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PROPAGATION OF SOUND WAVES IN UNCERTAIN ENVIRONMENT – NEW INTERVAL PERTURBATION METHODOLOGY

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The aim of the paper is to present applications of the new interval algebraic system in acoustic problems. The modified algebraic system operates over specifically strictly determine interval numbers with specially defined addition and multiplication. The introduced perturbation interval numbers are defined as ordered pairs of real numbers. Classical perturbation acoustic problems described by differential equations can be solved in the new interval algebraic system as easy as usual.

A novel 3D interval ray–tracing model of detailed representation of the indoor environment is presented. Interval perturbation ray tracing method is a technique based on geometrical optics with disturbance in parameters. The developed algorithms use the interval perturbation methodology where the perturbed images are applied to produce 3D – field of interval illumination zones. It can be easy considered how interval perturbations (small) of nominal parameters values change solutions of the considered problems.

Keywords: ray tracing, interval geometry, perturbation methods, room acoustic.

1. Introduction

In many acoustical analyses coefficients of a considered problem are not known exactly, but with only some approximation, say in interval form. It is the simplest way to introduce uncertainty into mathematical analysis.

From now it's assumed that values of boundary conditions, material properties, internal prescribed fields and the shape of a boundary are uncertain and will be modeled using the new methodology based on interval analysis [7–9]. A novel 2D and 3D interval ray-tracing model of detailed representation of the indoor environment is presented. Interval perturbation ray tracing method is a technique based on geometrical optics with interval perturbations in parameters, which can be considered as an easily applied approximation for estimating uncertain problems in acoustics [8–9].

2. Algebraic system of two-scale perturbation interval numbers

Interval numbers can by written as $\overline{z} = [z^-, z^+]$ and z^-, z^+, $z^- \leq z^+$ are called ends of the interval. If we note the center of interval as $\breve{z} = 0.5\,(z^- + z^+)$ and the radius: $\text{rad}\,(\overline{z}) = 0.5\,(z^+ - z^-)$, we can write the interval number as $\overline{z} = \left[\breve{z} - \text{rad}\,(\overline{z})\,, \breve{z} + \text{rad}\,(\overline{z})\right]$. Therefore, the interval number can be an ordered pair of real numbers $\left(\breve{z}, \Delta z\right)_r$, in simplicity $\Delta z = \text{rad}\,(\overline{z})$ and $\overline{\Delta} := [-\Delta z, \Delta z]$.

Further 2-scale perturbation numbers, i.e. ordered 3-couples of reals $(x_0, x_1, x_2) \in \mathbb{R}^3$ are in use. The first element x_0 of the 3-couple is called a main value and the following are the perturbation values. Denote $\varepsilon_1 = (0, 1, 0)$ and $\varepsilon_2 = (0, 0, 1)$, where symbols have the properties: $\varepsilon_1^2 = (0, 0, 0)$, $\varepsilon_2^2 = (0, 0, 0)$, $\varepsilon_1\varepsilon_2 = (0, 0, 0)$. Then for every $\zeta = (x_0, x_1, x_2) \in \mathbb{R}_{2\varepsilon}$ we can write $(x_0, x_1, x_2) = x_0 + \varepsilon_1 x_1 + \varepsilon_2 x_2$.

Assume now, that the radius of the interval $\overline{z}$ is a two-scale perturbation number and define two symbolic perturbation intervals $\overline{\varepsilon}_1 = [-\varepsilon_1, \varepsilon_1]$ and $\overline{\varepsilon}_2 = [-\varepsilon_2, \varepsilon_2]$. Then we write a 2-scale perturbation interval number as: $\overline{z} = \breve{z} + \delta z_1 \overline{\varepsilon}_1 + \delta z_2 \overline{\varepsilon}_2 \in \mathit{IR}_{2\varepsilon}$.

We have used the interval perturbation method based on the new algebraic system with specifically strictly defined interval perturbation numbers, cf. Skrzypczyk, Perturbation Methods for Acoustic Systems with Interval Parameters, this journal.

Two-scale perturbation interval value functions are defined for two-scale perturbation interval arguments as extensions of classical elementary functions.

3. The reflection of the perturbation ray from intervally two-scale perturbed surface

Assume that the ray equation (straight line) has the following parametric form,

$$\overline{\mathbf{r}} = \overline{\mathbf{r}}_1 + (\overline{\mathbf{r}}_2 - \overline{\mathbf{r}}_1)\,\overline{t}, \qquad \text{where} \quad \overline{\mathbf{r}} = (\overline{x}, \overline{y}, \overline{z})\,, \quad \overline{\mathbf{r}}_i = (\overline{x}_i, \overline{y}_i, \overline{z}_i)\,, \quad i = 1, 2, \quad \overline{t} \in \mathit{IR}_{2\varepsilon}. \tag{1}$$

Since all vectors take the perturbed form, we write further $\overline{\mathbf{r}} = \breve{\mathbf{r}} + \delta\mathbf{r}_1\overline{\varepsilon}_1 + \delta\mathbf{r}_2\overline{\varepsilon}_2$, $\overline{\mathbf{r}}_i = \breve{\mathbf{r}}_i + \delta\mathbf{r}_{i1}\overline{\varepsilon}_1 + \delta\mathbf{r}_{i2}\overline{\varepsilon}_2$, $i = 1, 2$, $\overline{t} = \breve{t} + \overline{\varepsilon}_1\delta t_1 + \overline{\varepsilon}_2\delta t_2$. The plane equation with interval parameters takes the form $\overline{\alpha}x + \overline{\beta}y + \overline{\gamma}z + \overline{\nu} = \overline{0}_{2\varepsilon}$, $\overline{\alpha}, \overline{\beta}, \overline{\gamma}, \overline{\nu} \in \mathit{IR}_{2\varepsilon}$.

If the uncertainty in the plane has the interval form $\overline{\varepsilon}_1 f_1\,(x, y) + \overline{\varepsilon}_2 f_2\,(x, y)$, we obtain,

$$\overline{\alpha}\,\overline{x} + \overline{\beta}\overline{y} + \overline{\gamma}\,\overline{z} + \overline{\nu} - \overline{\gamma}\left(\overline{\varepsilon}_1\overline{f}_1\,(\overline{x}, \overline{y}) + \overline{\varepsilon}_2\overline{f}_2\,(\overline{x}, \overline{y})\right) = \overline{0}_{2\varepsilon}. \tag{2}$$

Now we can determine the point $\overline{\mathbf{r}}_0 = (\overline{x}_0, \overline{y}_0, \overline{z}_0)$ of intersection between the ray (1) and the perturbed plane (2), assume $\overline{\mathbf{r}}_0 = \breve{\mathbf{r}}_0 + \delta\mathbf{r}_{01}\overline{\varepsilon}_1 + \delta\mathbf{r}_{02}\overline{\varepsilon}_2$. The same notation is taken for $\overline{x}_0$, $\overline{y}_0$, $\overline{z}_0$. Recall the extension form of $f_i(x_0, y_0)$ for perturbed arguments,

$$\overline{f}_i(\overline{x}_0, \overline{y}_0) = f_i(\breve{x}_0, \breve{y}_0) + \frac{\partial f_i(\breve{x}_0, \breve{y}_0)}{\partial \overline{x}_0}(\delta x_{01}\overline{\varepsilon}_1 + \delta x_{02}\overline{\varepsilon}_2) + \frac{\partial f_i(\breve{x}_0, \breve{y}_0)}{\partial \overline{y}_0}(\delta y_{01}\overline{\varepsilon}_1 + \delta y_{02}\overline{\varepsilon}_2), \quad (3)$$
$$\overline{x}_0 = \breve{x}_0 + \delta x_{01}\overline{\varepsilon}_1 + \delta x_{02}\overline{\varepsilon}_2, \quad \overline{y}_0 = \breve{y}_0 + \delta y_{01}\overline{\varepsilon}_1 + \delta y_{02}\overline{\varepsilon}_2.$$

To calculate $\overline{\mathbf{r}}_0 = (\overline{x}_0, \overline{y}_0, \overline{z}_0)$ one can solve the system of linear interval equation (2) for $\breve{x}_0, \breve{y}_0$. This system can be solved directly using interval algebra or as the hierarchical system of usual real-valued equations. Having the intersection point $(\overline{x}_0, \overline{y}_0, \overline{z}_0)$ in hand, we can form the plane tangent to the perturbed plane (2) as

$$\overline{z} - \overline{z}_0 = \frac{\partial \overline{z}}{\partial x}(\overline{x} - \overline{x}_0) + \frac{\partial \overline{z}}{\partial y}(\overline{y} - \overline{y}_0) = \left(-\frac{\overline{\alpha}}{\overline{\gamma}} + \overline{\varepsilon}_1 \frac{\partial \overline{f}_1(\overline{x}_0, \overline{y}_0)}{\partial x} + \overline{\varepsilon}_2 \frac{\partial \overline{f}_2(\overline{x}_0, \overline{y}_0)}{\partial x}\right)(\overline{x} - \overline{x}_0) + \left(-\frac{\overline{\beta}}{\overline{\gamma}} + \overline{\varepsilon}_1 \frac{\partial \overline{f}_1(\overline{x}_0, \overline{y}_0)}{\partial y} + \overline{\varepsilon}_2 \frac{\partial \overline{f}_2(\overline{x}_0, \overline{y}_0)}{\partial y}\right)(\overline{y} - \overline{y}_0). \quad (4)$$

4. Ray acoustic emission algorithm with interval perturbations

In the following, the analysis of new concepts of predicting the interval 2-scale perturbations of Acoustic Emission (2ε-IAE) signals, travelling within a room, is considered. Such disturbances can occur due to external stimulation or internal events. The energy reduction of these signals can be affected by perturbed material properties and by the perturbed geometry of the object. We assume wave propagation complex properties with respect to intricate shapes, with perturbed variations, and discontinuities in thickness and surface curvature. There is a strong analogy between the physical propagation of sound and light. The technique employed by the interval 2ε-RayAE algorithm exploits this analogy through classical steps reminiscent of rendering:

1. Generate a source of space perturbed interval vector field (2ε-IPVF) to present internal 2ε-IAE;
2. For each 2ε-IAEP, generate a series of weak edge segments that represent perturbed reflections/transmission of the 2ε-IAE ray;
3. For each sensor location test all 2ε-IAERs and record the number and values of time intersections between the single 2ε-IAER and the sensor neighbourhood.

A ray emitted from an omni-directional source with initial sound power $\overline{E}_{i0}$, $i = 1, 2, ..., n$, travels in its original direction. After it hits the boundary of the enclosure for the first time, the sound power carried by the ray will be reduced to $\overline{E}_{i1}$ due to the surface absorption. The time and distance travelled from the source to the boundary are $\overline{t}_k$ and $\overline{d}_k$, respectively. After n reflections, the rays sound power will be reduced to $\overline{E}_{in}$, the time and distance travelled from the last reflection point being $\overline{t}_n$

and $\overline{d}_n$, respectively. Thus, when a ray arrives at the receiver position after n reflections, its sound power is $\overline{E}_{in}$, and the time and distance travelled are $\overline{t}$ and $\overline{d}$, where $\overline{t} = \overline{t}_1 + \overline{t}_2 + \overline{t}_3 + ... + \overline{t}_n$, $\overline{d} = \overline{d}_1 + \overline{d}_2 + \overline{d}_3 + ... + \overline{d}_n$.

Due to 2-scale perturbation, we write $\overline{t}_k = \breve{t}_k + \delta t_{k1}\overline{\varepsilon}_1 + \delta t_{k2}\overline{\varepsilon}_2$, $\overline{d}_k = \breve{d}_k + \delta d_{k1}\overline{\varepsilon}_1 + \delta d_{k2}\overline{\varepsilon}_2$, $k = 1, 2, ..., n$,

$$\overline{E}_{in} = \overline{E}_{i0}\overline{e}^{-\overline{\xi}\,\overline{d}} \prod_{k=1}^{n} \left(\overline{1}_{2\varepsilon} - \overline{\alpha}_k\right), \tag{5}$$

where $\overline{E}_i$ is sound power carried by the i-th ray, $\overline{E}_{i0}$ is the original sound power of the ray, $\overline{\xi}$ is the air absorption attenuation, $\overline{\alpha}_i$ is the surface absorption coefficient of the i-th plane. Assume that $\overline{\xi} = \breve{\xi} + \delta\xi_1\overline{\varepsilon}_1 + \delta\xi_2\overline{\varepsilon}_2$, $\overline{\alpha}_k = \breve{\alpha}_k + \delta\alpha_{k1}\overline{\varepsilon}_1 + \delta\alpha_{k2}\overline{\varepsilon}_2$. If we use interval 2-scale perturbation algebra, then

$$\begin{aligned}\prod_{k=1}^{n} \left(\overline{1}_{2\varepsilon} - \overline{\alpha}_k\right) = \prod_{k=1}^{n} \left(1 - \breve{\alpha}_k\right) - \overline{\varepsilon}_1 \sum_{k=1}^{n} \delta\alpha_{k1} \prod_{\substack{i=1\\ i\neq k}}^{n} \left(1 - \breve{\alpha}_k\right) \\ - \overline{\varepsilon}_2 \sum_{k=1}^{n} \delta\alpha_{k2} \prod_{\substack{i=1\\ i\neq k}}^{n} \left(1 - \breve{\alpha}_k\right),\end{aligned} \tag{6}$$

$$\begin{aligned}\overline{e}^{-\overline{\xi}\,\overline{d}} = \left(1 + \overline{\varepsilon}_1\left(\delta\xi_1 \sum_{k=1}^{n} \breve{d}_k + \breve{\xi} \sum_{k=1}^{n} \delta d_{k1}\right) + \overline{\varepsilon}_2\left(\delta\xi_2 \sum_{k=1}^{n} \breve{d}_k + \breve{\xi} \sum_{k=1}^{n} \delta d_{k2}\right)\right) \\ \exp\left(-\breve{\xi} \sum_{k=1}^{n} \breve{d}_k\right).\end{aligned} \tag{7}$$

Finaly, from Eqs. (5–7) we can calculate interval 2-scale perturbation sound power $\overline{E}_{in}$ even if $\overline{E}_{i0}$ is the perturbation value as well. If the initial sound power E_0 is uniformly distributed between N 2ε-interval rays, then we have $\overline{E}_{i0} = \overline{E}_0 Q_i/N$, where Q_i is the directivity factor, N is the total number of the initial rays [4, 8].

5. Numerical example

Consider a reverberation chamber of irregular shape with an isotropic spherical sector-directional sound source placed inside. We analyze three cases of sound absorbing screen settings: 1) empty chamber; 2) screen in the middle of chamber; 3) screen on the wall (see Fig. 1). All dimensions, locations and absorbent coefficients of walls and screens can be perturbed. The values of interval 2-scale perturbed sound pressure level, in any point of the room, can be calculated as it was described above (see Fig. 2). The mean values of randomly perturbed absorbent coefficients of all walls and screens are determined. All numerical values are dimensionless.

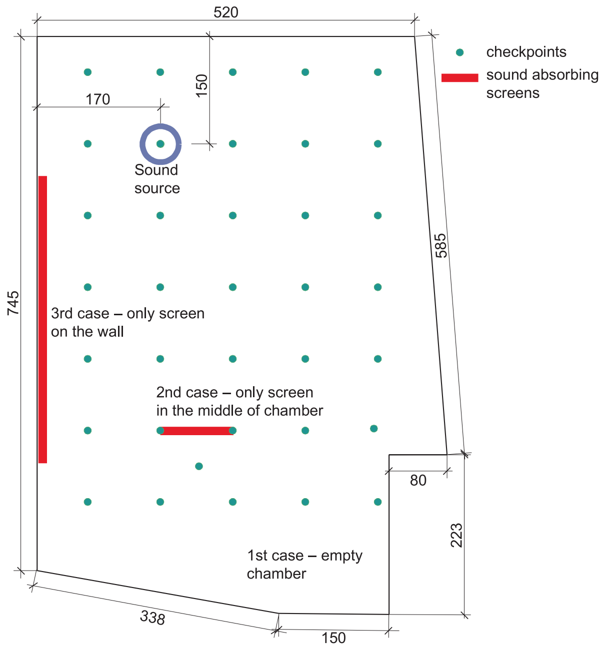

Fig. 1. View of the room.

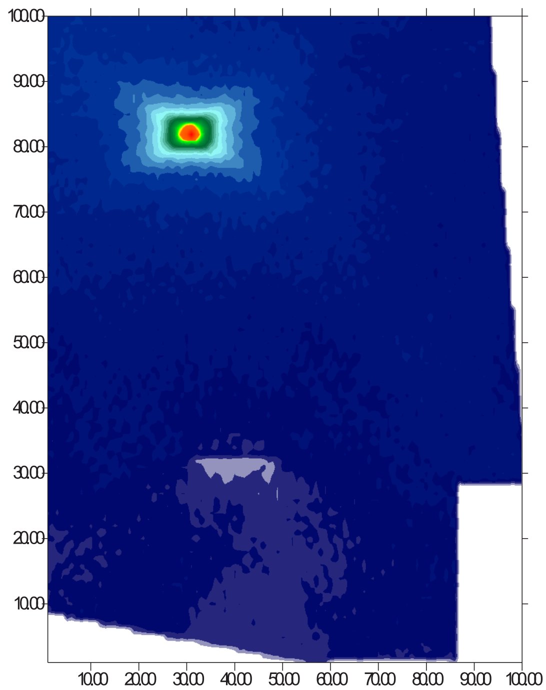

Fig. 2. The map of interval sound file – 2nd case.

6. Conclusions

This paper investigates the feasibility of predicting the interval perturbations of sound signals travelling within a indoor environment. The attenuation of these signals is affected by perturbed material properties and by the perturbed geomtry of the object. For example, wave propagation could be complex because of intricate shape with perturbed variations and discontinuities in thickness and surface curvature. In contrast to much of the reported results, this paper provides a perturbation ray firing procedure to model the interval transmission of rays both across the surface and through the interior of a complex rooms. With the new interval algebraic system we get very simple and useful mathematical tool which can be easy used in analysis of acoustic problems. This methodology can be applied problems with uncertain parameters.

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