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ACOUSTIC PROPERTIES OF BIOLOGICAL SUSPENSIONS WITH NON-SPHERICAL SUSPENDED DROPLETS

J. LEWANDOWSKI

Institute of Fundamental Technological Research,
Polish Academy of Sciences
(00-049 Warsaw, Świętokrzyska 21, Poland)

A theoretical study is made of the propagation properties of a suspension of viscous liquid droplets in a fluid medium with low viscosity. The droplets are of the form of oblate ellipsoids; the values of the material, structure and compositions parameters of the suspension are that of human blood. From the presented results of the analysis it can be seen that the propagation velocity and attenuation of ultrasound strongly depend on the blood composition, mechanical properties of the blood components as well as the ultrasonic frequency.

1. Introduction

In many areas of research and in engineering application are of interest the effective dynamic properties of some types of suspensions and emulsions. These properties are related to the acoustic (ultrasonic) wave velocities and attenuation (propagation parameters) in the materials being of interest and their structure. Therefore, some dynamic properties and structure parameters of suspensions and emulsions can be estimated on the basis of ultrasonic measurements. For ultrasonic waves to be used for this purpose, it is necessary to establish the factors which influence ultrasonic propagation in these inhomogeneous media and to relate the measurable ultrasonic propagation parameters as well as effective dynamic elastic constants of the media to the physical properties of medium components, their concentrations, size and shape. There are a variety of theoretical formulations that describe ultrasonic propagation in heterogeneous media. These differ from one another by the mathematical approaches used in their derivation and their underlying physical assumptions. However, analytical studies on the wave propagation through such composite materials, if they are closely related to the wave scattering theory, do not achieve formulae explicitly expressing the propagation properties in terms of physical properties of the constituents of the medium considered and its structure. Even in the best situation such analytical studies often lead to more or less complicated systems of algebraic equations for complex quantities which, although they have perfectly clear physical meanings, can not in general be calculated analytically from these equations. Consequently, results obtained in the form of the equation systems are not as satisfactory as could be hoped for; nevertheless they are of value because they can be thought of as supplying an algorithm for a numerical estimate of the propagation properties of the inhomogeneous material. Such a situation is presented in this paper where ultrasonic scattering was considered in the long-wavelength approximation and the BERRYMAN'S [1] self-consistent method of estimating effective dynamic elastic constants was used to identify the factors which influence ultrasonic propagation properties of blood.

2. Theoretical preliminaries

In the mathematical development which follows, it has been assumed:

1. That the effective propagation properties of blood can be deduced, in the long-wavelength approximation, from the Navier–Stokes equation of motion for a homogeneous isotropic viscous liquid called the equivalent homogeneous liquid.

2. That the dynamic properties of the equivalent liquid can be characterized by the effective dynamic material parameters: the density ϱ^*, and viscosities η^* (the dynamic viscosity) and ξ^* (the "second viscosity").

According to the two-phase model, blood may be regarded as an isotropic suspension consisting of the plasma liquid with low viscosity, in which are dispersed inclusions in the form of oblate ellipsoids (red cells) with random orientation made of a liquid with high viscosity. Throughout the paper, the effective material parameters of the suspension (blood) as a whole are labelled by the asteriks. Similarly, all the abbreviations with the sub- or superscripts f and p denote quantities referred to the isotropic suspending and suspended material, respectively. Finally, the sub- or superscripts s denote quantities referred to an isotropic solid material.

Due to the presence of acoustic field in the blood there exists a displacement field, which can be expressed as

$$\mathbf{u}^* = \frac{1}{i\omega}\mathbf{v}_0^* e^{i\omega t}, \tag{2.1}$$

where ω is the angular ultrasonic frequency and $\mathbf{v}_0^*$ is a complex quantity.

Then the velocity $\mathbf{v}^*$ and the acoustic pressure, Δp^*, is given by

$$\mathbf{v}^* = \mathbf{v}_0^* e^{i\omega t}, \qquad \Delta p^* = \frac{i}{\omega} K^* \operatorname{div}\mathbf{v}^*, \tag{2.2}$$

where K^* is the effective bulk modulus (adiabatic compressibility). If the above relations are applied to the description of the ultrasonic wave propagation in blood being represented by the homogeneous equivalent liquid, the equation of motion for blood (Navier–Stokes equation) will then become

$$-\mathbf{v}^* = (k_l^*)^{-2} \operatorname{grad}\operatorname{div}\mathbf{v}^* + (k_t^*)^{-2}(\Delta\mathbf{v}^* - \operatorname{grad}\operatorname{div}\mathbf{v}^*), \tag{2.3}$$

where

$$(k_l^*)^2 = \frac{\omega^2 \varrho^*}{K^* + i\omega\varrho^*\left(\xi^* + \dfrac{4}{3}\eta^*\right)}, \qquad (k_t^*)^2 = \frac{\omega^2\varrho^*}{i\omega\eta^*}. \tag{2.4}$$

The second viscosity ξ occurs as an effect of the internal degrees of freedom which are absent in the case under consideration. Thus, we put

$$\xi = 0. \tag{2.5}$$

At this point we turn our attention to the similarity of the equations of motion which describe the wave propagation in a homogeneous viscous Newtonian liquid (f, p) and isotropic solid (s). All these equations are subtantially identical in appearance, being obtained from Eqs. (2.1)–(2.4) after inserting $\xi = 0$ and replacing each of the asterisks by the superscripts f, p and s, respectively.

On taking into account the known relation

$$K^s = \lambda^s + \frac{2}{3}\mu^s \tag{2.6}$$

and after extending it formally to the phases $*$, f and p, Eqs. (2.4) arrive us at the following relations:

$$\lambda^j = K^j - \frac{2}{3} i\omega\eta^j, \qquad \mu^j = i\omega\eta^j, \qquad j = *, f, p, s, \tag{2.7}$$

where K^s, λ^s and μ^s are the bulk modulus (adiabatic compressibility) and Lamé constants, respectively, μ^s being the shear modulus.

The material parameters ϱ^*, K^* and μ^* determine the propagation properties of the suspension (blood) for the plane ultrasonic waves propagating and being polarized along the directions of the reference axes Ox_j, $j = 1, 2, 3$ of the macroscopic Cartesian coordinate system. In this case

$$v^*_{ij} = \frac{1}{Z^{*(a)}}, \qquad \alpha^*_{ij} = -\omega Z^{*(b)}, \tag{2.8}$$

$$Z^* = \left(\frac{\varrho^*}{\Gamma^*_{ij}}\right)^{1/2}, \tag{2.9}$$

where

$$\Gamma^*_{ij} = \Gamma^*_l = K^* + \frac{4}{3}\mu^* \quad \text{for} \quad i = j, \tag{2.10}$$

$$\Gamma^*_{ij} = \Gamma^*_t = \mu^* \quad \text{for} \quad i \neq j. \tag{2.11}$$

v^*_{ij} and α^*_{ij} denote the propagation velocity and amplitude attenuation coefficient, respectively, of the plane wave propagating in the direction of the axis Ox_i and being polarized in the direction of the axis Ox_j. Throughout the paper, the real and imaginary parts of complex quantities are denoted by the subscripts (a) and (b), respectively. Therefore, the problem considered in the paper consists in establishing the dependence of the quantities ϱ^*, K^* and μ^* on ϱ^f, K^f, μ^f, ϱ^p, K^p, μ^p, ω and some structure parameters. In other words, this problem consists in predicting the propagation parameters of the suspension on the base of knowledge of the dynamic properties of the suspension components and some parameters of its structure.

Evidently, if the hyphothesis of the possibility of finding the homogeneous equivalent medium is reasonable and the effective response of the medium is a plane attenuated wave with propagation parameters v^*_{ij} and α^*_{ij}, then

$$\Gamma^{*(a)}_{ij} = B(1-z^2), \qquad \Gamma^{*(b)}_{ij} = 2Bz, \tag{2.12}$$

$$B = (v^*_{ij})^2 \varrho^* (1+z^2)^{-2}, \qquad z = \left(\frac{\alpha^*_{ij}}{\omega}\right) v^*_{ij}. \tag{2.13}$$

Formulae (2.12) and (2.13) enable the effective complex moduli Γ^*_{ij} of the suspension to be determined from the measurements of the macroscopic parameters of the ultrasonic wave propagation, v^*_{ij} and α^*_{ij}, in the medium under examination.

In contrast to the simplicity of the above macroscopic relationships, which suggest the experimental assessment of the structure and frequency dependences of the propagation and material parameters of two-phase media, theoretical attempts of finding these dependences always involve problems of great complexity. The dynamics of the multi-phase media with non-spherical inclusions is so complicated that, for a wide range of the volume concentrations c of the inclusions, we would be content with performing a computational analysis of the problem of the propagation of ultrasonic waves in such media. The computational investigations, some results of which are presented in the next section of this paper, enable us to establish the desired dependences. To perform such numerical analysis we make use of the self-consistent approach proposed by BERRYMAN [1]. It should perhaps be noticed here that in Ref. [1] the self-consistent approach is presented in a generalized form to be applicable also for materials with complex material parameters.

BERRYMAN [1] arrived at an algorithm for computational investigation of N-phase media with ellipsoidal inclusions, with all the phase materials being characterized, in general, by complex Hooke's (stiffness) tensors. Of course, N is a natural number. Adopting Berryman's concept to the two-phase media in the form of suspensions with ellipsoidal inclusions, will achieve the below given algorithm, which is employed in our computational analysis. The adopting is possible due to the above mentioned similarity of the equations of motion which describe the wave propagation in the homogeneous viscous Newtonian liquid (f, p) and any isotropic solid (s). As it was stressed, all these equations are subtantially identical in appearance.

The numerical results of these calculations are presented in the next section. For making clear the physical meaning of the numerical results, it seems to be reasonable to point out shortly the adopting of the BERRYMAN'S [1] concept to the two-phase suspensions under considerations. For this purpose, consider a sphere of the volume V occupied by the suspending fluid f in which are dispersed numerous ellipsoidal inclusions made of the liquid p with very high viscosity as compared with that of fluid f. The ellipsoids are assumed to be randomly oriented. The volume concentrations of the both phases are n_f and n_p, respectively. The sphere, in turn, is imbedded in a homogeneous liquid whose acoustic properties may be varied freely in a controlled manner. The imbedding liquid and the liquids n_f, and n_p are assumed to be immiscible in each other. If the elastic and propagation constants of the suspension are identical to those of the imbedding liquid, there is no scattering from the composite sphere. Then we can say that the imbedding liquid is identical to the effective (homogeneous equivalent) liquid, say, the liquid of type $*$, which is to be determined. Now, continuing the thought experiment, replace the true composite sphere with a sphere composed of the matrix (suspending) liquid of type $*$ and of ellipsoidal inclusions of both type-f and type-p material in the same relative proportion as in the original suspension. Then, if the frequency is sufficiently small enabling the multiple scattering to be neglected to the lowest order of approximation, the equations for the effective material parameters ϱ^*, K^* and μ^* can be derived from the condition:

$$< u(\mathbf{x})_i^s >^* = 0. \tag{2.14}$$

$< u(\mathbf{x})_i^s >^*$ denotes the ensemble average of the of the displacement field fluctuation, $u(\mathbf{x})_i^s$, given by

$$u(\mathbf{x})_i^s = u(\mathbf{x})_i - u(\mathbf{x})_0^s. \tag{2.15}$$

$u(\mathbf{x})_i^s$ denotes displacement field scattered by a single scatterer, $u(\mathbf{x})_0^s$ denote the incident field. The left-hand side of Eq. (2.14) denotes the net scattered displacement field due to the scattering in the above described suspension with the self-consistently determined matrix liquid of type-$*$. Relation (2.14) states that the self-consistent effective medium is determined by requiring the net scattered, long-wavelength displacement field to vanish on the average. To calculate $u(\mathbf{x})_i$ in the single scattering approximation we must first perform the summation of the scattering effects over all the single scatterers which are present in the bulk sample of the composite. This summation and averaging lead to the following relations, enabling us to calculate numerically the effective material and propapagation parameters of the two-phase composite under study:

$$K^* = \frac{n_p K^p P^{*p} + n_f K^f P^{*f}}{n_p P^{*p} + n_f P^{*f}}, \tag{2.16}$$

$$\mu^* = \frac{n_p \mu^p Q^{*p} + n_f \mu^f Q^{*f}}{n_p Q^{*p} + n_f Q^{*f}}. \tag{2.17}$$

The formulae for P^* and Q^* are listed in the Appendix of [1]. These formulae are not rewritten here.

3. Numerical results

Numerical calculations were performed for frequencies $f = 4$, 8, 12 and 16 Mc/sec. The following values were taken for the material parameters of the emulsion under analysis:

$$\varrho^f = 1.021 \frac{\text{g}}{\text{cm}^3}, \qquad \varrho^p = 1.021 \frac{\text{g}}{\text{cm}^3}, \qquad n_p = 0.4, \tag{3.1}$$

$$K^f = \frac{1}{40.9} 10^{11} \text{ Pa}, \qquad K^p = \frac{1}{34.1} 10^{11} \text{ Pa}, \qquad \eta^f = 1.8\ 10^{-2} \text{ Poise}. \tag{3.2}$$

Some results of the numerical calculations are presented in Figs. 1–3. These results visualize how the propagation velocities and attenuation coefficients of the ultrasonic dilatation and shear modes depend on the dynamic viscosity of the ellipsoidal inclusions and the frequency of an ultrasonic mode. The calculalations were carried out for oblate ($a = b > c$) spheroids under the assumption that the shape of each inclusion in the suspension under examination is to be characterized by the same value of the shape factor $Z = a/c = 3.2$, independently of the inclusion size and orientation. a, b and c denote the principal axes of a spheroid. The shape factor is here a measure of the oblateness of an oblate pore.

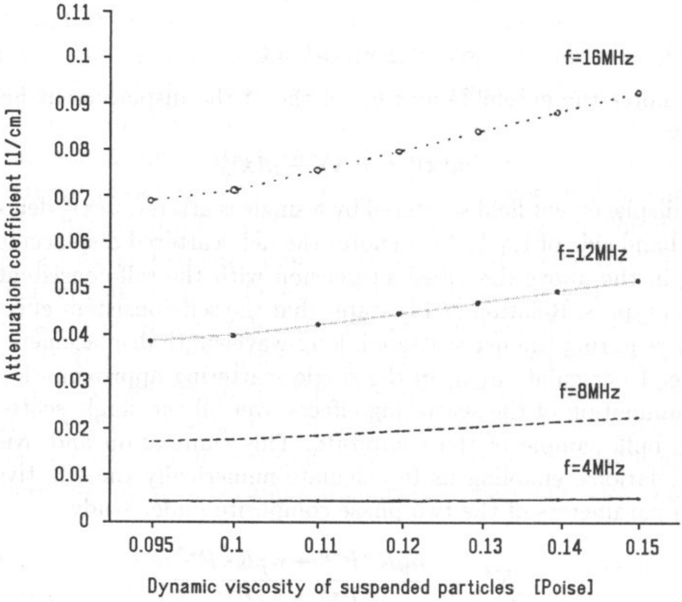

Fig. 1.

Figure 1 presents the dependency of the amplitude attenuation of the dilatation mode on the frequency and η^p. In Fig. 2, the propagation velocities of dilatation and shear modes are denoted by Cd and Cs, respectively. Similarly, in Fig. 3 the amplitude attenuation of dilatation and shear modes are denoted by ATs and ATd. The general tendency of the attenuation is to increase as frequency and η^p increase.

Although all the results presented above have been obtained under the assumption that the long-wavelength condition enables the single-scattering approximation to be used and that the non-spherical inclusions are randomly oriented in a macroscopic volume occupied by the suspension, it can be stated that the results show that the BERRYMAN'S [1] self-consistent method of estimating the effective dynamic elastic constants, if is applied to estimating the overall dynamical properties of the suspension, leads to rather strong dependence of the overall dynamic properties of the two-phase liquid on its composition, mechanical properties of the components as well as on the ultrasonic frequency.

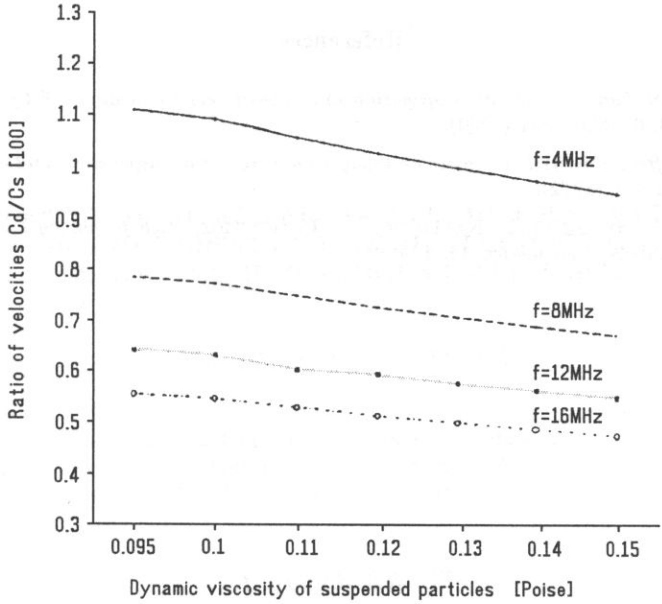
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
Ratio of velocities Cd/Cs [100]
f=4MHz
f=8MHz
f=12MHz
f=16MHz
0.095
0.1
0.11
0.12
0.13
0.14
0.15
Dynamic viscosity of suspended particles [Poise]

Fig. 2.

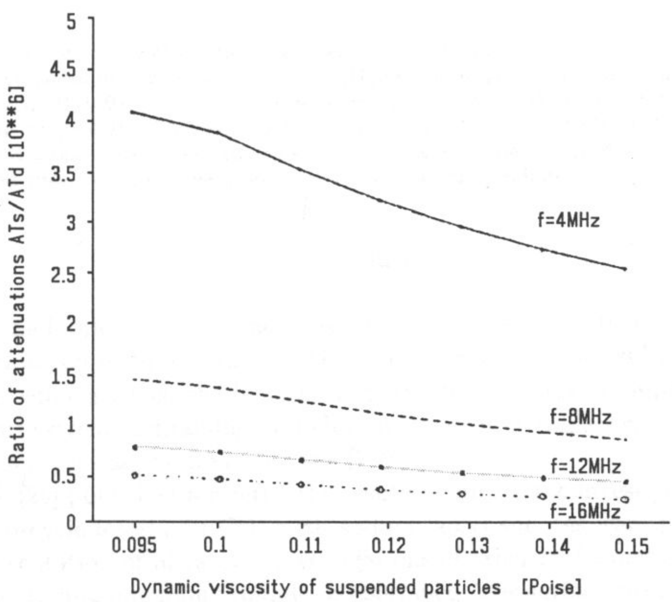
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Ratio of attenuations ATs/ATd [10**6]
f=4MHz
f=8MHz
f=12MHz
f=16MHz
0.095
0.1
0.11
0.12
0.13
0.14
0.15
Dynamic viscosity of suspended particles [Poise]

Fig. 3.

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