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BULK WAVE TRANSMISSION BY A MULTISTRIP COUPLER (MSC)

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Multistrip coupler (MSC) is a component of SAW devices that couples surface acoustic waves (SAWs) propagating in two acoustic channels on piezoelectric substrate. Little is known in SAW literature about simultaneous transmission of bulk waves by MSC; it is believed that MSC efficiently suppress bulk wave spurious signals in SAW devices, and this is one of reasons of application of MSC in SAW devices. In this paper we investigate the efficiency of bulk wave transmission by MSC. New theory of MSC is developed that accounts for the fact that MSCs comprise finite number of conducting strips spanning over two acoustic channels. Numerical analysis based on FFT algorithm shows that bulk waves can be quite efficiently transmitted by MSC. This means particularly, that MSC can be applied to couple the surface skimming bulk waves which, having large velocity, are attractive for higher frequency SAW devices. In this case, MSCs would make it possible to shape the device frequency response by two apodized interdigital transducers (IDTs) converting electric signal to ultrasonic surface waves.

1. Introduction

Growing applications of cellular telephones and mobile communication require still wider passband and higher frequency of operation of surface acoustic wave (SAW) devices, which are commonly applied for signal filtration, both in transponders, and in hand-held phones. To keep their price low, optical photolitography is prefered for their production thus, in order to get higher frequency SAW filters, one must apply faster SAWs, leaky or surface skimming bulk waves (SSBW) which are the fastest in given piezoelectric crystal substrate.

To obtain frequency characteristic of SAW devices required in modern electronic systems, designers frequently apply multistrip couplers that make it possible to shape the frequency response of the device by two apodized interdigital transducers (IDTs). A possibility of similar construction of SAW filters exploiting SSBW or leaky waves instead of SAW would be highly appreciated by the filter designers.

The theory of MSC, developed and applied in SAW literature is the coupled modes theory [1], in plane wave approximation (which is also valid in this paper because of the assumed large aperture width of acoustic channels with respect to a wavelength of bulk waves λ). Suppose the electrodes (thin metal metal strips) of MSC cover two identical acoustic channels (Fig. 1). There are two modes in such waveguide:

— symmetric mode where SAWs in both channels have equal amplitudes and phases,

— antisymmetric mode where SAWs in both channels, having equal amplitudes, have opposite phases.

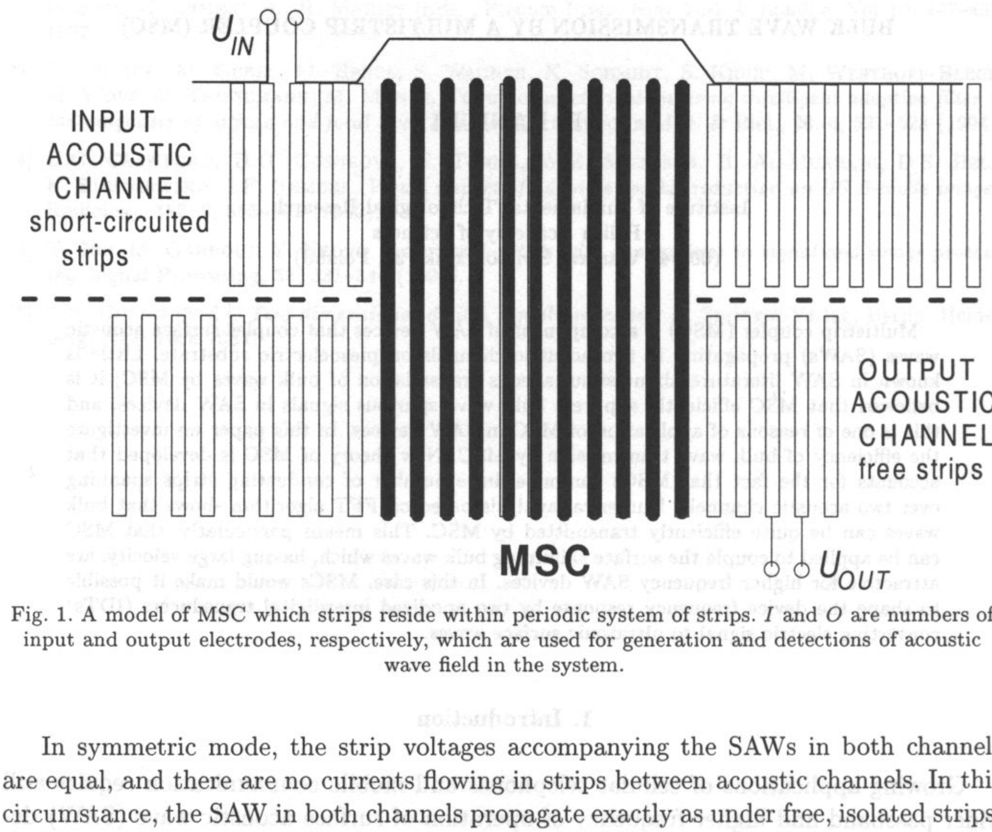

Fig. 1. A model of MSC which strips reside within periodic system of strips. I and O are numbers of input and output electrodes, respectively, which are used for generation and detections of acoustic wave field in the system.

In symmetric mode, the strip voltages accompanying the SAWs in both channels are equal, and there are no currents flowing in strips between acoustic channels. In this circumstance, the SAW in both channels propagate exactly as under free, isolated strips. And in antisymmetric mode, the strip voltages would be opposite if the corresponding strips are not connected between channels. But they are connected, thus the strip voltages must be zero and, as opposite to the previously considered case, a current flows in each strip between the channels. This means that, in both channels, the SAWs propagate in condition equivalent to short circuited, or grounded strips.

The theory of propagation of SAW under isolated, and grounded periodic strips is well developed and we can easily evaluate the SAW wave numbers in both cases which are r_v and r_o, correspondingly. These wave numbers depend on piezoelectric substrate parameters, on strip width w with respect to their period Λ, and on strip period with respect to the wavelength λ. Typically in MSCs, there are more than two strips falling in the shortest wavelength in the entire frequency band of operation of the SAW device. This is to avoid the Bragg scattering of SAWs into bulk waves. This will be assumed also in this paper, $K = 2\pi/\Lambda > 2r_o$.

The acoustic field in MSC is (Fig. 1)

$$\begin{aligned} &-\text{ in upper channel} \quad A(x) = a_v e^{-jr_v x} + a_o e^{-jr_o x}, \\ &-\text{ in lower channel} \quad A(x) = a_v e^{-jr_v x} - a_o e^{-jr_o x}, \end{aligned} \tag{1.1}$$

where a_v and a_o are amplitudes of SAWs having wave numbers r_v and r_o, correspondingly. Suppose that, at $x = 0$, $A(x) = 1$ in the upper channel, and $A(x) = 0$ in the lower channel, thus it must be $a_v = a_o = 1/2$ and, at $x > 0$, the SAW amplitudes are

$$\begin{aligned} |A(x)| &= \cos\frac{r_o - r_v}{2}x, \quad \text{in upper channel}, \\ |A(x)| &= \sin\frac{r_o - r_v}{2}x, \quad \text{in lower channel}. \end{aligned} \tag{1.2}$$

If we choose the length of MSC equal to $\pi/(r_o - r_v) = x$ then, at the output of MSC, the wave amplitudes will be

$$\begin{aligned} |A(x)| &= 0, \quad \text{in the upper channel}, \\ |A(x)| &= 1, \quad \text{in the lower channel}, \end{aligned}$$

what means that SAW is fully transmitted, by means of MSC, from the upper to the lower acoustic channels. This 0 dB MSC is the most frequently applied in SAW devices.

The theory of MSC working with SSBW cannot be such simple for one fundamental reason: there are not propagating modes of particular wave numbers. Instead, there are continuous spectra of bulk waves propagating in different directions inside the body, which spectra spans over the domain $(0, k_s)$, where k_s is the cut-off wave number of bulk waves (the backward bulk waves have the spectra spanning from 0 to $-k_s$). This means that there is not such simple representation of acoustic wave field under MSC as given in Eq. (1.1). Instead, we will have certain integral representation over the whole spectra of bulk waves propagating under MSC.

In this paper, we will adopt the theory of bulk wave excitation, propagation and detection in the system of periodic strips developed in [2]. The main objectives in developing theory of MSC working with SSBW are the following

1. evaluation of bulk wave field generated by certain electrode of the periodic system of strips that is supplied from external voltage source. This wave field "isonifies" the MSC placed in some distance from the exciting electrode (that is from the source of bulk waves),

2. evaluation of electric potentials and currents of strips that make MSC in the periodic system of strips and spanning over both coupled acoustic channels (Fig. 1),

3. and finally, evaluation of the voltage excited on certain strip residing in some distance of MSC in the other acoustic channel. This voltage is a measure of transmission of bulk waves from one to the other acoustic channels; without MSC, the acoustic field in the other channel would not exist.

2. Characterization of piezoelectric substrate

In this paper, we assume that strips of MSC are perfectly conducting and weightless, they influence the electric field on the substrate surface only, leaving it mechanically free, without any surface tractions. This results in purely electric boundary value problem to be considered in this paper, and only electric characterization of the body is required in the theory.

The useful and sufficient characterization is given by the so-called effective surface electric permittivity of the substrate which, originally introduced for the case of SAWs [3], was further generalized to account for bulk waves [4]. This is the characterization in spectral domain of wave number k of an electric wave field on the substrate surface

$$e^{-jkx}e^{j\omega t} \tag{2.1}$$

for given angular frequency ω, x is spatial coordinate on the substrate surface, along the wave propagation direction. It is assumed independent on the other coordinate on this plane. The wave field depends on the coordinate perpendicular to the surface (in depth of the body), and this dependence is fully accounted for in the planar characterization described below.

In piezoelectric body, the elastic waves are coupled to electric field. We will be particularly interested in electric field tangential to the substrate surface $E_{\|}$, and induction perpendicular to this surface $D_{\perp}$. For given $E_{\|}$, the corresponding induction in vacuum over the substrate can be easily evaluated, and it is useful to introduce the electric flux discontinuity at the substrate surface $\varDelta D_{\perp}$. In these notations, the above mentioned generalized effective surface permittivity is the following [4]

$$\varDelta D_{\perp}/E_{\|} = -j\epsilon_e \frac{k}{\sqrt{k^2}} \frac{\sqrt{k^2-k_s^2}-\beta\sqrt{k^2}}{\sqrt{k^2-k_s^2}-\alpha\sqrt{k^2}}, \tag{2.2}$$

where square roots are evaluated uniquely by applying cuts of complex k-plane as shown in Fig. 2 (in lossy media), k_s has small negative imaginary value, this makes that the corresponding branch points lay outside the real axis of k; and argumentation concerning electrostatic approximation [4] allows us to replace $\sqrt{k^2}$ with $\sqrt{k^2-o^2}$, with o small, that moves also these branch points outside the real axis of the complex k-plane.

The approximation parameters, α, β, and ϵ_e are evaluated by standard analysis of the corresponding boundary value problem for piezoelectric halfspace governed by its

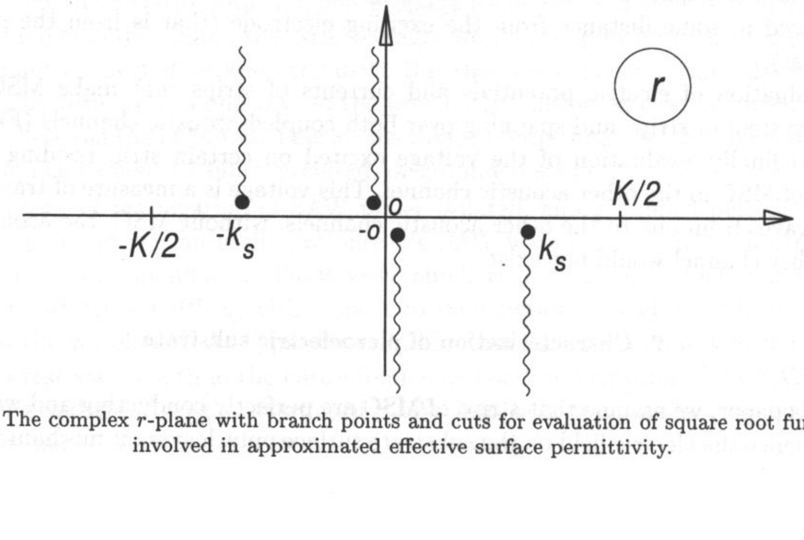

Fig. 2. The complex r-plane with branch points and cuts for evaluation of square root functions involved in approximated effective surface permittivity.

constitutive equations and the equations of motion [5]. The approximation (2.2) is the simplest one accounting for bulk waves. Generally, α and β can be complex, and $(k - k_s)^p$, $p = 1/n$, $n < 6$ can appear instead of $(k - k_s)^{1/2}$. In this paper we apply the approximation (2.2) because it is valid for most practical cases of piezoelectric substrates.

It results immediately from Eq. (2.2) that

$$\begin{aligned} k_v &= k_s/\sqrt{1-\beta^2}, \quad \text{provided that} \quad \beta > 0, \\ k_o &= k_s/\sqrt{1-\alpha^2}, \quad \text{provided that} \quad \alpha > 0, \end{aligned} \tag{2.3}$$

are the wave numbers of SAW propagating on free, and metallized substrate surface, correspondingly; the surface wave exists provided that the specified condition is satisfied.

3. Theory of periodic strips

The boundary value problem for periodic strips residing on the piezoelectric halfspace is following: find electric field in the plane of strips knowing that

$$\begin{aligned} E_{\|}(x) &= 0 \quad \text{on strips}, \\ \Delta D_{\perp} &= 0 \quad \text{between strips}, \end{aligned} \tag{3.1}$$

which are the local boundary conditions on the substrate surface. The global conditions resulting from the strip interconnections, concern voltages of strips and their total currents; they will be accounted for later below.

The convenient way of solving the above problem is to expand the wave field in the Fourier series that appear natural in periodic system of strips considered here

$$\begin{aligned} E_{\|} &= \sum_{-\infty}^{\infty} E_n e^{-j(r+nK)x}, \\ & \qquad\qquad\qquad\qquad r \in (-K/2, K/2), \\ \Delta D_{\perp} &= \sum_{-\infty}^{\infty} D_n e^{-j(r+nK)x}, \end{aligned} \tag{3.2}$$

where each pair (E_n, D_n) satisfies Eq. (2.2) taken for $k = r + nK$. In Eq. (3.2) we choose r in the first Brillouin zone to remove the representation redundancy.

The subsequent (BIS) expansion introduced in [6], in finite limits of m

$$\begin{aligned} D_n &= -j\epsilon_e \frac{1-\beta}{1-\alpha} \sum_m \alpha_m P_{n-m}(\cos\Delta), \\ E_n &= S_n \sum_m \alpha_m P_{n-m}(\cos\Delta), \quad S_\nu = \begin{cases} 1, & \nu \leq 0, \\ -1, & \nu < 0 \end{cases} \end{aligned} \tag{3.3}$$

yields following system of equations, for $n \in (-\infty, \infty)$ and for finite number of unknowns α_m

$$\sum_m \alpha_m [1 - S_{r/K+n} S_{n-m} g(r+nK)] P_{n-m}(\cos\Delta) = 0, \tag{3.4}$$

where $\Delta = \pi w/\Lambda$ and P_ν is a Legendre function of rank zero, and

$$g(k) = \frac{1-\alpha}{1-\beta}\frac{\sqrt{k^2-k_s^2}-\beta\sqrt{k^2}}{\sqrt{k^2-k_s^2}-\alpha\sqrt{k^2}}, \qquad g(\pm\infty) = 1. \tag{3.5}$$

Happily, most of these equations are satisfied identically provided that we apply approximation that $g(|k| > P) = 1$ for certain large, but finite P (see discussion in [7]).

In what follows, we asume $K > 2k_s$, and applying $P \approx 1.2k_s$ that is a reasonable value for small values of α and β, and $|\alpha-\beta| \ll 1$ for all practical piezoelectric substrates, the nonzero solution to Eqs. (3.2) is following

$$\alpha_1/\alpha_0 = \frac{1-g(r)}{1+g(r)}, \qquad r \in (0, K/2) \tag{3.6}$$

with arbitrary α_0 (it will be later evaluated from the above mentioned global boundary conditions), and for $r > 0$.

It can be verified by inspection that, for $r = -\tau < 0$, the solution to α_m is α_{1-m} obtained for $\tau > 0$ from Eq. (3.4). Indeed, applying substitutions $r \to -\tau$, $n \to -n$ and $m \to 1-m$, Eq. (3.4) transforms into

$$\sum_m \alpha_{1-m}[1-(-S_{\tau/K+n})(-S_{n-m})g(-(\tau+nK))]P_{n-m} = 0$$

because $S_{-1-n} = -S_n$, $P_{-1-n} = P_n$, and $g(-k) = g(k)$.

The strip voltage which is the integral of tangential electric field, and the strip current that is proportional to the integral of electric flux discontinuity over the strip width, can be partially evaluated by summing up the field Fourier components. This yields the potential of the strip residing at $x = 0$ [8]

$$V(r) = \alpha_0\frac{\pi}{jK\sin(\pi r/K)}V_r, \qquad V_r = \sum_m \frac{\alpha_m}{\alpha_0}P_{-r/K-m}(-\cos\Delta), \tag{3.7}$$

and for next strip, at $x = \Lambda$, this potential is $V_1(r) = V(r)\exp(-jr\Lambda)$.

For further convenience, we introduce the voltage difference between two neighbouring strips $U(r) = V - V_1$. Finally, this voltage difference U, and the current flowing to the strip residing at $x = 0$ are

$$\begin{aligned} U(r) &= \alpha_0\Lambda V_r e^{-jr\Lambda/2}, \\ I(r) &= \alpha_0\Lambda\omega\epsilon I_r, \qquad I_r = \sum_m \frac{\alpha_m}{\alpha_0}P_{-r/K-m}(\cos\Delta) \end{aligned} \tag{3.8}$$

($\epsilon = \epsilon_e(1-\beta)/(1-\alpha) \approx \epsilon_e$). In the above equations, U and I depend on spectral variable $r \in (-K/2, K/2)$, and α_0 that is a function of r to be evaluated later.

The discrete set of potentials of electrodes V_m or their differences $U_m = V_m - V_{m+1}$ defined above, is the inverse Fourier transform of $U(r)$ [8]

$$U_m = \frac{1}{K}\int_{-K/2}^{K/2} U(r)e^{-jrm\Lambda}dr. \tag{3.9}$$

For known set of U_m, one can evaluate $\alpha_0(r)$. Indeed, applying

$$\alpha_0(r) = \frac{1}{\Lambda V_r} e^{jr\Lambda/2} U_m e^{jrm\Lambda} \tag{3.10}$$

in Eq. (3.8), and then applying Eq. (3.9) for k-th strip, we obtain

$$U_k = U_m \int_{-K/2}^{K/2} e^{jr(k-m)\Lambda} dr/K = \delta_{km} U_m .$$

The same inverse Fourier transform allows us to evaluate I_n for known set of U_m, included in α_0 evaluated above

$$\begin{aligned} I_n &= y_{nm} U_m , \qquad y_{nm} = y_{n-m} , \\ y_k &= \omega\epsilon \int_{-K/2}^{K/2} R(r) e^{-jr(k-1/2)\Lambda} dr/K, \\ R(r) &= \frac{I_r}{V_r} = \frac{\sum_m \alpha_m P_{-m-r/K}(\cos\Delta)}{\sum_m (-1)^m \alpha_m P_{-m-r/K}(-\cos\Delta)} . \end{aligned} \tag{3.11}$$

Analogously we can solve a reciprocal problem, for given set of currents flowing to strips I_n and for searched voltage differences U_m

$$\begin{aligned} U_m &= z_{mn} I_n , \qquad z_{mn} = z_{m-n} , \\ z_k &= \frac{1}{\omega\epsilon} \int_{-K/2}^{K/2} \frac{1}{R(r)} e^{-jr(k+1/2)\Lambda} dr/K, \end{aligned} \tag{3.12}$$

with $R(r)$ as evaluated in the equation above; it results from the discussion below Eq. (3.6) that

$$R(-r) = -R(r). \tag{3.13}$$

4. Circuit theory of MSC

Figure 1 presents a model of MSC which electrodes are placed inside the periodic system of strips, just to make Eqs. (3.11) and (3.12) directly applicable to the model. In practical MSC, there are not strips ouside it, and electric field is slightly different at its edge strips. There are many strips in MSC however, and except the edge strips, they are correctly described by the above mentioned equations, thus small inaccuracy concerning edge strips does not change much the MSC performance.

Accordingly to Fig. 1, the current excited in this part of MSC strips which reside in the upper acoustic channel is

$$I_n = y_{nI} U_I + y_{nm} U_m , \qquad n, m \in \text{MSC}, \tag{4.1}$$

where U_I is the voltage difference applied to some strips outside MSC in the upper channel to excite acoustic waves that propagate towards the MSC, and U_m, I_n are voltage difference on MSC strips and currents flowing to the upper part of MSC strips. Because all strips are isolated, the corresponding current flowing to the lower part of MSC strips (the parts residing in the lower acoustic channel) is $-I_n$, and U_m is the same in both parts of strips (except, may be, the edge strips, but we neglect this). Thus the corresponding equation for the lower part of MSC strips is

$$U_k = z_{kn}(-I_n), \tag{4.2}$$

where k can take value outside MSC, for example U_O is the output voltage difference in the lower channel, or $k \in$ MSC, in this case U_k is the same as involved in Eq. (4.1). Both the above equations give the system of equations

$$[\delta_{nk} + y_{nm} z_{mk}] I_k = y_{nI} U_I\,, \tag{4.3}$$

that allows us to evaluate the MSC strip currents I_k, and subsequently to evaluate the transmitted signal to the output electrodes in the lower channel placed somewhere outside the MSC

$$U_O = z_{On} [\delta_{nk} + y_{nm} z_{mk}]^{-1} y_{kI} U_I\,, \tag{4.4}$$

δ_{mn} is the Kronecker delta, and there are sums over repeating indices, over the strips belonging to the MSC.

The output signal U_O would be zero if there was not MSC making coupling of both upper and lower acoustic channels. However, certain part of U_O is transmitted by means of strip mutual capacitances, which part of the signal is not interesting from SAW devices point of view. It is instructive however to discuss it in some details.

Purely electrostatic transmission will take place when the system is placed on dielectric substrate, which effective permittivity is that given in Eq. (2.2) with parameters $\alpha = \beta$. In this case [9]

$$R(r) = S_r P_{-r/K}(\cos\Delta)/P_{-r/K}(-\cos\Delta) \tag{4.5}$$

which is 1 for $\cos\Delta = 0$ applied here for simplicity. It is easy to evaluate the "capacitive" parts of y and z

$$\begin{aligned} I_n &= y^C_{n-m} U_m\,, \qquad y^C_k = -j\frac{\omega\epsilon}{\pi}\frac{1}{k-1/2}\,, \\ U_n &= z^C_{n-m} I_m\,, \qquad z^C_k = -j\frac{1}{\pi\omega\epsilon}\frac{1}{k-1/2}\,. \end{aligned} \tag{4.6}$$

It will be shown in next sections that the "acoustic" part of y or z are much smaller than y^C and z^C for not excessively large k, that is for strips of MSC. This means particularly, that for MSC strips $y_{mn} \approx y^C_{m-n}$ and $z_{nm} \approx z^C_{n-m}$, and subsequently that, for MSC having not too few strips (typical MSCs have 50 or more strips that is sufficient number for current considerations),

$$[y_{nm} z_{mk}] \approx [y^C_{nm} z^C_{mk}] \approx [\delta_{nk}] \tag{4.7}$$

and the inverse matrix involved in Eq. (4.4) can be put approximately equal half of the identity matrix (for typical MSC; if this is not the case, there will be certain "structural" losses in signal transmission by MSC, including losses caused by radiation of bulk waves in backward direction).

Summarizing, we see that it is easy to evaluate purely electrostatic transmission of the signal by MSC, however in practical cases the distance between input and output electrodes are chosen sufficiently large, or certain other isolation of these electrodes is applied to avoid such signal. In the rest of the paper we will neglect this signal by substracting "electrostatic" part from $R(r)$ in integrals describing y_{kI} and z_{On}.

5. Piezoelectric coupling of SAWs

This section is introduced in this paper to clarify the concept of signal transmission coefficient of MSC. The ratio U_O/U_I cannot be applied as a measure of this transmission because, due to weak piezoelectric coupling of elastic waves to electric field in piezoelectrics, even for known 0 dB coupling MSC, $U_O/U_I \ll 1$ (as shown below for the SAW case, this ratio is proportional to the piezoelectric coupling coefficient of SAW).

Applying, for simplicity reasons, that $\cos\Delta = 0$ and $K \gg r_o$, we have [8]

$$R(r) = S_r \frac{r^2 - r_v^2}{r^2 - r_o^2},$$

and without "electrostatic" contribution it is

$$R(r) - S_r R(\infty) = S_r \frac{r_o^2 - r_v^2}{r^2 - r_o^2},$$

which substituted into Eq. (3.11), allows us to evaluate the signal transmission between I-th and O-th strips in the upper acoustic channel by "acoustic" means

$$I_O = \omega\epsilon U_I \int_{-K/2}^{K/2} S_r \frac{r_o^2 - r_v^2}{r^2 - r_o^2} e^{-jr(O-I)\Lambda} dr/K.$$

For $O - I > 0$, and K sufficiently large, we can extend the integration limits to $\pm\infty$. Neglecting integration resulting from branch points at $\pm o$ ($S_r = r/\sqrt{r^2 - o^2}$) and using Jordan's lemma in the lower plane of complex r, we finally obtain

$$I_O/U_I = -j2\pi\omega\epsilon \frac{r_o^2 - r_v^2}{2r_o K} e^{-jr_o(O-I-1/2)\Lambda}. \tag{5.1}$$

Similar considerations allow us to evaluate the signal transmission between I-th and O-th strips in the lower system of isolated strips

$$U_O/I_I = -j\frac{2\pi}{\omega\epsilon} \frac{r_v^2 - r_o^2}{2r_v K} e^{-jr_v(O-I-1/2)\Lambda}. \tag{5.2}$$

Finally we arrive at the following averaged nondimensional parameter that characterizes the electric signal transmission, by "acoustic" means of elastic waves in piezoelectrics, between equally distant strips in upper, and lower channels (in short-circuited, and free systems of strips, correspondingly)

$$\kappa = \left\| \left(\frac{I_O}{U_I}\right)_{\text{short}} \left(\frac{U_O}{I_I}\right)_{\text{free}} \right\|^{1/2} = (r_o - r_v)\Lambda \tag{5.3}$$

where indices "short" and "free" mark the cases of evaluation of corresponding terms. The term $(r_o - r_v)/r_o$ is known in SAW literature as the piezoelectric coupling of SAWs in periodic system of strips; it is usually small quantity, of an order of 1% or less. The above signal transmission coefficient is proportional to this coupling coefficient.

This result shows that, when evaluating strip voltages and currents of MSC strips which are not very distant, we can neglect contributions of elastic waves, replacing y, z by y^C, z^C.

6. MSC for SAWs

Equations (5.1), (5.2), taken for n, m instead of O, I, and applied in Eq. (4.4) with the inverse matrix approximated by the identity matrix divided by 2, allow us to obtain the following relation for signal transmission between upper and lower acoustic channels by means of elastic waves and the MSC

$$\begin{aligned} U_O/U_I &= \frac{1}{2} z_{On} y_{nI}, \\ |U_O/U_I| &= \frac{1}{2}\left[(r_o - r_v)\Lambda\right]^2 \left|\sum_{\text{MSC}} e^{-j(r_o - r_v)n\Lambda}\right| \approx (r_o - r_v)\Lambda \left|\sin\frac{r_o - r_v}{2} N\Lambda\right|, \end{aligned}$$

where N is the number of strips in MSC.

In relation to the average transmission coefficient κ, the SAW transmission by MSC is

$$T = \frac{1}{\kappa}|U_O/U_I| = \left|\sin\frac{r_o - r_v}{2} N\Lambda\right|, \tag{6.1}$$

which T characterizes only the MSC, and it is independent of parameters of piezoelectric substrate. The length of MSC is $x = N\Lambda$, and if x is chosen such that $\sin x(r_o - r_v)/2 = 1$ than $T = 1$ and we have 0 dB MSC, in agreement with coupled modes theory presented in Introduction.

7. Transmission of bulk waves

7.1. The wave field of bulk waves

As opposite to the SAW case, in this section we consider somewhat artificial piezoelectric substrate that does not support SAWs at all; only bulk waves can propagate in the system. In such cases $\alpha < 0$ and $\beta < 0$ in Eq. (2.2) and in consequence, $R(r)$ has no a pole neither a zero in the integration domain in Eq. (3.11) and (3.12). This allows us to evaluate these integrals numerically, by applying the algorithm of fast Fourier transform, for example

$$\int_{-K/2}^{K/2} R(r)e^{-jr(k-1/2)\Lambda}dr/K$$

$$\approx \frac{1}{M}\sum_{l=0}^{M-1}\left\{\begin{array}{ll} R(Kl/M)e^{j\pi l/M}, & l < M/2 \\ R(K(M-l)/M)e^{-j\pi l/M}, & l \geq M/2 \end{array}\right\} e^{-j2\pi kl/M}. \qquad (7.1)$$

The transmission of bulk wave signal from I-th strip to O-th strip in the same channel can be evaluated using Eqs. (3.11) and (3.12), with $R(r)$ appropriately modified to elliminate the signal resulting from purely electrostatic interactions between strips. This can be done by substracting term evaluated after substitution $\alpha = \beta = 0$ in Eq. (2.2).

Figure 3 shows the evaluated $|I_O/U_I|$ in the upper channel (with short circuited stips, curve a), and $|U_O/I_I|$ in lower channel of isolated strips (curve b), for different values of $O - I > 0$ and for $K = 2.5k_s$. These bulk wave signals decay with the distance between strips ($|O-I|\Lambda$). This is due to bulk wave diffraction making the acoustic field weaker at the substrate surface with growing distance from the source, which surface field interact with the output strip.

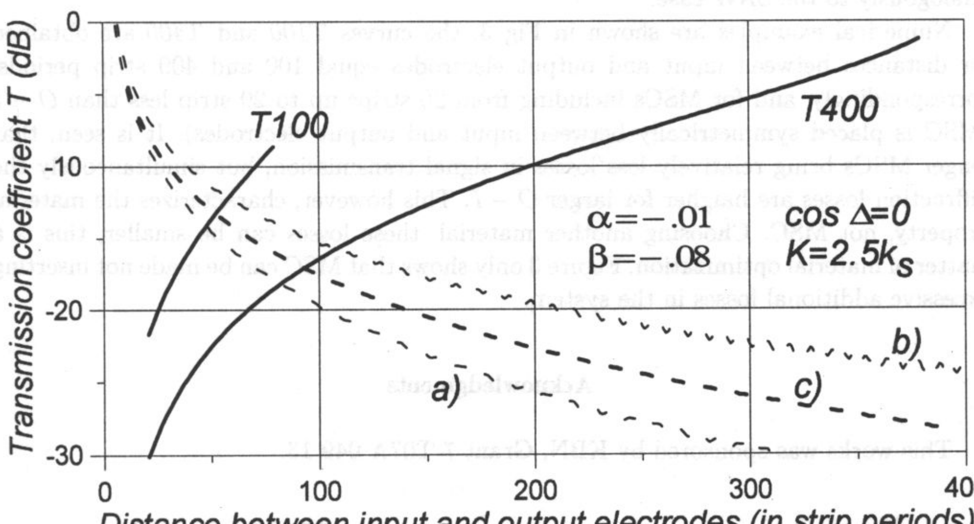

Fig. 3. Bulk wave transmission in acoustic channels of a) short-circuited strips and b) isolated strips, and c) their average κ. Curves present relative decay of the detected signal dependent on the distance (in strip periods) between input and output electrodes. *T100* and *T400* are the transmission coefficient of bulk waves by MSC, as dependent on the number of strips it includes, for $O - I = 100$ and $O - I = 400$ correspondingly; MSC includes from 20 to $O - I - 20$ strips placed symmetrically between I-th and O-th strips.

Analogously to the SAW case, we define the coupling coefficient of bulk waves by Eq. (5.3), with I_O/U_I and U_O/I_I evaluated numerically as discussed above (curve c).

This "coupling coefficient" κ includes the effect of SAW diffraction between input and output strips thus, when applied in the relation for bulk wave transmission by MSC, both these effects will be removed from the transmission coefficient characterizing only MSC (and not piezoelectric coupling coefficient or bulk wave diffraction), analogously to the SAW case considered previously.

7.2. Relative bulk wave transmission by MSC

We again assume weak piezoelectric substrate and sufficiently long MSC to make the inverse matrix in Eq. (4.4) equal to $[\delta_{nk}]/2$. Subsequently neglecting purely electrostatic interaction between input and output electrodes and MSC strips, we obtain

$$U_O/U_I = \sum_{\text{MSC strips}} z_{O-m} y_{m-I} \tag{7.2}$$

where y and z are evaluated numerically using FFT.

Finally, the transmission coefficient of bulk waves by MSC is defined by

$$T = \frac{1}{\kappa}|U_O/U_I|, \qquad \kappa = \left\| \left(\frac{I_O}{U_I}\right)_{\text{short}} \left(\frac{U_O}{I_I}\right)_{\text{free}} \right\|^{1/2}, \tag{7.3}$$

analogously to the SAW case.

Numerical examples are shown in Fig. 3: the curves *T100* and *T400* are obtained for distances between input and output electrodes equal 100 and 400 strip periods, correspondingly, and for MSCs including from 20 strips up to 20 strip less than $O - I$ (MSC is placed symmetrically between input and output electrodes). It is seen, that longer MSCs bring relatively less losses in signal transmission, but simultaneously the diffraction losses are haigher for larger $O - I$. This however, characterizes the material property, not MSC. Choosing another material, these losses can be smaller, this is a matter of material optimization. Figure 3 only shows that MSC can be made not inserting excessive additional losses in the system.

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