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ONLINE IDENTIFICATION OF MATHEMATICAL MODEL OF CIRCULAR PLATE FOR VIBRATION CANCELLATION

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Identification is the first step in designing of the controller for suppressing vibrations of considered circular plate. It constructs a parametric model of the system dynamics from measured input and output data. This data-driven technique can describe systems that are not easily modelled from the first principles or specifications. Identification methods can handle a wide range of the system dynamics without knowledge of the actual system physics. In this paper the ARX model structure and algorithm called *online identification* have been considered. In this procedure, model parameters are calculated and updated during the process using the data measured a few steps earlier. Before output data can be resorted for identification it should be prefiltered by using the digital Butterworth filter. This step removes mean values and high-frequency noise.

The second order transfer function, obtained by using described identification algorithm, was implemented in procedure of designing the optimal PID controller for suppressing vibrations of the plate. Simulations of working this regulator for *sin* and *chirp* signals shows that the controller causes substantial reduction of the vibrations.

Keywords: online identification (RLSM), ARX model, plate vibration, PID controller, Butterworth filter.

1. Introduction

As an object of research the author considered a circular plate whose small amplitude vibrations should be suppressed. This problem can be resolved using a PC computer as a regulator of the system with implemented designed control algorithm. Such algorithm can be projected by using equations which describes actual system physics (*modelling*) or by constructing parametric model of system dynamics from measured input and output data (*identification*). In many cases the first approach is hard to obtain or the result of investigation does not conform with real system behavior because of used simplifications. The other, data-driven technique, can handle system dynamics without the knowledge of system physics. These two methods were widely discussed by L. LENIOWSKA [1, 2].

System model can be obtained using the results of preliminary test. In this procedure unknown parameters of a model with selected structure are appointed only once for all measured data. The accuracy of such technique is limited but computational complexity is also low. That is why it could be executed on low performance computers. The development of IT technologies software gives opportunity to use more advanced methods like adaptive controllers which update parameters during the process. In such regulator, system identification is an integral part of algorithm and calculates the model using data measured a few steps before the actual step of procedure loop. It means that the model depends on the actual state of a real system. The accuracy of this technique is higher than previously described one. It also increases effectiveness of an adaptive controller. This technique, called *online identification*, is considered by the author in this paper.

2. Plant description

An active vibration control system for suppressing vibration of circular plate can be built as it is shown in Fig 1. The process of cancelling vibration is regulated by PC Computer with the Real Time Application Interface (RTAI) installed on it. This extension of Linux operating system changes priority of running process – real time applications are built as kernel modules and if they run, they always have higher priority than ordinary Linux programs. That is why it guarantees deterministic response time of *rtai process* and significantly increases the rate of executing them. The described way of running software used by the author gives opportunity to use more advanced control algorithm which involves online identification as a main step. Data used in calculating of mathematical system model are measured by the application of strain sensors. Intelligent materials such as 2-layer piezo disk elements are used as the actuators. Primary excitation is provided by a low frequency loudspeaker installed centrally at the bottom of the cylinder.

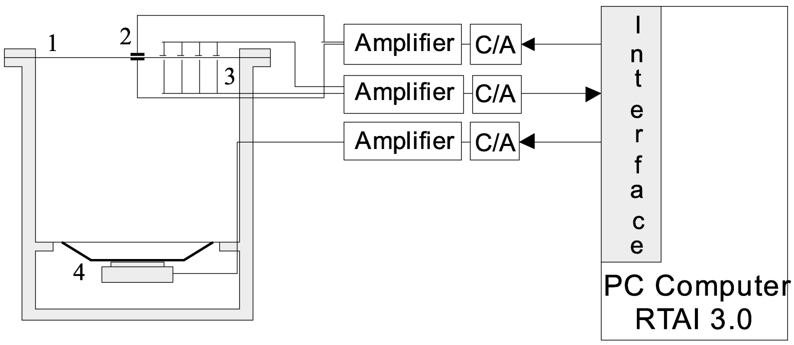

Fig. 1. Research position: 1 – circular plate, 2 – PZT elements, 3 – strain sensors, 4 – loudspeaker.

3. Preparing data for system identification

The process of identification calculates parametric model of the system based on input-output data. The analog signal measured by sensors must be amplified and transformed to digital form, which is used by computer – controller. These steps involve adding some mistakes which should be removed before identification. It can be done by using digital filter. It removes mean values and high-frequency noise. In addition to minimizing the noise, prefiltering lets focus model on specific frequency bands.

The author has chosen second order Butterworth filter because it is characterized by a magnitude response that is maximally flat in the passband and monotonic overall. It is assumed that for the considered system, the output signal should be filtered in frequency bands from 10 Hz to 500 Hz. Desired transfer function of filter can be described by Eq. (1).

$$H(z) = \frac{B(z)}{A(z)} = \frac{0.0581 - 0.0581z^{-2}}{1 - 1.8836z^{-1} + 0.8839z^{-2}}. \tag{1}$$

Example result of online prefiltering measured data are presented in Fig. 2.

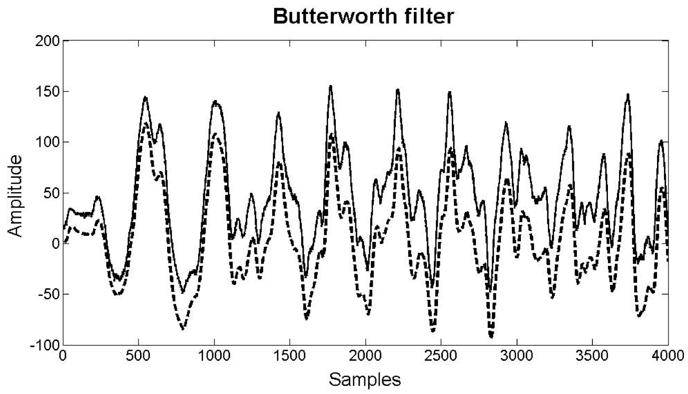

Fig. 2. Designed Butterworth filterfiltering output data: '—' real system output, '- - -' filtered data.

4. Online identification

Parametric models describe dynamical systems in terms of differential equations and transfer functions. Generally, the process can be described using Eq. (2) [3]:

$$A(z)y(k) = B(z)u(k-d) - C(z)e(k), \tag*{(2)$_1$}$$

where $A(z)$, $B(z)$ and $C(z)$ are polynomials in the delay operator:

$$A(z) = 1 + a_1z^{-1} + \cdots + a_{na}z^{-na}, \tag*{(2)$_2$}$$

$$B(z) = b_1 + b_2z^{-1} + \cdots + b_{nb}z^{-nb+1}, \tag*{(2)$_3$}$$

$$C(z) = 1 + c_1z^{-1} + \cdots + c_{nc}z^{-nc}, \tag*{(2)$_4$}$$

and $y(k)$ represents the output at time k, $u(k)$ represents the input at time k, na – number of poles, nb – number of zeros plus 1, nc – number of C coefficients, d is the number of samples before the input affects the system output and $e(k)$ is the white-noise disturbance. Equations (2) describe model structure which is known as ARMAX structure. In many cases one can use simpler method such as used by the author ARX model.

4.1. Auto-recursive extensive (ARX) model

The ARX model structure is a process model used in the generation of models where outputs are dependent on previous system inputs, outputs and current white noise sample:

$$A\,(z)\,y(k) = B\,(z)\,u(k-d) - e(k). \tag{3}$$

After reorganizing Eq. (3) and using vectors: φ which represents the input and output observed from real process and θ - unknown parameters of model, one could get the following form:

$$y(k) = \varphi(k)\theta, \qquad \forall k, \tag*{(4)$_1$}$$

where:

$$\varphi(k) = [-y(k-1), \ldots, -y(k-na), u(k-d), \ldots, u(k-d-nb+1)], \tag*{(4)$_2$}$$

$$\theta = [a_1, \ldots, a_{na}, b_1, \ldots, b_{nb}]^{\mathrm{T}}. \tag*{(4)$_3$}$$

4.2. Recursive last squares method (RLSM)

The main task of online identification is to obtain model parameters basing on the actual state of process. One of the procedures called *Recursive Last Squares Method* (*RLSM*) was proposed by L. LJUNG, T. SÖDERSTRÖM [3]. It can be described by Eqs. (5):

$$\begin{aligned}
\boldsymbol{\theta}(k) &= \boldsymbol{\theta}(k-1) + \frac{\alpha(k)\mathbf{P}(k-1)\boldsymbol{\varphi}(k)}{1+\boldsymbol{\varphi}^{\mathrm{T}}(k)\mathbf{P}(k-1)\boldsymbol{\varphi}(k)}\varepsilon(k),\\
\mathbf{P}(k) &= \mathbf{P}(k-1) - \frac{\alpha(k)\mathbf{P}(k-1)\boldsymbol{\varphi}(k)\boldsymbol{\varphi}^{\mathrm{T}}(k)\mathbf{P}(k-1)}{1+\boldsymbol{\varphi}^{\mathrm{T}}(k)\mathbf{P}(k-1)\boldsymbol{\varphi}(k)},\\
\varepsilon(k) &= y(k) - \boldsymbol{\varphi}(k)\boldsymbol{\theta}(k-1),\\
\alpha(k) &= \begin{cases} a, & |\varepsilon(k)| > \delta\sqrt{1-a},\\ 0, & |\varepsilon(k)| \le \delta\sqrt{1-a}, \end{cases}\\
\mathbf{P}_0 &= \rho\mathbf{I}.
\end{aligned} \tag{5}$$

Equations presented in this paragraph were used by the author for building program for calculating parameters of ARX model of the considered process. The results of working design application are presented below.

4.3. Result of online identification

The accuracy of the described method mostly depends on choosing a model order (numbers *na* and *nb*): the higher the order the higher precision of the model. On the other hand, high order increases the complexity of calculation, which should be done in every step of adaptive regulator procedure to obtain their parameters. The author, after making several tests, has chosen $na = 2$ and $nb = 2$. The precision of algorithm depends also on values of parameters: $0 < a < 1$, $\delta, \rho > 0$.

An example of online identification result is shown in Fig. 3 (outputs of the system were prefiltered before calculation). As it can be seen the procedure works very well: the result of calculation is comparable with the filtered output. The error of identification process is limited and constitutes a small part of data.

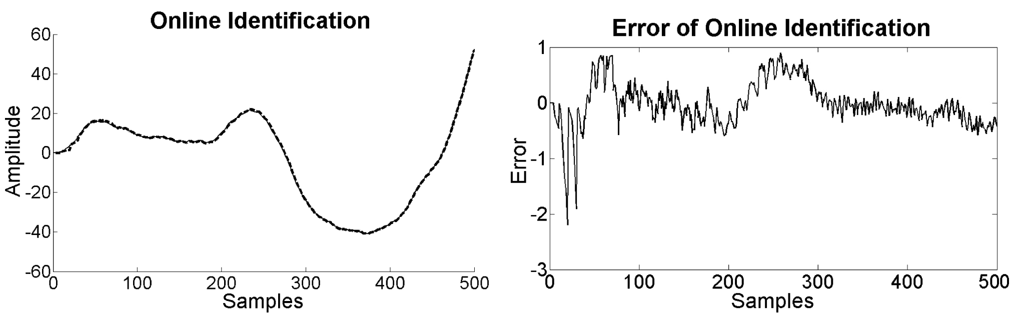

Fig. 3. Online identification process for 500 samples (sampling frequency: 25 kHz) '—' output of real process, '- - -' output of model.

The result of conducted examination shows also that the accuracy of the method increases when sampling rate of process increase as well. It is possible only after building the optimal code for control application and using more advanced software like real time operating system (for example RTAI).

5. Conclusions and simulation result

The author has chosen an example result of identification process (described by Eq. (6)) and built a "static" PID controller for vibration cancellation using *rltool* toolbox. The Simulation of working designed regulator is shown in Fig. 4.

$$y(k) - 1.944y(k-1) + 0.944y(k-2) = 0.007u(k-1) - 0.007u(k-2). \qquad (6)$$

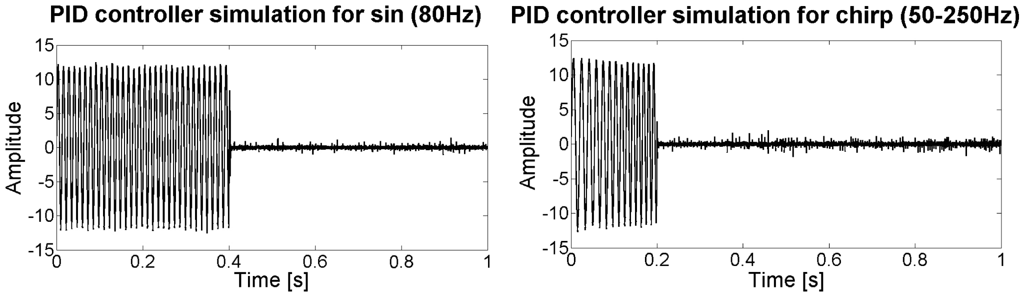

Fig. 4. Response of close-loop system with PID controller for *sin* and *chirp* signal (with noise).

It can be seen that the uncontrolled plate (to 0.4 s) response vibrates significantly while the controller causes substantial reduction of the plate vibrations, even if identified model is only of second order.

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