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TRANSVERSAL EQUIVALENT CIRCUIT MODEL OF A FILTER WITH A SURFACE ACOUSTIC WAVE

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The paper suggests a way of analysis of the operation of the SAW oscillator basing on a computer simulation programme of the electronic systems of the SPICE type. In compliance with the requirements of this programme, an equivalent diagram of a surface wave filter has been presented in a form convenient for computer analysis.

1. Introduction

A filter with an acoustic surface wave (SAW) in the feed back loop of a wide-band amplifier consists of an oscillator applied in a gas analyser of the SAW type (Fig. 1). Various research centres all over the world try to design new kinds of gas sensors. Similar investigations on the design of an apparatus for the SAW method [1, 2, 3] have also been carried out at the IFTR in Warsaw.

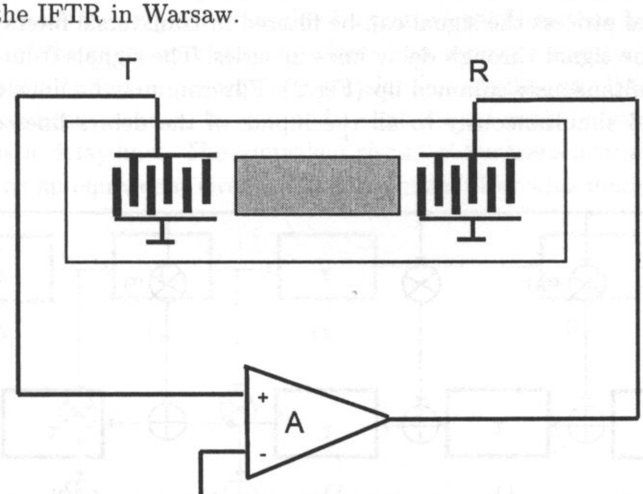


Fig. 1. Oscillator with an acoustic surface wave and sensor layer.

The sensor layer placed on the surface of an acoustic wave-guide changes its physical properties (mass and electric conductivity) due to its interaction with the surrounding atmosphere, and affects a change in the velocity of wave propagation, and thereby a change in the frequency of oscillations in the system. A bandpass filter consists of a transmitting transducer (T) with N electrodes and a receiving transducer (R) with M electrodes. Adapting them to the structure of the gas sensor, the transducers are produced as not-weighted transducers with constantly overlapping electrodes. The transducers are separated from each other by an acoustic delay line of length L and provided with a sensoric layer on the surface.

The paper suggests an analysis of the operation of the oscillator basing on a computer simulation programme of electronic systems of the SPICE type. In compliance with the requirements of this programme, an equivalent diagram of a surface wave filter has been presented in a form convenient for the computer analysis. The necessity of analysing the real system of an oscillator results from the necessity of choosing optimal values of its elements and systems matching the impedance of the acoustic-wave transducers with the impedance of the amplifier, as well as of applying optimal levels of supply voltages. Such an analysis aims at the achievement of a single-mode operation of the oscillator in a possibly wide frequency band [4].

The way of analysing an interdigital transducer as well as a whole SAW filter basing on the Smith's equivalent system [7, 8] (cross-field model), derived from the Mason's model for bulk wave transducers [9] makes it possible to determine the fundamental parameters of the transducer and filter including the amplitudinal and phase characteristics; it requires, however, numerical calculations, and the equivalent diagram obtained in thus way does not suit the purpose of constructing a model of an oscillator which might be applied in simulative analysis by means of the SPICE programme. In the equivalent system the elements of controlled sources of voltage and current cannot be modelled in the SPICE environment.

In the physical process the signal can be filtered in transversal filters [5, 10], i.e. the propagation of the signal through delay lines in series. The signals from the outputs of the lines are simultaneously summed up (Fig. 2). Filtering may be done also by passing the filtered signal simultaneously to all the inputs of the delays lines connected in a

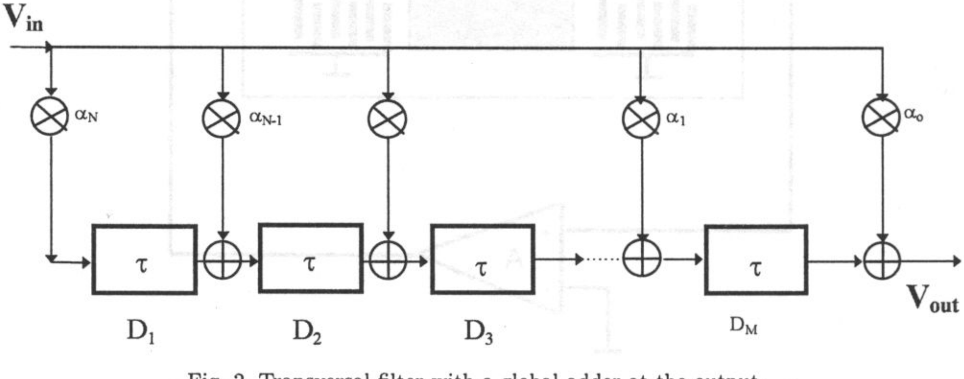


Fig. 2. Transversal filter with a global adder at the output.

cascade (Fig. 3) [5, 10]. The filtered output signal is obtained at the output of the last delay line. A filter of the first kind is called a filter with a global adder at the output, a filter of the second kind – a filter with many adders. With respect to their structure, those are filters with a finite impulse response. The number of delaying elements applied in a given filter determines the length of the impulse response. The response time of a transversal filter results from the choice of the number of delay lines, the delay τ_n and the weight factors α_n.

The signal at the output of the system presented in Fig. 2 takes the following form:

$$V_{\text{out}}(f) = V_{\text{in}}(f) \sum_{n=1}^{N} \alpha_n \exp(-i2\pi f \tau_n), \tag{1}$$

whereas the frequency response looks as follows:

$$H(f) = \frac{V_{\text{out}}(f)}{V_{\text{in}}(f)} = \sum_{n=1}^{N} \alpha_n \exp(-i2\pi f \tau_n). \tag{2}$$

At first, the required considerable length of the delay lines produced by a concentric cable impeded the application of transversal filters. This problem could be solved by the application of acoustic delay lines in which the required retardations can be achieved at much shorter distances (small velocity of propagation of the acoustic wave if compared with the velocity of electromagnetic waves). Modern transversal filters are realised in digital circuits' [11].

2. Interdigital transducers used as transversal filters

The interdigital transducer [6, 7, 8] applied for the generation and receiving of a surface acoustic wave is a typical example of a transversal filter that is distributed over a given surface of the medium of propagation of an acoustic wave in the piezoelectric body. In the transmitting transducer the input signal is applied simultaneously to all the electrodes. It has been assumed in the model that the intervals between the electrodes consists of acoustic delay lines. The equivalent circuit of the transmitting transducer may be represented as an equivalent circuit of a transversal filter with many adders (Fig. 3).

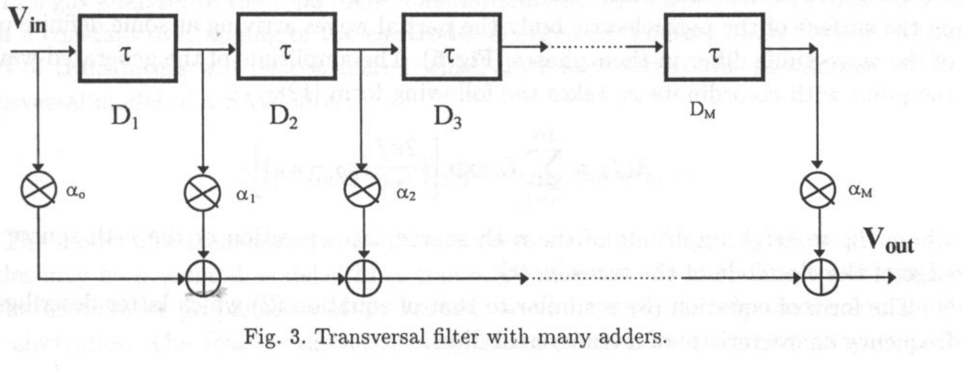


Fig. 3. Transversal filter with many adders.

In the receiving transducer of the surface wave, the acoustic signal is converted into an electric one, while the acoustic signal (of the elastic wave) gradually passes under the set of electrodes of the transducer. This process corresponds to the passing of the signal through the delay lines connected in series. The output signal, which is the sum of delayed signals of the respective electrodes, is collected from joint rails clenching the interdigital electrodes of the transducer. These rails play the role of a global adder of the signal. The equivalent circuit of a transversal filter with a global adder (Fig. 2) may, therefore, represent the equivalent circuit of the receiving transducer.

The input voltage applied to the transducer is the source of the distribution of the electric field in a shape that is similar to Dirac's delta function distributed along the edges of the electrodes (Fig. 4). This description complies with the delta-function model of the transducer known from literature [12].

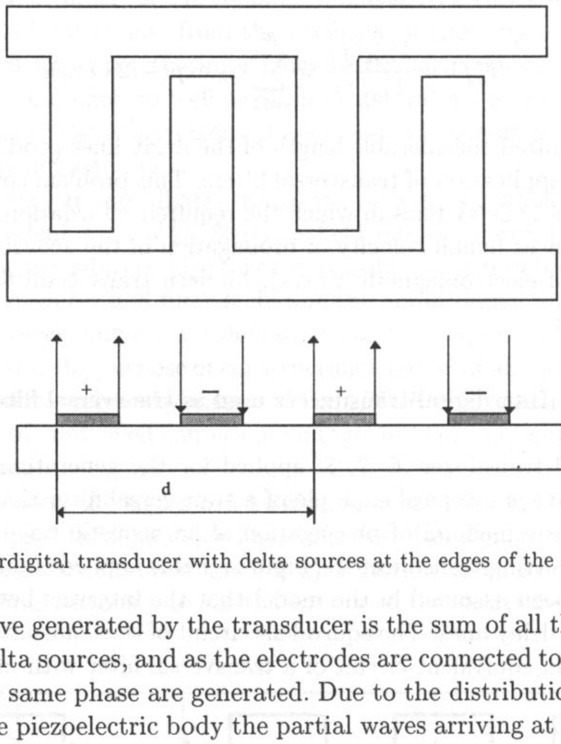


Fig. 4. Interdigital transducer with delta sources at the edges of the electrodes.

The acoustic wave generated by the transducer is the sum of all the waves generated by the individual delta sources, and as the electrodes are connected to a common rail, the partial waves of the same phase are generated. Due to the distribution of the electrodes on the surface of the piezoelectric body the partial waves arriving at some definite point of the wave-guide differ in their phases (Fig. 5). The amplitude of the generated wave at the point with co-ordinate x_0 takes the following form [12]:

$$A(f) = \sum_{n=1}^{2N} I_n \exp\left[i\frac{2\pi f}{v}(x_0 - x_n)\right] \tag{3}$$

where: $I_n = \pm|I_n|$ amplitude of the n-th source, x_n – position of the n-th source (the edge of the electrode of the transducer).

The form of equation (3) is similar to that of equation (2) which latter describes the frequency characteristic of a transversal filter.

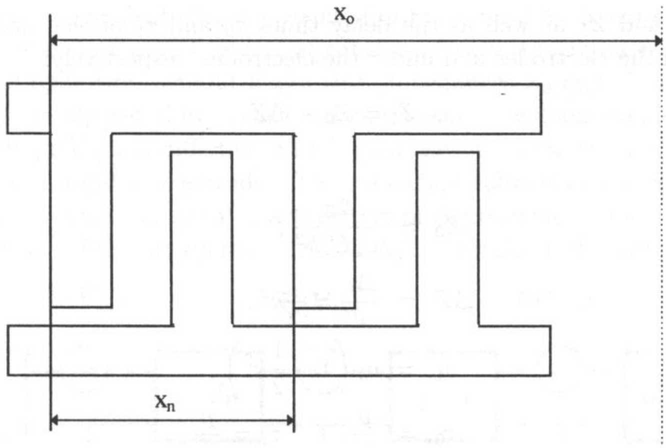


Fig. 5. Surface acoustic wave transducer.

After the acoustic wave as a mechanical perturbation of the piezoelectric surface, has reached the receiving transducer, it is detected by respective electrodes of this transducer, while the wave is passing beneath it. The receiving transducer, probes so to say, the proceeding acoustic wave at each edge of the electrode, the result being summed up on common rails. The frequency characteristic of a SAW filter $H(f)$ is like that [12]:

$$H(f) = \left[\sum_{n=1}^{2N} I_n \exp\left(-i\frac{2\pi f}{v}x_n\right)\right]\left[\sum_{m=1}^{2M} I_m \exp\left(i\frac{2\pi f}{v}y_m\right)\right], \tag{4}$$

where: $2N$ – number of sources in the transmitting transducer, $2M$ – number of sources in the receiving transducer, x_n, y_m – position of the n-th and m-th edge (of the source) in the transmitting and receiving transducers, respectively.

The expressions describing the frequency characteristics of a SAW filter are analogical to those of a transversal filter. Thus, an equivalent model of the SAW filter may be suggested in the shape of the model of a transversal filter. The equivalent circuit consists of a model of the transmitting transducer, the receiving one and the delay line corresponding to the path of propagation of an acoustic wave between the two transducers.

In a gas analyser of the type SAW, unapodized transducers of acoustic surface waves with a constant overlapping of the electrodes are applied. Further on, a filter with that kind of transducers will be considered which does not restrict the possibilities of the transversal model of a SAW filter.

2.1. Model of a transmitting transducer

The model of a transducer with $2N$ sources distributed along the edges of the electrodes may be presented as delay lines connected in a cascade modelling the fragments of the medium of propagation of an acoustic wave between the electrodes and under the electrodes. The feature distinguishing the respective lines is their characteristic impedance Z_0 and Z_1 as well as the delay times τ_0 and τ_1 of the medium of propagation between the electrodes and under the electrodes, respectively:

$$Z_1 = Z_0 - \Delta Z \tag{5}$$

where:

$$Z_0 = \frac{2\pi}{\omega_0 C_s k^2}, \tag{6}$$

$$\Delta Z = \frac{\Delta v}{v} = \frac{1}{2}k^2, \tag{7}$$

$$v_1 = v_0\left(1 - \frac{1}{2}k^2\right), \tag{8}$$

$$\tau_0 = \frac{p}{v_1}, \qquad \tau_1 = \frac{p}{v_0} \tag{9}$$

k – coefficient of electromechanical coupling of the medium of propagation, C_s – static capacitance of the electrode [6], p – width of the electrode, ω_0 – resonance frequency ($\omega_0 = 2\pi v_0/4p$), v_0, v_1 – velocity of propagation of the acoustic wave between the electrodes and under the electrodes, respectively.

The assumption that the velocity of the wave propagation between the electrodes differs from the that of the propagation under the electrodes of the transducer allows us to take into consideration the phenomenon of the reflection of the acoustic wave from the transducer. This effect depends on the value of the coefficient of electromechanical coupling of the piezoelectric medium. It is of essential importance in the case of lithium niobate, which is characterised by a high coefficient of electromechanical coupling, whereas in a quartz medium it is rather low.

Taking into account the distribution of the electric field on the edges of the electrodes (Fig. 4), the equivalent circuit of one section of the transmitting transducer is presented in Fig. 6.

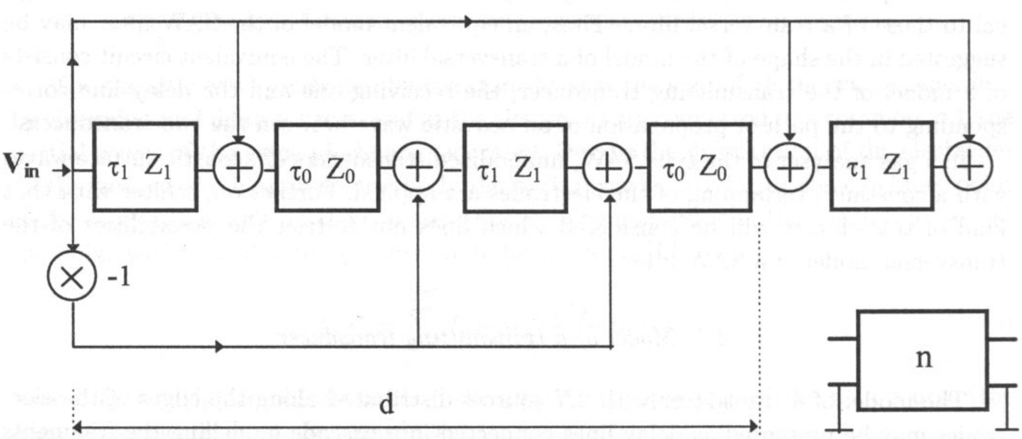


Fig. 6. Equivalent circuit of a section of the transmitting transducer in the form of a filter with many adders (the length d corresponds to one pair of electrodes).

The idea of the equivalent circuit presented in Fig. 6 can be simulated in the SPICE programme as shown in Fig. 7. This circuit consists of ideal transmission lines connected in a cascade and operating without losses with impedances Z_0 and Z_1 and delay times τ_0 and τ_1. They are separated from each other by means of voltage sources E controlled by the input voltage V_{in} and differing in their polarisation due to the different potentials of the positive and negative electrodes. The controlled voltage sources permit a simple realisation of the transmittance (3). Each section of the transducer may be defined as a subcircuit by means of the instructions: *.SUMCKT ... ENDS* in the SPICE programme.

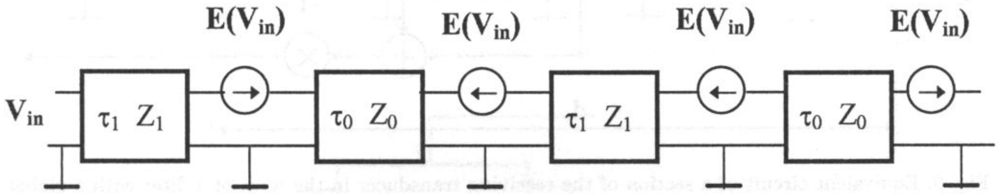


Fig. 7. Equivalent circuit of one section of the transmitting transducer in SPICE.

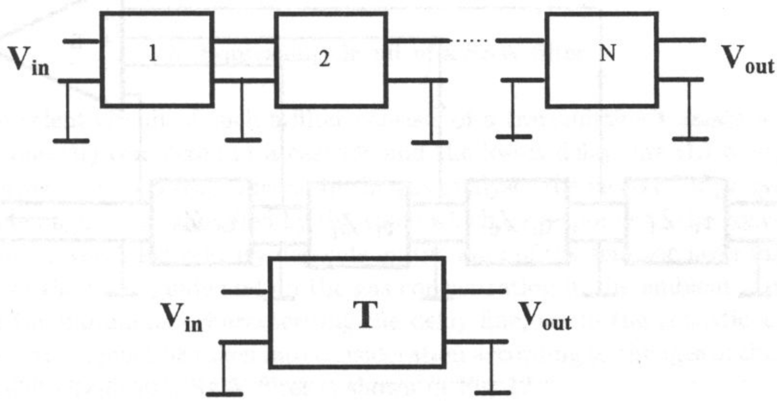


Fig. 8. Equivalent circuit of a transmitting transducer.

A transmitting transducer consisting of N sections is shown in Fig. 8.

2.2. Model of the receiving transducer

The equivalent circuit of the receiving transducer, which would reproduce the physical reality, is a system with a global adder as shown in Fig. 2. Taking into account the polarisation of the electrodes, the equivalent system of one section of the transducer looks like that presented in Fig. 9. The idea of such a system may be simulated in the SPICE programme in the form shown in Fig. 10.

Besides the delay lines, the system comprises a controlled multi-dimensional voltage source summing up the voltages from the outputs of the individual lines. Each section

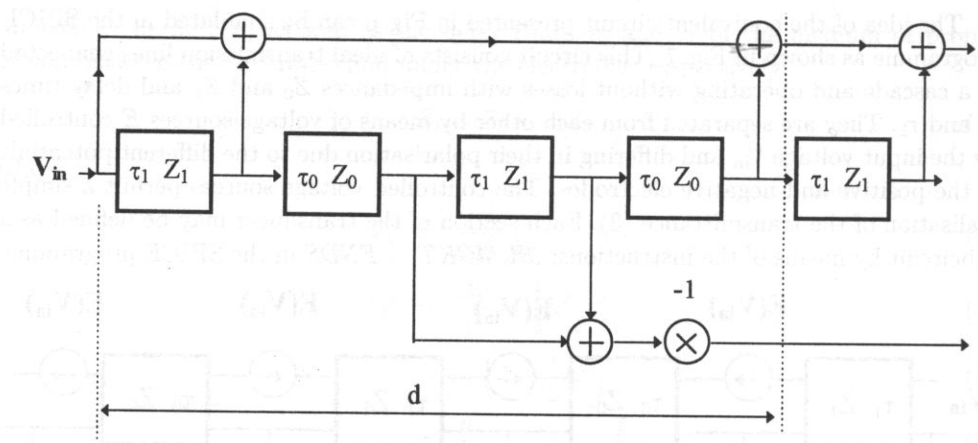


Fig. 9. Equivalent circuit of a section of the receiving transducer in the form of a filter with a global adder (the length d corresponds to one pair of electrodes).

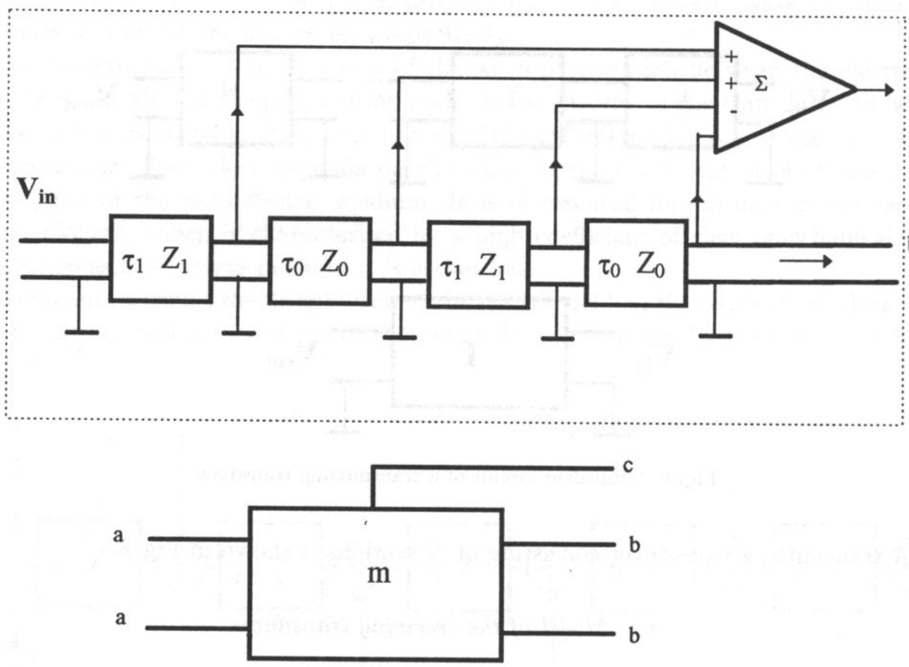
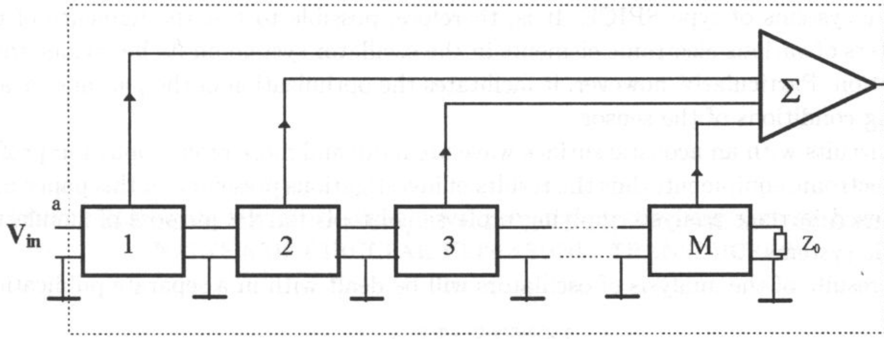


Fig. 10. Equivalent circuit of one section of a receiving transducer in SPICE.

of the transducer may be defined as a subcircuit in compliance with the instructions *.SUMCKT ... ENDS* in the SPACE programme.

A receiving transducer consisting of M sections has been presented in Fig. 11.



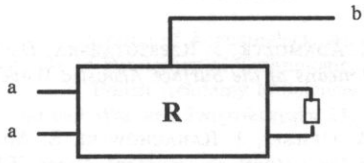


Fig. 11. Equivalent circuit of a receiving transducer in SPICE.

3. Equivalent circuit of a SAW filter

The equivalent circuit of such a filter consists of a transmitting transducer (T) and a receiving one (R) connected in a cascade and the losses delay line (L) modelling the distance between the two transducers. In the gas analyser the velocity of propagation in the delay line ought to be modified by the value which corresponds to the concrete state of the sensoric layer. This velocity depends on the kind of the sensoric layer imposed on the surface of the wave-guide and on the gas concentration in the ambient atmosphere. Changes of the parameters characterising the delay line, i.e. in the acoustic impedance and the delay time, must be taken into consideration according to changes of this velocity. The equivalent circuit of a SAW filter is shown in Fig. 12.

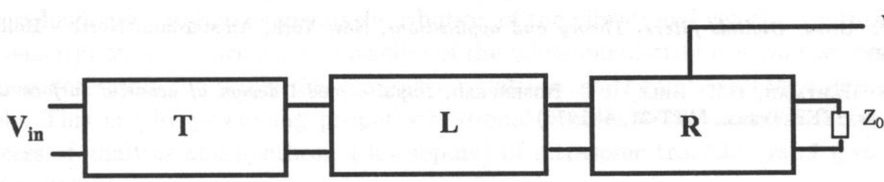


Fig. 12. Equivalent circuit of a SAW filter.

4. Conclusions

The paper deals with an equivalent transversal circuit of a SAW filter. Such a description of the filter makes it possible to analyse an oscillator system with an acoustic surface wave making use of a professional programme of a computer-aided simulation of electronic systems of type SPICE. It is, therefore, possible to test the influence of the parameters of various electronic elements in the oscillator system on its behaviour while in operation. Particularly, however, it facilitates the optimization of the parameters and operating conditions of the sensor.

Subcircuits with an acoustic surface wave are more and more often applied in professional electronic equipment; thus the results of investigations presented in this paper may be also used in their analysis applying professional tools for the purpose of simulating electronic systems.

The results of the analysis of oscillators will be dealt with in a separate publication.

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