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PREDICTION OF TURBOFAN ENGINE NOISE CONSIDERING DIFFRACTION AT THE DUCT OUTLET

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The sound field produced by jet engine is extremely complicated due to nonlinear and turbulent effects and thus cannot be predicted exactly. The rise in air-transport causes a strong demand to evaluate the field with better accuracy than that given by baffled outlet and Kirchhoff integral. In the paper the radiation of sound from the inlet is considered with respect to results obtained for semi-infinite rigid cylindrical duct. Introduction of such a model, assuming the duct to be empty inside, is justified by increase of the by-pass ratio and the energy distribution inside the duct. The subsonic mean flow is assumed and the adequate wave equation is considered. The Prandtl–Glauert transformation is adopted.

Keywords: turbofan engine inlet noise, cylindrical unbaffled duct, Prandtl–Glauert transformation.

1. Introduction

Aircraft turbofan engines radiate high level noise from unbaffled cylindrical outlets. The increasing demand on reducing the noise pollution results in strong attempts on lowering the effective perceived noise level (EPNL) by means of active and passive noise control methods. The methods most promising and frequently considered in the literature are as follows: mounting arrays of secondary sources inside the duct [1] or lining the walls with sound absorbing material [2]. Both methods demand prediction of the conditions inside the duct, such as the pressure, the velocity, the temperature, the density *etc*. The sound field produced by jet engine is extremely complicated due to nonlinear and turbulent effects and thus cannot be predicted exactly. Anyhow the rise in air-transport causes a strong demand to evaluate the sound field with better accuracy than that given by the baffled outlet and Kirchhoff integral.

In modern aircrafts the engine is mounted inside a cylindrical casing with a huge fan in front and a hub in the back, as shown in Fig. 1. The sound radiation looks different at the duct inlet (the fan side) [3] and the exhaust, where jets of hot gases flow out from the nozzle [4]. Geometry of the exhaust, with the hub exceeding the cylindrical casing calls for modeling it mathematically as an annular duct. The exhaust radiation will not be analyzed because this topic does not conform to the scope of the paper.

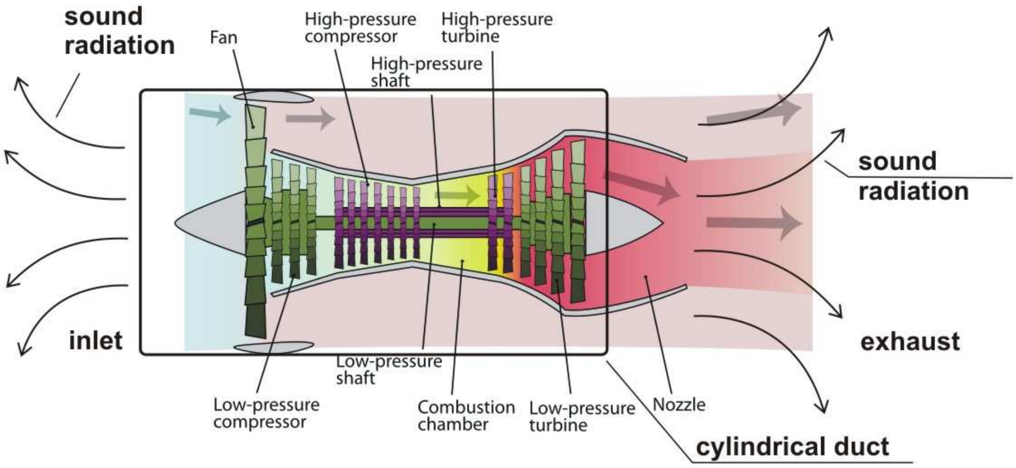

Fig. 1. Scheme of modern high by-pass ratio turbofan engine.

The transmission of harmonic sound inside the duct and its radiation from the inlet is considered with respect to results obtained for semi-infinite rigid cylindrical duct [5–7]. Introduction of such a model, assuming linearity of the sound field and the duct to be empty inside (where in fact it contains the engine, as shown in Fig. 1) is justified by increase of the by-pass ratio (the mass flow rate of air drawn in by the fan but bypassing the engine core to the mass flow rate passing through the engine core) and the way in which the energy is distributed inside the duct [1] – the energy density of the higher modes is minor in the vicinity of the duct axis. The by-pass ratio in GE-90 turbofan engine is about 9, the duct length is 7.3 m and the diameter – 3.1 m. The extension of the by-pass ratio and the duct diameter in modern turbofan engines resulted in lowering the jets velocity and, in turn, the emitted sound power, depending, according to the Lighthill law, on the eight power of the velocity [8]. The subsonic mean flow (Mach number $M < 1$) is assumed and the adequate wave equation is considered. For such a flow the Prandtl–Glauert transformation allows to adopt the results obtained for the reflection and transformation coefficients, the intensity directivity function and the power gain function obtained for the duct without flow [5, 7, 9]. No coupling is assumed between duct inlet and exhaust.

2. Governing equations in a rigid duct with flow

As was mentioned in the introduction, the phenomena in the turbofan engine are, in general, nonlinear and turbulent. This especially concerns the region behind the fan, where the rotor-stator interaction takes place, and which is called the fluid-dynamic computational region [10], while the area close to the inlet is considered as computational areoacoustics propagation region and can be described by velocity potential wave equation with mean flow [5]

$$\left(\triangle - \frac{1}{c^2}\frac{D^2}{Dt^2}\right)\Phi(\mathbf{r},t) = 0, \tag{1}$$

where $\dfrac{D}{Dt} = \dfrac{\partial}{\partial t} + \mathbf{u}\cdot\nabla$, is the convected derivative operator and **u** is the flow velocity [5].

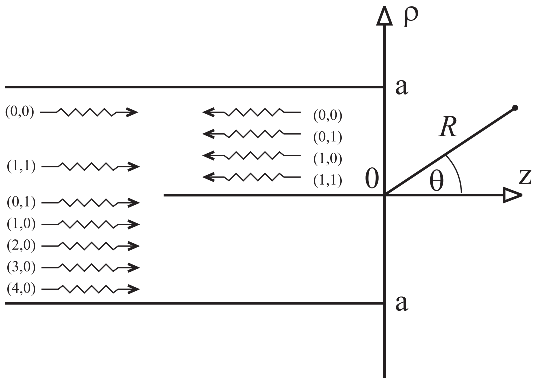

Fig. 2. Geometry of the semi-infinite duct and schematic presentation of some incident and reflected/transformed propagating modes.

The duct geometry, with the inlet at $z = 0$ is presented in Fig. 2. For harmonic sound of frequency ω, described by $e^{-i\omega t}$ time-factor, assuming the flow in the z direction and introducing the Mach number $M = u/c$, one obtains in the cylindrical co-ordinates (ϱ, φ, z) [1]

$$\frac{1}{\varrho}\frac{\partial}{\partial\varrho}\left(\varrho\frac{\partial\Phi}{\partial\varrho}\right) + \frac{1}{\varrho^2}\frac{\partial^2\Phi}{\partial\varphi^2} + \frac{\partial^2\Phi}{\partial z^2} - \left(-ik + M\frac{\partial}{\partial z}\right)^2\Phi = 0. \tag{2}$$

The axial wave numbers calculated when solving Eq. (2)

$$\gamma_{mn}^{fl} = \left[\mp kM + \left(k^2 - (1-M^2)\mu_{mn}^2/a^2\right)^{1/2}\right]\frac{1}{1-M^2}, \tag{3}$$

with μ_{mn}/a representing the radial wave number fulfilling the hard-wall duct boundary condition [5], describe waves propagating in the direction of flow (minus sign) or opposite (plus sign). For radial wave number μ_{mn}/a and the diffraction parameter (reduced frequency) $ka > \mu_{mn}\sqrt{1-M^2}$ the axial wave number γ_{mn}^{fl} is also real and represents propagating mode (m,n). Otherwise its imaginary part depicts mode attenuation coefficient. The cut-off frequencies (critical) $\omega_{mn}^{cr} = c\mu_{mn}\sqrt{1-M^2}/a$, are lower than in the duct without flow by the factor $\sqrt{1-M^2}$ and so, for a given ka, the number of propagating modes increases. For the plane wave $\mu_{00}=0$ what leads to $\gamma_{00}^{fl}=k/(1+M)$ for waves propagating in the positive direction and $\gamma_{00}^{fl}=k/(1-M)$ for waves propagating in the negative direction.

The m,n subscripts represent the m-th circumferential and n-th radial mode of the velocity potential. In the infinite duct [6]

$$\Phi_{mn}^{fl}(\varrho,\varphi,z,t)=A_{mn}e^{im\varphi}J_m\left(\mu_{mn}\varrho/a\right)e^{i(\gamma_{mn}^{fl}z-\omega t)}, \tag{4}$$

where A_{mn} means amplitude, $J_m()$ – Bessel function of the first kind with the derivative $J_m'(\mu_{mn})=0$. For aircraft engine noise the ka parameter is big enough to allow propagation of many modes [1] and so the velocity potential is a summ of potentials expressed by (4)

$$\Phi(\varrho,\varphi,z,t)=\sum\Phi_{mn}^{fl}(\varrho,\varphi,z,t), \tag{5}$$

summation covering all pairs of indices (m,n) representing propagating modes.

Apart from different axial and radial wave numbers the potential formula for infinite duct with subsonic mean flow (4) is the same as for a hard duct without flow [6]. Thus, one may expect this relation to extend on results obtained for the semi-infinite unbaffled duct, when diffraction at the opening occurs. If so, than the results obtained for the semi-infinite duct without flow could have been applied to the duct with flow.

3. Application of the Prandtl–Glauert transformation

The Prandtl–Glauert transformation serves as a tool for reducing the problem of sound transmission and radiation from a duct in which the medium flows with uniform velocity profile to the no-flow problem. The wave number components and the space variable have to be changed as follows [3]

$$\widetilde{k}=\frac{k}{\sqrt{1-M^2}},\qquad \widetilde{\gamma}_{mn}=\sqrt{\widetilde{k}^2-\frac{\mu_{mn}^2}{a^2}}, \tag{6}$$

$$\widetilde{z}=\frac{z}{\sqrt{1-M^2}},\qquad \widetilde{x}=x,\qquad \widetilde{y}=y. \tag{7}$$

It is seen that the Prandtl–Glauert transformation brings about expressions for the duct with flow. The next step is to transform formulae for the velocity potential (inside and outside the duct) and other physical quantities, such as the acoustic pressure, the noise level and the power-gain function. Thus, according to [6] the velocity potential of amplitude A_{mn} inside the duct with flow is given by

$$\begin{aligned}\Phi_{mn}^{fl}(\varrho,\varphi,z)=&A_{mn}e^{i(m\varphi-M\widetilde{k}\widetilde{z})}\\&\cdot\left[J_m\left(\mu_{mn}\frac{\varrho}{a}\right)e^{i\widetilde{\gamma}_{mn}\widetilde{z}}+\sum_l\widetilde{R}_{mnl}J_m\left(\mu_{ml}\frac{\varrho}{a}\right)e^{-i\widetilde{\gamma}_{ml}\widetilde{z}}\right],\end{aligned} \tag{8}$$

where $\widetilde{R}_{mnl}$ means the transformation ($l \neq n$) or reflection ($n = l$) coefficient [6]. As their evaluation is computationally complicated some authors apply the baffled duct solution to predict the field inside [1]. Contrary to the field inside the duct, the field outside – the intensity or the sound power level directivity characteristics, have to be calculated in the frame of unbaffled-duct model [1, 7, 9]

$$\Phi_{mn}^{fl}(R,\theta,\varphi) = A_{mn}\widetilde{d}_{mn}(\theta,\varphi)\frac{e^{i(\widetilde{k}\widetilde{r}-M\widetilde{k}\widetilde{z})}}{\widetilde{r}}, \tag{9}$$

where $\widetilde{d}_{mn}$ is the (m,n) mode directivity function and according to (7), $\widetilde{r} = \sqrt{x^2+y^2+\widetilde{z}^2}$. In the last two equations the harmonic time factor $\exp(-i\omega t)$ was omitted for brevity. Explicite formulae for R_{mnl} and d_{mn} can be found in [6, 7, 11].

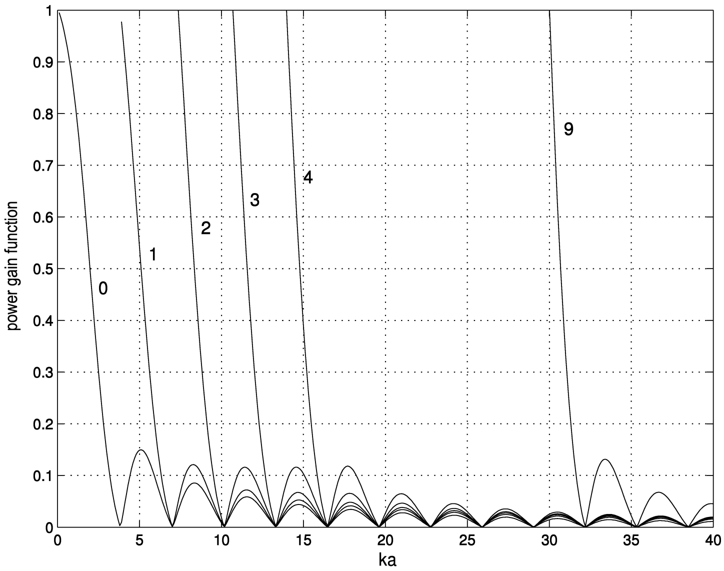

Fig. 3. Power gain function for plane wave and radial modes for $\theta = 90°$.

The baffled-duct directivity characteristics, obtained in the Kirchhoff approximation are much easier to compute, but unsatisfactory in the considered application, when radiation in directions "to ground" is important. First, they become inaccurate at angles greater than 70° and are undefined at angles beyond 90°. As an example, Fig. 3 presents the power-gain function, calculated along the definition given in [11] by Levine and Schwinger, for the plane wave and consecutive axis-symmetric modes for $\theta = 90°$, when the Kirchhoff approximation gives zero value.

4. Conclusions

The paper presents a procedure of extending the canonical results obtained for the sound field of semi-infinite cylindrical duct without flow to the duct with uniform velocity profile, what in turns allows to model the inlet turbofan engine radiation, as the engine casing is very much like a cylindrical duct. The results were obtained under the assumption that the field is harmonic and single-frequency, the duct is empty inside and the flow is uniform, what resulted in applying the Prandtl–Glauert transformation to homogenous wave equation for sound velocity potential. The theoretical model presented above serves also as validation of achievements in the numerical modeling (Boundary Element Method, Finite Element Method, Computational Aero-Acoustics methods, Green's Function Discretization technique *etc.*) of noise radiated from aero-engines [1, 3, 12, 13]. The results are valuable as noise produced by modern aircrafts is recognized as one of the most important and difficult environmental pollution problem.

References

[1] JOSEPH P., NELSON P.A., FISHER M.A., *Active control of fan tones radiated from turbofan engines. I. External error sensors*, J. Acoust. Soc. Am., **106**, 2, 766–778 (1999).

[2] CAMPOS L.M.B.C., OLIVEIRA J.M.G.S., *On the acoustic modes in a cylindrical duct with an arbitrary wall impedance distribution*, J. Acoust. Soc. Am., **116**, 6, 3336–3347 (2004).

[3] LIDOINE S., BATARD H., TROYES S., DELNEVO A., ROGER M., *Acoustic radiation modelling of aeroengine intake comparison between analytical and numerical methods*, AIAA-2001-2140 AIAA/CEAS Aeroacoustics Conference and Exhibit, 7th, Maastricht, Collection of Technical Papers **1** (2001).

[4] DRUON Y., LIDOINE S., ROGER M., *Acoustic radiation modeling of engine exhaust. Comparison between analytical and numerical methods*, 10th AIAA/CEAS Aeroacoustics Conference 10–12 may 2004, Manchester, U.K., AIAA Paper, 2004–2990 (2004).

[5] SNAKOWSKA A., *The energy distribution in the far field of an arbitrary Bessel mode radiating from the semi-infinite unflanged cylindrical wave-guide*, Archives of Acoustics, **17**, 4, 525–542 (1992).

[6] SNAKOWSKA A., WYRZYKOWSKI R., *Calculations of the acoustical field in a semi-infinite cylindrical wave-guide by means of the Green function expressed in cylindrical coordinates*, Archives of Acoustics, **11**, 3, 261–285 (1986).

[7] SNAKOWSKA A., *The acoustic far field of an arbitrary Bessel mode radiating from the semi-infinite unflanged cylindrical wave-guide*, Acustica, **77**, 2, 53–62 (1992).

[8] LIGHTHILL M.J., *Waves in fluids*, Cambridge University Press, Cambridge 2007.

[9] SNAKOWSKA A., IDCZAK H., BOGUSZ B., *Modal anlysis of the acoustic field radiated from an unflanged cylindrical duct – theory and measurement*, Acustica/Acta Acustica, **82**, 2, 201–206 (1996).

[10] OVENDEN N.C., RIENSTRA S.W., *Mode matching strategies in slowly varying engine ducts*, AIAA Journal, **42**, 9, 1832–1840 (2004).

[11] LEVINE H., SCHWINGER J., *On the radiation of sound from an unflanged circular pipe*, Phys. Rev., **73**, 19483, 83–406.

[12] CASALINO D., GENITO M., *Achievements in the numerical modeling of fan noise radiation from aero-engines*, Aerospace Science and Technology, **12**, 105–113 (2008).

[13] ZHANG X., CHEN X.X., MORFEY C.L., NELSON P.A., *Computation of Spinning Modal Radiation from an Unflanged Duct*, AIAA Journal, **42**, 9, 1795–1801 (2004).