ARCHIVES OF ACOUSTICS
23, 1, 51–66 (1998)

SYMMETRY AND ASYMMETRY AS A PHYSICAL AND PERCEPTUAL FEATURE OF THE COMPLEMENTARY PAIR OF BEATING SINUSOIDS PART I. AMPLITUDE AND FREQUENCY ENVELOPE RELATIONS

L. RUTKOWSKI

Institute of Acoustics,
Adam Mickiewicz University
(60-769 Poznań, ul. Matejki 48/49)

Beating sinusoids are an interesting case of a simultaneous change of intensity and frequency achieved without the need of a modulator. Studies of the perception of beats provide numerous data concerning also the sound pitch perception. Hitherto, the following conclusions have been made from those studies: i) if the amplitude of one tone is much larger than the amplitude of the other one, of the two-tone complex, the pitch shifts towards the frequency of the larger amplitude tone; ii) if the amplitudes of the two tones are the same, the pitch is localized precisely at the arithmetic average of the two tone frequencies. These statements imply therefore, that a symmetry with respect to the arithmetic average frequency of the two-tone beatings is present in the pitch localization on the frequency scale. Most recent studies show, however, that this symmetry is not always maintained. In the current study, divided into Part 1 and Part 2, an attempt is made, basing on the discussion and numerical analysis of the functions which describe the beatings, to determine the cause of this asymmetry. One of the arguments may come from the fact that the narrow-band condition for beating waveforms is only partially satisfied. This implies that the consequences of the relative rate of changes of the amplitude envelope to the resultant frequency envelope should be considered in the analysis of the beatings signal. The lack of symmetry is evidenced by the functions which reflect the influence of the magnitude of the ratio of the amplitudes of two signal components on the values of the normalised parameters EWAIF (Envelope Weighted Average of Instantaneous Frequency) and IWAIF (Intensity Weighted Average of Instantaneous Frequency) correlated with the sound pitch. In Part 2, two psychoacoustic experiments are described that aimed at the examination of the pitch of beatings in view of the symmetry arguments mentioned above. Main conclusions obtained in this part of the study are used throughout together with the literature available on this subject.

1. Introduction

In spite of its rudimentary character, the phenomenon of beating sinusoids continues to be the subject of interest of both the theory of modulation [9, 15, 16] as well as the studies of the perception of sounds with time-varying parameters.

The beating waveform, also referred to as TCC (Two-Component Complex [4]) as the superposition of two tones with very similar frequencies, may also, from the analytical standpoint, be regarded as an example of the elementary modulation process known as TTSS (Two-Tone Single-Sideband) [15]. The basic feature of the beating is the time-variation of the amplitude envelope which resembles that of the modulated waveforms. Usually, this variation does not arise from nonlinearities of the amplitude (AM) or the frequency modulation (FM) process, but follows from superposition of two signals which is evidently a linear process. In the signal resulting from superposition the features of the two components, in particular their individual time variations, cease to be important. The Fourier spectrum of the beating waveform contains two components of pre-determined frequencies and amplitudes. The width of the Fourier spectrum of beats is equal to the frequency difference of its two components. All these arguments out the elementary properties of the beats phenomenon as long as we confine ourselves to spectral analysis.

The beating waveform manifests itself as a periodic variation of the sound pressure level (SPL) with the repetition time equal to the inverse of the frequency difference of the two components. The SPL variations (envelope of the beating) can be monitored on an oscilloscope or directly perceived as an audible loudness change. It occurs however, that the beating cycle also features an instantaneous frequency variation [1, 2, 4, 8, 14, 15, 16], perfectly synchronised with the variations of the sound pressure level. But from the perceptual point of view and its physical detection, the former is not perceived as easily as the changes in the loudness. Both these features of the beating waveform, i.e. the change of the amplitude envelope and the instantaneous frequency variation, are not resolvable in the Fourier spectrum. Determination of the instantaneous frequency variations in the beating cycle requires frequency demodulation, an instrumental task which is not straightforward within the acoustic frequency range [10]. Hence, the procedure of the demodulation is usually performed employing Hilbert transform algorithms [1, 9] or time-frequency distributions TFD [9]. The perceptual complexity of the beatings results mainly from the simultaneous occurrence of changes of the amplitude envelope together with the instantaneous frequency variations, including the occurrences of extreme frequency changes at the moments which correspond to the minima of the sound amplitude envelope. Perception of the beatings is a part of the studies of signals of variable frequency and the amplitude envelope and provide an insight into problems of simultaneous perception of the loudness and the pitch of sounds.

Investigation of the physical properties and the perception of beatings is also important for the room acoustics, particularly with sounds of time-varying frequency propagating in a room. A beating-like effect occurs in a room [12] when two waves superpose, for instance, the direct wave of an advancing frequency and the reflected one having the same frequency changes but reproducing its previous history of the frequency change. In a specific point of the room there is some frequency difference which causes the beatings and which depends on the velocity, character of the frequency changes and the delay of the reflected wave.

While it is difficult to accept that sound propagation in our environment may involve nonlinear processes of amplitude and frequency modulation, it must be quite natural to expect, for the envelope changes and instantaneous frequency variations, that it results from acoustic waves superposition; there is little doubt that the two features should appear even in geometric structures of moderate complexity. Hence, it seems purposeful and justified to devote some attention to this kind of phenomena.

This study consists of two parts. Part I is mainly devoted to the physical aspect of variations of the beatings signal and its results establish some grounds for the subject of the perception of the beatings pitch that is dealt with in Part II. In its principal contents, the study deals with the so called complementary pair of signals usually referred to as SL and SH [3]. The signal abbreviated as SL (Stronger Low) consists of a pair of tones in which the lower frequency signal is of a higher sound pressure level whereas the reverse is implied for the Stronger High, abbrev. SH. The subject of particular interest of the authors is the intuitive possibility of a symmetry or its absence for the two complementary pairs of signals: SL-SH; those will be discussed on the physical grounds of the beatings (Part I) as well as the perception of the pitch of the two-tone complexes SL-SH. There is already a report by DAI [2] who notifies the problem of the asymmetry in the discrimination of the pitch. Thus far, the reported asymmetries in the perception of the sound pitch remain unexplained.

The analytical representation of the beatings waveform embraces its two cardinal attributes: the changes of amplitude envelope and the instantaneous frequency variations. There is a number of articles in which the changes of the amplitude envelope and the instantaneous frequency variations of the beats have been analysed [1, 3, 8, 14, 15]. In some of these articles the final formulae, describing the changes of amplitude envelope and instantaneous frequency variations of beatings, have been obtained in terms of the analytic signal concept. It looks however that in the majority of the contributions to this subject, the physical aspects of the beats have been treated only marginally. Consequently, the available description has been only superficial and has routinely ignored the essential elements of the problem pursued by the present investigation.

2. The beatings – its analytical description and properties

To derive a formula for the changes of the amplitude envelope and instantaneous frequency variations of a beatings signal, let us assume the real signal $r(t)$ to be a sum of two tones of different amplitudes x_L and x_H and the frequencies f_L and f_H (the subscripts L and H refer to the lower and the higher frequency tones, respectively).

$$r(t) = x_L \cos 2\pi f_L t + x_H \cos 2\pi f_H t\,. \tag{I.1}$$

Applying the Hilbert transform to the real signal $r(t)$ we obtain

$$\mathrm{Hi}\,\{r(t)\} = x_L \sin 2\pi f_L t + x_H \sin 2\pi f_H t\,. \tag{I.2}$$

The analytic signal associated with the real one (I.1) will be

$$r_a(t) = r(t) + j\,\mathrm{Hi}\,\{r(t)\}. \tag{I.3}$$

The envelope of the analytic signal is obtained from the formula:

$$|r_a(t)| = \sqrt{[r(t)]^2 + [\mathrm{Hi}\,\{r(t)\}]^2}\,. \tag{I.4}$$

This envelope is a real function equal to the real signal envelope. For the beatings (I.1), the envelope calculated with the formula (I.4) is

$$e(t) = |r_a(t)| = x_L\sqrt{1 + \delta^2 + 2\delta \cos 2\pi\Delta f t}\,, \tag{I.5}$$

where $\delta = x_H/x_L$ is the amplitude ratio, whereas $\Delta f = f_H - f_L$ is the frequency difference of the superposed signals which effect the beats.

The phase of the analytic signal equals

$$\varphi_a(t) = \operatorname{arctg}\left[\frac{\operatorname{Hi}\{r(t)\}}{r(t)}\right] \quad \text{[rad]}. \tag{I.6}$$

The instantaneous frequency IF(t) is calculated as the time derivative of the phase (I.6) of the analytic signal

$$\operatorname{IF}(t) = \frac{1}{2\pi}\frac{d\varphi_a(t)}{dt} \quad \text{[Hz]}. \tag{I.7}$$

In practice, for the complex signals covering a certain frequency range, the phase changes are defined with reference to a fixed, constant frequency, for instance f_L, its phase being a linear function of time $\varphi(f_L, t) = 2\pi f_L t$. Then, the instantaneous frequency variations can be written as

$$\operatorname{IF}(t) = \frac{1}{2\pi}\frac{d\widetilde{\varphi}_a(t)}{dt} + f_L\,, \tag{I.8}$$

where $\frac{1}{2\pi}\frac{d\widetilde{\varphi}_a(t)}{dt}$ is the time dependent part of the instantaneous frequency.

Hence, the resulting signal, associated to the real signal $r(t)$ (Eq. (I.1)) may be rewritten in the following form:

$$r(t) = |r_a(t)|\cos\left[2\pi\int \operatorname{IF}(t)\,dt\right] = \operatorname{Re}\left\{\exp\left[\ln|r_a(t)| + j2\pi\int \operatorname{IF}(t)\,dt\right]\right\}. \tag{I.9}$$

This equation points out the possibility of a generalisation of the signal phase (see also [13]) by regarding the changes of the amplitude envelope as a factor in the phase of the signal $r(t)$.

Therefore, the expression

$$\operatorname{CI\Phi}(t) = \ln|r_a(t)| + j2\pi\int \operatorname{IF}(t)\,dt = \ln|r(t)| \tag{I.10}$$

is the complex phase of the resultant signal. The complex instantaneous frequency of the signal becomes

$$\operatorname{CIF}(t) = \frac{1}{2\pi}\frac{d\operatorname{CI\Phi}(t)}{dt} = \left[\frac{1}{2\pi}\frac{1}{|r_a(t)|}\frac{d|r_a(t)|}{dt} + j\operatorname{IF}(t)\right]$$
$$= \frac{1}{2\pi}\frac{1}{|r(t)|}\frac{d|r(t)|}{dt} \quad \text{[Hz]}. \tag{I.11}$$

It follows from Eq. (I.11) that the complex instantaneous frequency is a function which most generally describes the changes of the real signal $r(t)$, disregarding them if they are associated with phase or the amplitude envelope changes. The magnitude of the complex instantaneous frequency (a real function describing the changes of frequency) of the resulting signal $r(t)$ can be calculated according to the equation

$$|\operatorname{CIF}(t)| = \sqrt{\left[\frac{1}{2\pi}\frac{1}{|r_a(t)|}\frac{d|r_a(t)|}{dt}\right]^2 + [\operatorname{IF}(t)]^2} \quad \text{[Hz]}. \tag{I.12}$$

Note that only when the amplitude envelope is constant, the magnitude of the complex instantaneous frequency is equal to the instantaneous frequency IF(t).

Applying Eq. (I.8) and (I.11), the complex instantaneous frequency of the beatings will be

$$\mathrm{CIF}\,(t) = -\frac{\delta \Delta f \sin(2\pi \Delta f t)}{1+\delta^2+2\delta\cos(2\pi\Delta f t)} + j\frac{\delta\Delta f(\cos(2\pi\Delta f t)+\delta)}{1+\delta^2+2\delta\cos(2\pi\Delta f t)} + jf_L\,. \tag{I.13}$$

The first part of the complex instantaneous frequency (the real part) results exclusively from the amplitude envelope changes; the second and the third terms (imaginary part) involve instantaneous frequency variations and the constant tone frequency f_L. The real part of the instantaneous frequency has been usually omitted in the literature, however, for acoustic signals of a variable amplitude envelope or for signals which do not satisfy the narrow band condition, it ought to be taken into account.

The magnitude of the complex instantaneous frequency of the beats (a real function describing frequency variations) equals according to the equation (I.12)

$$|\mathrm{CIF}\,(t)| = \sqrt{\left[\frac{\delta\Delta f\sin(2\pi\Delta f t)}{1+\delta^2+2\delta\cos(2\pi\Delta f t)}\right]^2 + \left[\frac{\delta\Delta f(\cos(2\pi\Delta f t)+\delta)}{1+\delta^2+2\delta\cos(2\pi\Delta f t)} + f_L\right]^2}\,. \tag{I.14}$$

Generally, the variability of the beats is described with two basic real functions: the amplitude envelope (I.5),that is the magnitude of the analytic signal and the frequency envelope (I.14) which is the magnitude of the complex instantaneous frequency of the analytic signal.

We now differentiate $r(t)$ Eq. (I.9) against time

$$\frac{dr(t)}{dt} = r(t)\,\mathrm{Re}\left\{\left[\frac{1}{|r_a(t)|}\,\frac{d|r_a(t)|}{dt} + j\,\mathrm{IF}\,(t)\right]\right\}. \tag{I.15}$$

The expression in square brackets (I.15) is the complex instantaneous frequency. An essential physical interpretation follows from the Eq. (I.15), namely that the complex instantaneous frequency can be regarded as a generalised, relative rate of changes of the signal $\dfrac{1}{r(t)}\,\dfrac{dr(t)}{dt}$ involving both the envelope and/or the frequency changes. At the same time, it is a quantity which, to certain extent, describes the real time evolution of any signal because it contains only the first derivative of the signal with respect to time. Under certain conditions, however, namely for a narrow-band signal, the complex instantaneous frequency (Eq. (I.15)) may adequately account for the signal variability.

Generally, using polar coordinates, we may now write

$$\mathrm{CIF}\,(t) = |\mathrm{CIF}\,(t)|\exp\left[\varphi_{\mathrm{CIF}}(t)\right], \tag{I.16}$$

where

$$\varphi_{\mathrm{CIF}}(t) = \mathrm{arctg}\left[-\frac{\delta\Delta f(\cos(2\pi\Delta f t)+\delta) + f_L(1+\delta^2+2\delta\cos(2\pi\Delta f t))}{\delta\Delta f\sin(2\pi\Delta f t)}\right] \tag{I.17}$$

is the phase angle between the imaginary and real components of the complex instantaneous frequency variations of beatings. The nonlinear dependence of φ_{CIF} on time points out the existence of modulation of the phase angle of the complex instantaneous frequency.

3. Results of calculations of the beat changes

For the analysis of physical properties of the two-tone beatings, the formulae derived in section 2 were used in the numerical calculations. The outcome of these calculations, obviously limited to a few selected parameters of the beatings, present some cognitive value, and will be applied in Part II of the present study in the interpretation of results of the investigation of perception processes.

In Fig. 1, envelope changes $e(t)$ (normalised to unity) (a), changes of the real part of the complex instantaneous frequency Re CIF (t) (b), and the imaginary part of the

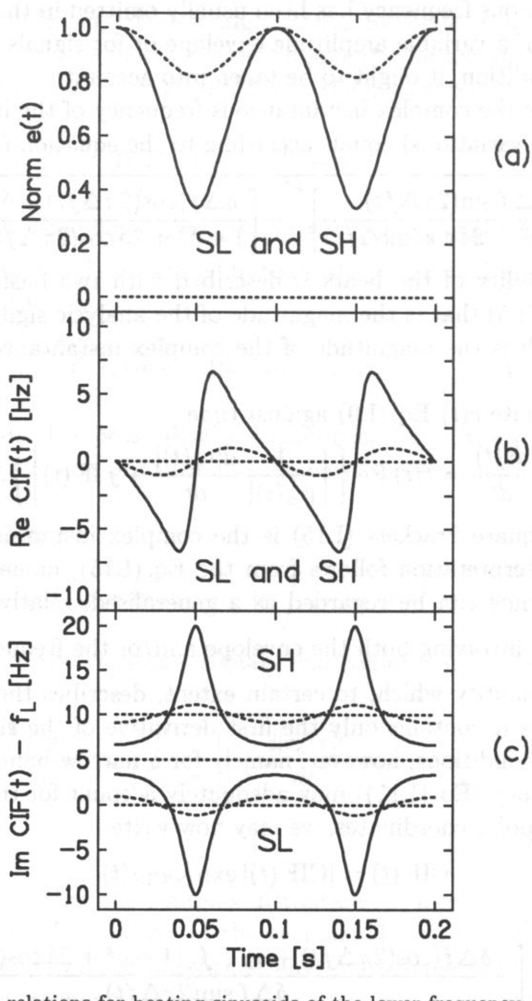

Fig. 1. Basic temporal relations for beating sinusoids of the lower frequency $f_L = 1000$ Hz and the frequency difference of 10 Hz for the complementary pairs SH – stronger high and SL – stronger low (see text); (a) the normalised envelopes, (b) the real parts of the Complex Instantaneous Frequency, (c) the imaginary parts of the Complex Instantaneous Frequency diminished by f_L. (SL – amplitude ratio 0.5 (solid line), amplitude ratio 0.1 (dashed line)), (SH – amplitude ratio 2 (solid line), amplitude ratio 10 (dashed line)).

complex instantaneous frequency changes Im CIF (t) (c) diminished by $f_L = 1000\,\mathrm{Hz}$, the lower tone frequency; are shown. The frequency difference of the beats signal equals 10 Hz, the ratio of the amplitudes δ with SL marks is 0.5 (solid line), 0.1 (dashed line), whereas those marked with HL are $\delta = 2$ and 10 (solid and dashed lines, respectively). Thus, the situations discussed here, i.e. when $\delta < 1$ and $\delta > 1$ (or $\delta = 1/\delta$), refer to the two complementary pairs of signals which were labelled earlier SL ($\delta < 1$) and HL ($\delta > 1$).

The following conclusions can be drawn from the discussion of the plots in Fig. 1:

- variations of the imaginary part of the complex instantaneous frequency (Fig. 1c) exhibit a symmetry with respect to the arithmetic average of the two frequencies of the beating components SL and SH, both for small and large magnitudes of δ,
- at $\delta = 0.5$ (with the pair denoted SL) and at $\delta = 2$ (with the pair marked SH), the variations of the imaginary part of the complex instantaneous frequency are non-symmetrically displaced from the frequency f_L for $\delta < 1$ and from the frequency $f_H = f_L + 10\,\mathrm{Hz}$ for $\delta > 1$; respective (see also [3]) average values of these variations equal f_H and f_L,
- the changes of the amplitude envelope (Fig. 1a) are identical for the two complementary pairs of signals SL and SH; the minimum of the normalised amplitude envelope curve is 0.818 for $\delta = 0.1$ (both SL and SH) and 0.33 for $\delta = 0.5$ (SL and SH),
- identical changes of the amplitude envelope lead to identical variations of the real part of the complex instantaneous frequency (Fig. 1b) of zero mean value; it does not depend on which pair of signals, SL or SH, is discussed,
- at $\delta = 0.1$ with the SL pair and $\delta = 10$ with the SH pair, the graphs of all the functions presented in Figs. 1b and 1c look like sinusoidal curves, i.e. they oscillate more or less around f_L at $\delta < 1$ and $f_H = f_L + 10\,\mathrm{Hz}$ at $\delta > 1$,
- at the moments corresponding to the maximum value of the envelope amplitude, the magnitudes of the imaginary part of the complex instantaneous frequency (Fig. 1c) reach a maximum for the SL pair and a minimum for the SH pair.

4. Weighted changes of the frequency envelope

The concepts of amplitude-envelope weighted or squared envelope weighted signal frequency variations, is widely used, mainly in studies concerning the perception of the sounds in which changes of amplitude and frequency coexist. FETH *et al.* [4] proved a considerable correlation between the perceived pitch of the beatings and envelope-weighted average of instantaneous frequency (EWAIF). Next, ANANTHARAMANN *et al.* [1] proposed that the application of sound intensity (correctly: squared envelope) variations (IWAIF: *Intensity-Weighted Average of Instantaneous Frequency*) as a weighting function for evaluation of the pitch for beatings yields better accord with subjective investigations. IWAMIYA *et al.* [6, 7] used the signal envelope function to weight the instantaneous frequency variations in the determination of the so called principal pitch of sounds which were simultaneously amplitude and frequency modulated; a model for the vibrato achieved by musicians during instrumental performance was achieved in this way. The comparison of Figs. 1a and 1c shows also that the instantaneous frequency variations taking place in the vicinity of maximum of beatings amplitude has a more pronounced effect on the perception of the pitch than those occurring within the intensity minimum. From the perceptual point of view, these problems will later be discussed more thoroughly in Part II.

On the frequency axis, the envelope weighted average of instantaneous frequency [1, 2] defines the coordinate of a centre of gravity of the figure set out by the signal spectrum which, in the time domain, can be determined from the formula

$$\mathrm{EWAIF} = \frac{\int_0^t e(t)f(t)\,dt}{\int_0^t e(t)\,dt}, \tag{I.18}$$

where $e(t)$ is the amplitude envelope (weighting function) and $f(t)$ describes the frequency variations. For periodical changes of amplitude and frequency, the upper bound of integration ought to be made a multiple of the period of these changes. Similarly, the squared envelope (intensity) weighted average of the instantaneous frequency can be defined with [1, 3]

$$\mathrm{IWAIF} = \frac{\int_0^t e^2(t)f(t)\,dt}{\int_0^t e^2(t)\,dt}. \tag{I.19}$$

Some problems may arise, when attempting numerical calculations with the formulae (I.18) and (I.19), for instance, when the envelope amplitude approaches the zero value (two-tone beatings of amplitude ratio close to 1). For this reason ANANTHARAMAN [1] recommends a spectral method of the calculation of EWAIF and IWAIF. Given a preset resolution of analysis, the spectral method, however, is accurate only for large frequency separations of the two tones of the beatings. The error may be significant when these separations are small. The usage of the spectral method may be attributed rather to the wide availability of FFT algorithms. Investigation of the time evolution of the frequency $f(t)$ requires a demodulation of the signal frequency, involving the calculation of the Hilbert transform; the latter is not so easily available and so popular as the Fast Fourier Transform, FFT. From the perceptual point of view, not only the spectrum of the beatings, but also the pattern of the time variations of the instantaneous frequency is important.

The functions defined by Eq. (I.14) and illustrated as examples in Fig. 1 are necessary if the two weighting representations of the beatings, (I.18) and (I.19), have to be determined. For the appropriate complementary pairs of signals SH and SL the beatings envelopes are identical, whereas the frequency variations are described by distinctive functions. Also, it is of importance which function will be adopted for the description of the frequency dependence on time $f(t)$. For signals of constant amplitude and variable frequency, for example for FM signals, $f(t)$ describes directly the variations of frequency effected by the modulation process. However, if the signal, for which the weighted values of instantaneous frequency are to be evaluated, exhibits both envelope and frequency variations, like the beatings, the formulae (I.18) and (I.19) have to be modified by the frequency function expressed through the changes of the frequency envelope (I.14).

In order to demonstrate the importance of the frequency envelope variations in evaluating the weighted values defined by (I.18) and (I.19), the two quantities (normalised through dividing by $\Delta\omega$) were calculated as a function of the amplitude ratio of the two components SL and SH of beatings in the $< 0.5, 1 >$ and $< 1, 2 >$ ranges, respectively. Later in the text, we shall refer to the normalised values of EWAIF and IWAIF as NEWAIF and NIWAIF. They are shown subsequently in Figs. 2–4 with the frequency $f_L = 480\,\text{Hz}$ and the frequency difference $\Delta f = 40\,\text{Hz}$.

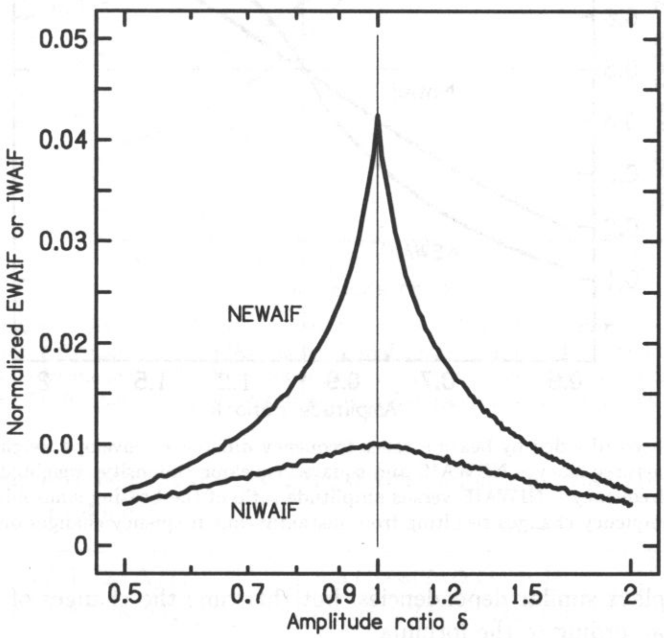

Fig. 2. Normalised (divided by beating tones frequency difference) envelope weighted average of instantaneous frequency – NEWAIF and squared envelope (intensity) weighted average of instantaneous frequency – NIWAIF versus amplitude ratio of the beating sinusoids. Numerical calculations for frequency changes resulting from the relative envelope changes only (Eq. (I.20)).

In Fig. 2 there are plots of the normalised envelope weighted average of the instantaneous frequency and of the normalised intensity weighted average of instantaneous frequency vs. δ, i.e. vs. the ratio of the two amplitudes; the frequency changes are linked to the changes of the amplitude envelope of the beatings via the formula:

$$f(t) = \sqrt{\left[\frac{\delta\Delta f(\sin(2\pi\Delta ft)}{1+\delta^2+2\delta\cos(2\pi\Delta ft)}\right]^2 + f_L^2} - f_L\,. \tag{I.20}$$

Thus, in the plots of Fig. 2 only the real part of complex instantaneous frequency (I.13) was accounted for, while the imaginary part was omitted. One may see from these plots that both relations reach maximal values at $\delta = 1$ (largest modulation depth of the beatings). It is also interesting that for the complementary pairs of the signals SL and SH these functions display the same values (parity).

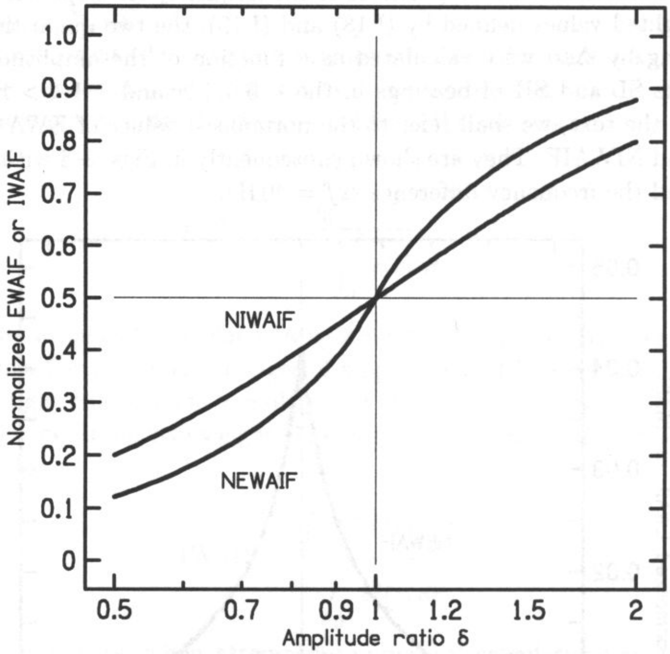

Fig. 3. Normalised (divided by beating tones frequency difference) envelope weighted average of instantaneous frequency – NEWAIF and squared envelope (intensity) weighted average of instantaneous frequency – NIWAIF versus amplitude ratio of the beating sinusoids. Numerical calculations for frequency changes resulting from instantaneous frequency changes only (Eq. (I.21)).

Figure 3 displays similar dependencies, but this time the changes of the frequency are determined according to the formula

$$f(t) = \sqrt{\left[\frac{\delta\Delta f(\cos(2\pi\Delta f t) + \delta)}{1 + \delta^2 + 2\delta\cos(2\pi\Delta f t)} + f_L\right]^2} - f_L\,. \tag{I.21}$$

In this case only the imaginary part of the frequency envelope (I.13), usually referred to as instantaneous frequency IF(t), (Eq. (I.7)), was accounted for. The coordinates of the point, with respect to which the symmetry may be discussed, can be set at $\delta = 1$ on the abscissa and at NEWAIF (NIWAIF) $= 0.5$ on the ordinate. The following relation is valid: the magnitude of NEWAIF and NIWAIF for the pair denoted as SH equals 1 minus the value of NEWAIF and NIWAIF for the pair denoted as SL. At $\delta = 1$, thus when the amplitudes of the two signals are equal (SL = SH), both functions amount to 0.5, i.e. the resultant frequency yields $f_L + 0.5\Delta f$ which corresponds to arithmetic average frequency of the two component signals of the beatings. The functions describing the graphs displayed in Fig. 3 are odd with regard to the argument $\delta = 1$.

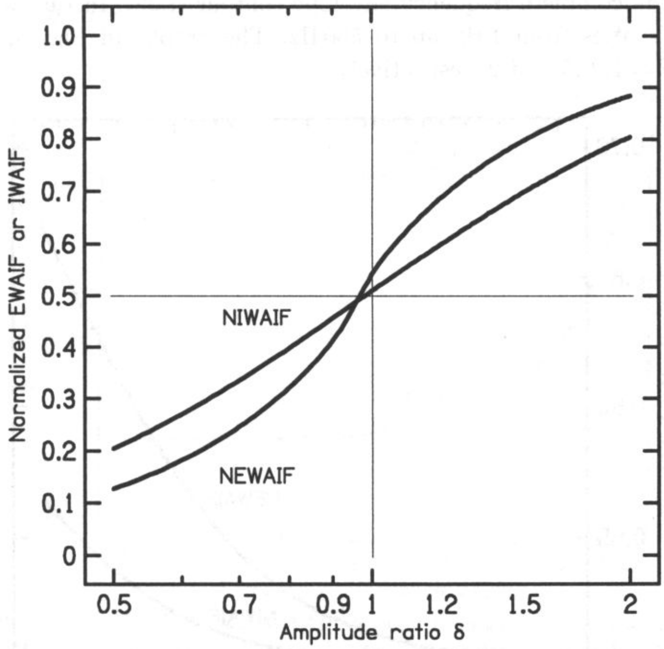

Fig. 4. Normalised (divided by beating tones frequency difference) envelope weighted average of instantaneous frequency – NEWAIF and squared envelope (intensity) weighted average of instantaneous frequency – NIWAIF versus amplitude ratio of beating sinusoids. Numerical calculations for frequency changes resulting from frequency envelope changes (Eq. (I.22)).

Next, similarly to the plots of Figs. 2 and 3, the complete formula $f(t)$ for the changes of the frequency envelope, i.e.

$$f(t) = \sqrt{\left[\frac{\delta\Delta f \sin(2\pi\Delta f t)}{1+\delta^2+2\delta\cos(2\pi\Delta f t)}\right]^2 + \left[\frac{\delta\Delta f(\cos(2\pi\Delta f t)+\delta)}{1+\delta^2+2\delta\cos(2\pi\Delta f t)} + f_L\right]^2} - f_L\,, \qquad \text{(I.22)}$$

was employed to produce the dependencies shown in Fig. 4.

The appearances of the graphs in the Figs. 3 and 4 are identical. A detailed analysis of the numerical data points out certain differences, especially in the vicinity of $\delta = 1$. The crossing point of the two curves, NEWAIF(δ) and NIWAIF(δ), is slightly shifted to the left from the value $\delta = 1$, whereas the magnitudes of the two functions are less then 0.5. With the amplitude ratio δ equal one, both the normalised functions exceed 0.5.

Hence, following the results of calculations, an asymmetry of the NEWAIF(δ) and NIWAIF(δ) curves occurs with regard to $\delta = 1$. Till now, at the special point $\delta = 1$ NEWAIF and NIWAIF were believed to be exactly equal to an average frequency of the two beating tones (normalised functions equals 0.5). The observed asymmetry results from taking into account the real part of the complex instantaneous frequency. Let us take a closer look at the dependence of NEWAIF and NIWAIF on the frequency separation Δf of the two tones. Figures 5 and 6 show the results of calculations of the two functions NEWAIF(Δf) and NIWAIF(Δf) performed for the frequency variations weighted with the complete frequency envelope contour (I.22). In the two figures $f_L = 500\,\text{Hz}$ and Δf varies from 1 Hz up to 500 Hz. The graphs in Figs. 5, 6 and 7 were obtained with $\delta = 1$, 0.5 and 2, respectively.

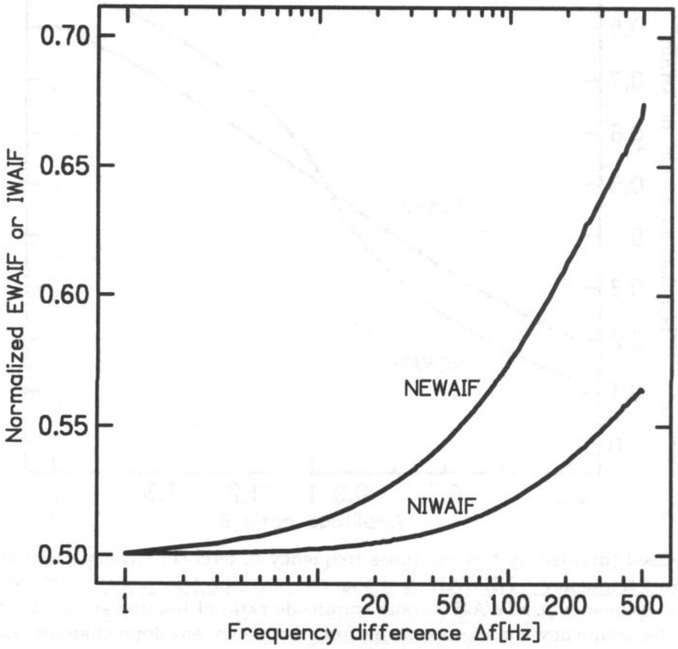

Fig. 5. Illustration of the NEWAIF and NIWAIF changes as function of the frequency difference between the beating sinusoids. The frequency changes for calculations are taken from Eq. (I.22). The lower frequency is constant and equals 500 Hz. The amplitude ratio of two components $\delta = 1$.

If there was a full symmetry for the data illustrated in Fig. 5, then the values of NEWAIF and NIWAIF should be constant and equal to 0.5, independent of the magnitude of Δf. Similarly, for the data presented in Fig. 6 and Fig. 7, the values of EWAIF and IWAIF would not be related to Δf and correspond to the appropriate ordinates read at the lowest range of frequencies (here, $\Delta f = 1\,\text{Hz}$).

The graphs in all three Figs. 5, 6 and 7, illustrate the essential problem often disregarded in the context of acoustic waveforms: the problem of a narrow band property of signals. The narrow band signal, in the case of beatings, is a signal for which $\Delta f / f_L \ll 1$, whereas the broadband criterion is the similarity of the magnitudes of the frequency difference Δf and of the lower frequency of the complex signal, i.e. $\Delta f / f_L \sim 1$. Figures 5, 6 and 7 demonstrate that a significant effect of the real part of the complex instantaneous frequency on the frequency envelope is observed when the beatings do not obey the narrow band condition (this can be extended to other signals, too). It remains an open question, if the mathematically correct calculations of the envelope and squared envelope averages of the instantaneous frequency values would, in the entire Δf range, match the subjective impression of the pitch. Eventually, these values are intended for the evaluation of the latter.

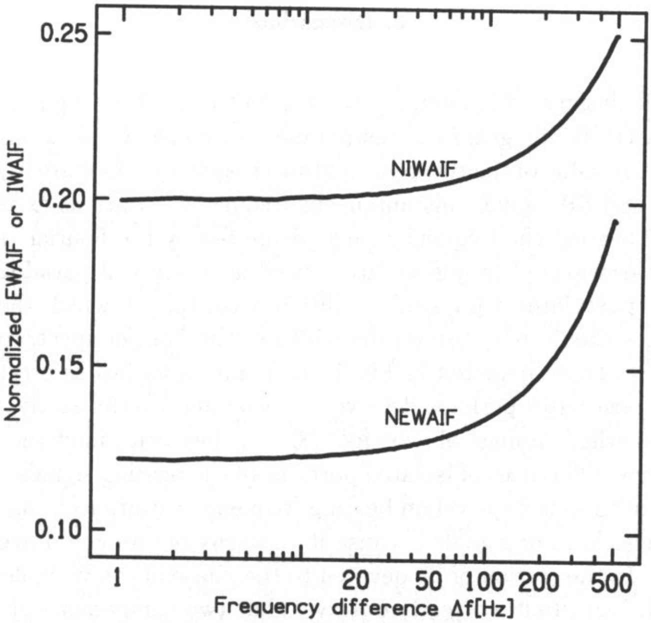

Fig. 6. Illustration of the NEWAIF and NIWAIF changes as function of the frequency difference between the beating sinusoids. The frequency changes for calculations are taken from Eq. (I.22). The lower frequency is constant and equals 500 Hz. Amplitude ratio $\delta = 0.5$.

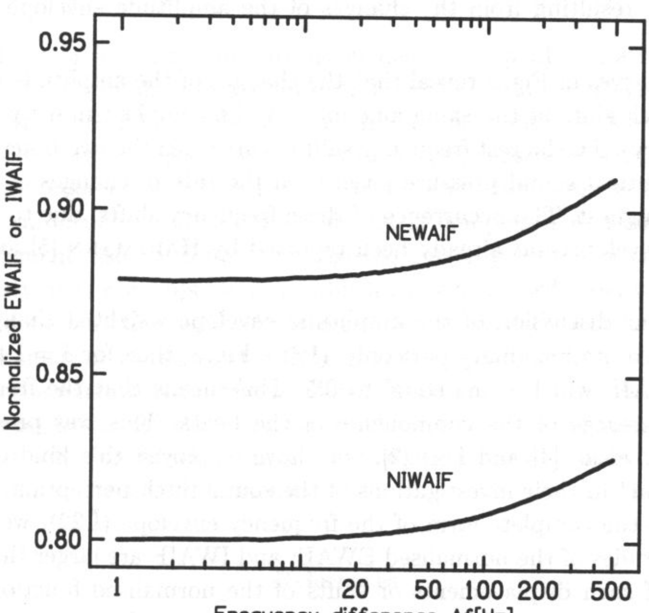

Fig. 7. Illustration of the NEWAIF and NIWAIF changes as function of the frequency difference between the beating sinusoids. The frequency changes for calculations are taken from Eq. (I.22). The lower frequency is constant and equals 500 Hz. Amplitude ratio $\delta = 2$.

5. Discussion

From the calculations which employ the concept of analytic signal, it follows, that there is a symmetry in the graphs of instantaneous frequency variations IF(t) with respect to the average value of the two-tone beatings frequency (compare Fig. 1c). However, for both the SL and SH signals, instantaneous frequency variations exceed and, in case of $\delta \approx 1$, go well beyond the frequency range delimited by the Fourier transform of the beatings. Such variations of frequency are viewed as a physical paradox by LOUGHLIN *et al.* [9] and he postulates a physical condition according to which the instantaneous frequency changes should not surpass the width of the Fourier spectrum of the signal. Changes similar to those depicted in Fig. 1c Loughlin describes as erratic. Though his suggestions and postulates perhaps deserve a special and careful study, they are not in accord with the earlier findings of JEFFRES [8], i.e. they contradict the experimentally observed frequency differences of isolated portions of the beating signals. Loughlin's idea of the physical limitations imposed on beating frequency variations remains closer to the perception of these kind of signals because it concerns the weighted average frequency of the two tones. In the section of [9] devoted to the phase of the variable amplitude and frequency signal, LOUGHLIN correctly observes that two components of the phase have to be considered, i.e. φ_F, the derivative of which yields the instantaneous frequency IF, and φ_A effected by the amplitude envelope changes (see also RUTKOWSKI [13]). Such an approach was also adopted in the present study (Eq. (I.9) and (I.10)) and the frequency patterns, resulting from the changes of the amplitude envelope, are displayed in Fig. 1b.

Normalised curves in Fig. 2 reveal that the changes of the amplitude envelope alone may cause a pitch shift of the same amounts for the complementary pairs of beating signals (symmetry). The largest frequency shift occurs when the two tones of the beating signal are of identical sound pressure level; then the rate of changes of the amplitude envelope is the highest. The occurrence of these frequency shifts, due to the changes of the amplitude envelope, has already been reported by HARTMANN [5] and ROSSING *et al.* [11].

If we limit our discussion of the amplitude envelope weighted changes of the frequency envelope to its imaginary part only, (IF) – Fig. 3, then for $\delta = 1$ the normalised EWAIF and IWAIF will become equal to 0.5. This means that the frequency will be the arithmetic average of the components of the beats. This was presumed, among others, by FETH *et al.* [4] and DAI [2], who have employed this kind of signal as an "adjustable signal" in their investigations of the sound pitch perception. However, taking into account the complete form of the frequency envelope (I.22), we found that at $\delta = 1$ the magnitudes of the normalised EWAIF and IWAIF are larger than 0.5 (Fig. 4). The existence of such displacements or shifts of the normalised functions magnitudes proves that the narrow band criterion is only partially satisfied by the beatings (compare Fig. 5). These shifts occur also with other values of the amplitude ratio (compare Figs. 6 and 7) and ought to be considered every time when the condition $f_L \gg \Delta f$ is not fulfilled.

6. Conclusions

Summing up the above analysis of physical features of beatings, it can be stated that the signal variability is mainly described by two functions: the amplitude envelope (I.5) and the frequency envelope (I.14). As the amplitude envelope, or its square, is directly related to the changes of the signal sound pressure level, the frequency envelope tells us the rate with which phase variations occur as well as the relative speed of the amplitude envelope variations. Both the rate of phase variations and the relative rate of the envelope changes exhibit the same dimension-frequency. Association of the time evolution of the amplitude envelope of beats and the variations of the frequency gives evidence (see Fig. 1) that at values of the amplitudes ratio approaching 1, the frequency variations occurring near the maximum of the amplitude envelope ought to be estimated more consequently than those in the vicinity of the envelope minimum. In the light of this finding, the concept of two sets of values, the envelope and squared envelope weighted averages of instantaneous frequency, appears to be correct and entirely justified. The performed analysis indicates, however, that it is not trivial which form of the frequency variations is used in the calculations. Using the so called complete formula for the frequency changes, i.e. the enveloped frequency (I.14), leads to the evidence of asymmetries (Fig. 4) in the course of the normalised EWAIF(δ) and IWAIF(δ) curves.

The above conclusions are quite general and may be applied to analysis of changes of the frequency envelope of arbitrary signals, featuring concurrent variations of amplitude and frequency envelopes known as MM (Mixed Modulation) or CM (Combined Modulation).

In the analysis of beatings, due attention has to be paid to narrow-band and/or wide-band aspects of signals that are important in the question of symmetry or asymmetry of the complementary pairs of two-tone complex signals. As demonstrated in section 4, when the distance between the beating components increases, the trend of the variations of the frequency envelope is to an increasing extent controlled by the rate of changes of the amplitude envelope (the real part of complex instantaneous frequency). The above statement also holds for other signals, not only for the beatings.

Unfortunately, it is not possible to establish the exact border between narrow-band and broad-band signals. Consequently, it must be accepted that each signal (this concerns especially the acoustic signals) with time-varying parameters always exhibits some departure from the narrow-band criteria. When determining the attributes of the signals variability, one ought to use the function which describes the frequency envelope of a sound. It is this function which permits to establish to what extent the narrow-band attribute determines the signal variability.

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