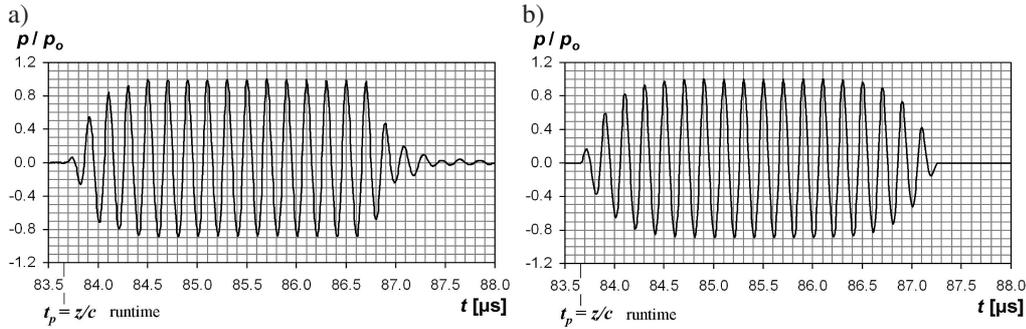


Fig. 3. Nonlinear *burst* type pulse transmitted through a biological medium: a) measured, b) calculated.

Figure 4 shows the comparison of a *burst* type sending pulse spectrum with a measured nonlinear receiving pulse spectrum from Fig. 3a and modelled nonlinear receiving pulse spectrum from Fig. 3b.

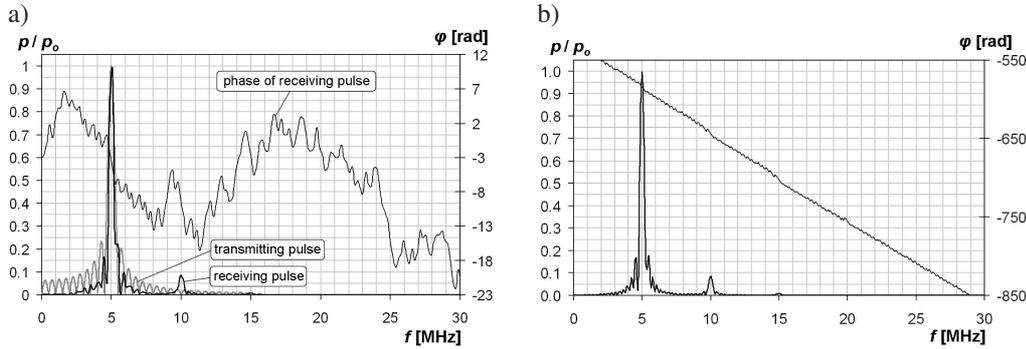


Fig. 4. Spectrum of: a) *burst* type sending pulse and measured receiving pulse from Fig. 3a, b) modelled receiving pulse from Fig. 3b.

Figure 5 shows a comparison of three pulses' spectra from Fig. 4 in a narrow band around resonance frequency and around frequencies of two harmonic components of measured and modelled pulses from Fig. 3.

The differences can be observed in the form of visible harmonic components in case of receiving pulse spectrum (Fig. 4a). Additionally, resonance band in the area of send-

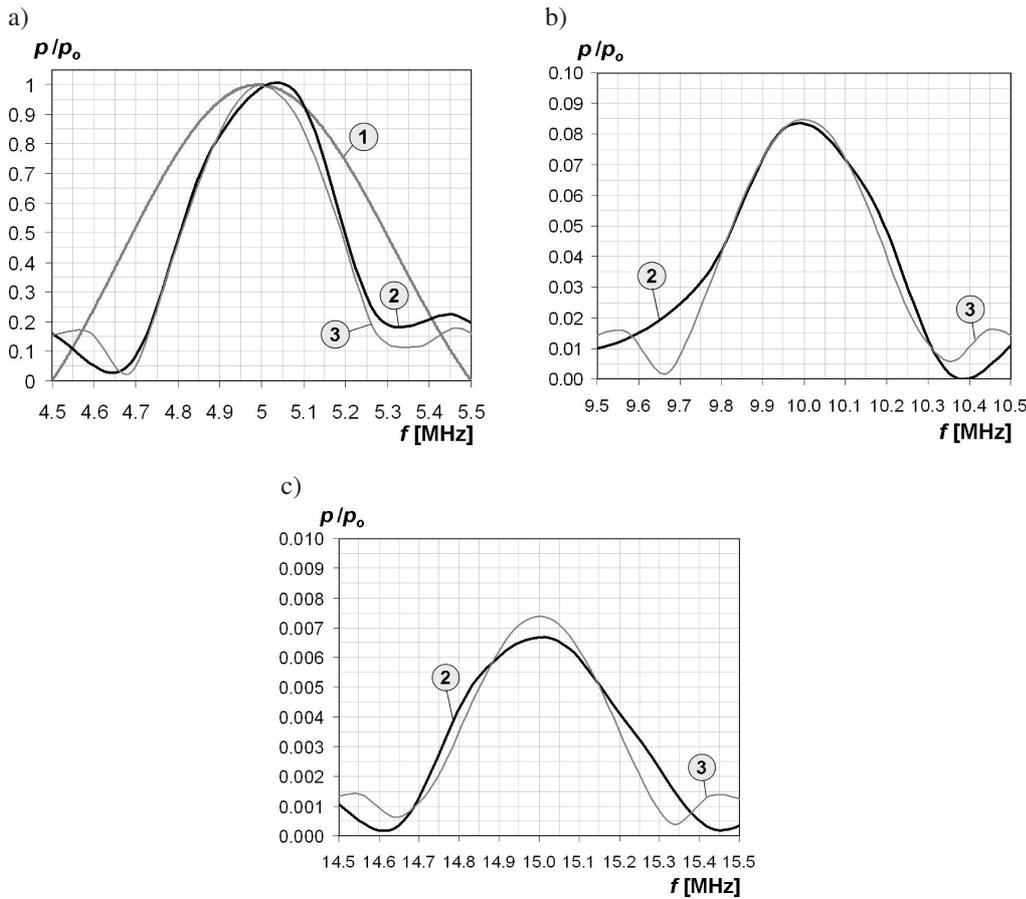


Fig. 5. Spectrum of: a) *burst* type sending pulse (1), measured receiving pulse (2) from Fig. 3a and modelled pulse (3) from Fig. 3b, b) the first harmonic component of measured and modelled pulse, c) the second harmonic component of measured and modelled pulse.

ing transducer operation frequency (5 MHz) is slightly thinner, and signal components of frequencies around the resonance are attenuated, which can be explained by finite time of receiving pulse attack and decay (Fig. 5a). Receiving pulse phase decreases linearly for successive harmonics. Figure 4b shows spectrum of a modelled nonlinear receiving pulse from Fig. 3b (amplitude and phase). Determined by means of FFT, the mid frequency of receiving pulse 2,  $f_o = 5.040$  MHz shows slight shift in the direction of higher frequencies (around 40 kHz) in comparison to sending signal frequency  $f_o = 5.000$  MHz, which, however, can be caused by an error resulting from interpolation (low FFT resolution, i.e. small number of samples and the lack of sample for maximum amplitude – see Fig. 5a). No visible decrease of mid frequency of the receiving pulse, which would result from attenuation coefficient dispersion in medium, can be explained in this case by a low value of attenuation coefficient derivative on frequency.

#### 4. Conclusions

Analysed of the presented waveforms and their spectra it is possible to observe very good correspondence between measured and modelled pulses, which indicates that the developed equations are useful for modelling of ultrasonic wave propagation in biological media. Envelopes determined from formula (1) for  $m > 1$  correspond in terms of shape with pulses generated in a biological medium by round surface ultrasonic transducers (Fig. 3). In case of small rectangular piezoelectric transducers, which are parts of transducer arrays [1], in order to numerically determine the shape of a pulse (Fig. 1) it is necessary to use an adequate envelope composition in formula (1), with exponent  $m = m_1 > 1$  for pulse attack time and  $1 \geq m = m_2 > 0$  for pulse decay time.

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