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MATHEMATICAL MODELLING OF AN ULTRASONIC FLOWMETER PRIMARY DEVICE

S. WALUŚ

Institute of Automatic Control,
Technical University of Silesia
(44-100 Gliwice, ul. Akademicka 16)

The role of the primary device of an ultrasonic flowmeter is described. The author proposed a classification of the ultrasonic flowmeter primary devices based on: 1) the principle of operation of the flowmeter sensor, 2) the method of obtaining information about the measured quantity from the flow phenomenon, 3) the kind of heads (inserted in the pipe wall and clamped-on), 4) the number of ultrasonic paths and their configurations. Mathematical models of flowmeter primary devices with a point velocity measurement are calculated (for laminar and turbulent flow). Also the model for an average velocity measurement over the surface of the ultrasonic transducer inserted in flow area is given. The next models are derived for a primary device with average velocity measurements over a single segment (most frequently in pipe diameter) or over a few segments (in the multi-path ultrasonic flowmeter primary device).

1. Introduction

Ultrasounds are of great importance in flow measurements. Flow-rate measurements are employed in industry, in water supply systems, as well as in medicine [1, 3, 6, 10, 15, 16, 18, 20]. They are used to determine the blood flow-rate in blood vessels by means of a Doppler ultrasonic flowmeter [7] and the flow-rate of water in large rivers by means of a transit-time ultrasonic flowmeter [10, 18, 19]. There are many kinds of ultrasonic flowmeters in which different constructions of the primary devices are used [11, 22, 27]. They are used in measurements of liquid flow-rates (in most cases) and also in measurements of gas flow-rates [1, 4].

The main purpose of the mathematical modelling of a flowmeter primary device [12, 17] is to describe the flow phenomenon under various conditions in order to reproduce a measuring value (measurand), and to estimate the total error for a real flowmeter under rated operating conditions [22]. The primary device of an ultrasonic flowmeter consists of a meter tube and heads (with probes and transducers) as shown in Fig. 2. The most commonly used devices for the ultrasonic transducers (emitters and receivers) are piezoelectric sensing elements (ultrasonic sensors). The greatest influence on the mathematical model of the flowmeter has the primary device because the measured quantity is converted in it into a signal that can be detected by a secondary device (measuring transmitter). The modelling of the ultrasonic flowmeter primary device under conditions which differ from the rated operating ones is more difficult [22]. It is possible to choose a kind of an ultrasonic flowmeter for concrete conditions [21] on the ground of a mathematical model of the flowmeter primary device. The mathematical model enables us to use some flowmeters without experimental calibration [9, 22]. The greatest influence on the model of the flowmeter primary device has the velocity distribution shape, although the shape of the ultrasonic beam [16, 20] influences the primary device model. The modelling of the velocity distribution changes (with flow changes and changes of the conditions of the measurement) enables a correction of the systematic error [24]. The volume flow-rate is estimated in many cases in virtue of the velocity measurement at a defined place (or places) of the stream. In this case, a place (places) must be found in that the measured value is representative for the whole cross-sectional area of the conduit (pipe or open channel).

2. Importance of the primary device in the flow-rate measurement with an ultrasonic flowmeter

The flow phenomenon is a space-time one and the measurement can be usually treated as a static measurement in the presence of fluctuations of the measured quantity (measurand). The velocity vector in a turbulent flow shows fluctuations of about 30% of the average value; the flowmeter measures the average velocity on the grounds of many single measurements. The scheme of the flow-rate measurement is shown in Fig. 1.

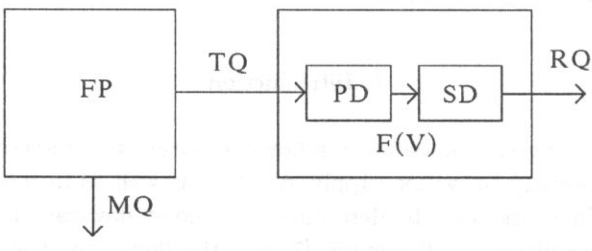

Fig. 1. Scheme of the flow-rate measurement: FP – flow phenomenon, PD – primary device, SD – secondary device (measuring transmitter), F(V) – flowmeter (velocitymeter), MQ – measured quantity (measurand), TQ – taken quantity, RQ – reproduced quantity (result of measurement).

The flow phenomenon induces the state of the fluid as well as the state of flow. The fluid can be transparent or nontransparent for ultrasonic beam, it can be homogenous or can contain gas bubbles (solid particles). This determines the kind of the ultrasonic flowmeter that has to be applied; transit-time ultrasonic flowmeters are used for clean liquids, the Doppler-type ultrasonic flowmeters are used for two-phase liquids [3, 6, 22]. The flow can be laminar, turbulent or of an intermediate regime.

Two main parts can be distinguished in the flowmeter [13]: a primary device and secondary one (measuring transmitter, transmitter [27], control unit [11]). Sometimes third part is distinguished: a data output unit; however, the last two parts are usually in a single device. A simplified diagram of ultrasonic flowmeter is shown in Fig. 2.

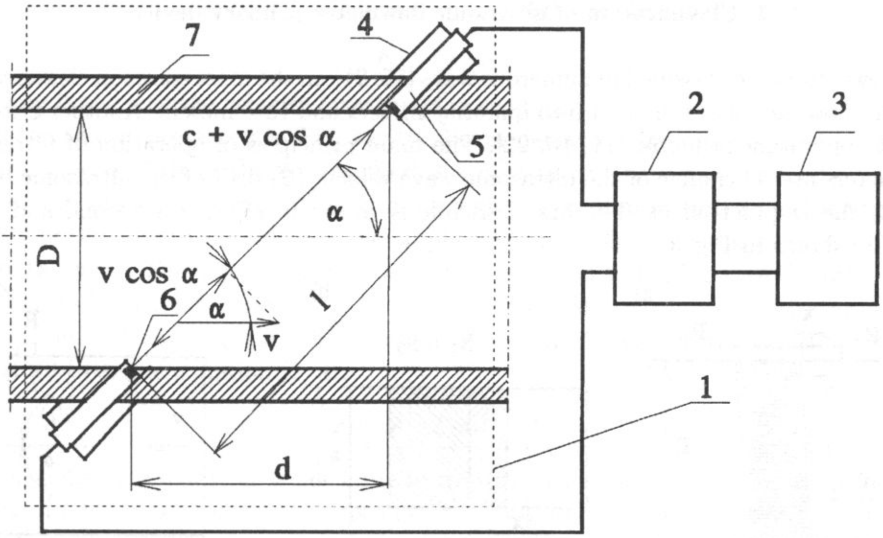

Fig. 2. Simplified diagram of the ultrasonic flowmeter: **1** – primary device, **2** – secondary device, **3** – output data unit, **4** – head, **5** – probe, **6** – ultrasonic transducer, **7** – meter tube, D – internal pipe diameter, l – distance between the centres of the ultrasonic wave emitting surfaces, d – projection of l on the pipe axis, α – angle between the ultrasonic beam direction l and the pipe axis, c – velocity of sound in the fluid at rest, v – velocity of the fluid.

The primary device takes a quantity (velocity, volume flow-rate) from some place of the flow. In the secondary device, the output value is calculated on virtue of the mathematical model of the flow phenomenon and the primary device construction. Modelling is a process of formulating a mathematical equation which gives an approximate description of the phenomenon in the measurement system [14].

The flowmeter is very often composed of many parts, and many factors must be taken into account: the nominal cross-sectional area of the conduit or the open channel, the methods and measuring procedures, the result of the measurement, the goal for which the results of the measurement are to be used, the measuring installation, influencing quantities, reference signals, standards and simulators.

The measured quantity (MQ in Fig. 1) can be the velocity v, the velocity profile $v(r)$, the velocity distribution $v(x, y)$, the volume flow-rate q_v or mass flow-rate q_m, the volume V or mass m. The taken quantity (TQ in Fig. 1) is sometimes only the representation of the measured one (for example, the velocity measured by means of two ultrasonic transducers at a given point of the cross-section (Fig. 5)), and is the basis for the calculation of the volume flow-rate. The reproduced quantity (RQ in Fig. 1) is calculated on the grounds of the taken quantity and of mathematical model of flow, as well as in virtue of the ultrasonic flowmeter primary device construction.

From the metrological point of view, the primary device is of greatest importance because it causes the main contribution to the total error of the results of the measurement [22]. Ultrasonic flowmeter transducers do not influence the measured quntity and do not disturb the flow phenomenon.

3. Classification of ultrasonic flowmeter primary devices

Flowmeters are classified in numerous ways [15, 21]; in the most common classification system, flowmeters are divided into quantity meters and rate meters. Another criterion is their operating principle [11, 16, 22]. The main principles of operation of ultrasonic flowmeters are: 1) change of the ultrasonic wave velocity, 2) drift of the ultrasonic beam, and 3) the Doppler effect. The first principle is shown in Fig. 2, the second and third ones are shown in Fig. 3.

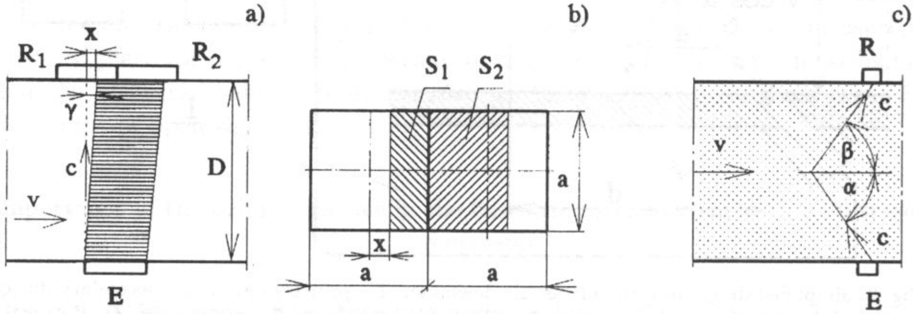

Fig. 3. Principles of the operation of the ultrasonic flowmeters: a) b) drift of the ultrasonic beam (beam deflection principle), c) Doppler effect, E – emitter (sender) of the ultrasound, R – receiver of the ultrasound, γ – angle between the drifted ultrasonic beam and the axis perpendicular to the pipe, x – drift of the incidence point of the ultrasonic beam, α – angle between the emitted ultrasonic beam and the pipe axis, β – angle between the received ultrasonic beam and the pipe axis.

For the ultrasonic flowmeters based on the first principle (change of the ultrasonic wave velocity) shown in Fig. 2, the downstream and upstream travel times are:

$$t_1 = \int_0^l \frac{dx}{c + v(x)\cos\alpha}, \tag{3.1}$$

$$t_2 = \int_0^l \frac{dx}{c - v(x)\cos\alpha}. \tag{3.2}$$

From (3.1) and (3.2), the average fluid velocity in the ultrasonic path (v_l) of velocity $v_l \ll c$ can be expressed by

$$v_l = \frac{c^2}{2l\cos\alpha}(t_2 - t_1) = \frac{c^2}{2l\cos\alpha}\Delta t, \tag{3.3}$$

where v_l is equal to the average velocity on the pipe diameter v_D,

$$v_l = v_D = \frac{1}{D}\int_0^D v(x)\,dx. \tag{3.4}$$

Volume flow-rate is the product of v_l, the velocity distribution shape coefficient K, and the cross-sectional area S:

$$q_v = S\,K\,v_l\,. \tag{3.5}$$

The velocity distribution shape coefficient K is defined as the ratio of v_S to v_D,

$$K = v_S/v_D\,, \tag{3.6}$$

where v_S is the average velocity over the pipe section.

In the sensor presented in Fig. 3 a, the beam deflection principle is used [2, 8, 14]. The sonic beam drift in the flowing fluid is a measure of the average velocity over the path of the ultrasonic beam. The emmiter sends a continuous ultrasonic wave; for a velocity of the liquid equal to zero, the amplitudes of the signals at the receivers R_1 and R_2 are the same. The signals from ultrasonic receivers go to the input of the differential amplifier and, for fluid velocity $v = 0$, the output signal is zero. For $v > 0$, the drift of the incidence ultrasonic beam will appear as shown in Fig. 3 a and the value of x can be calculated from the equation [2, 8]:

$$x = \sin\gamma D = \frac{v_D}{c}D. \tag{3.7}$$

When the pipe diameter is not large so that the ultrasonic beam is in the near field, the amplitudes of the received singnals can be calculated from the relations (see Fig. 3 a and Fig. 3 b):

$$A_1 = kS_1\,, \tag{3.8}$$

$$A_2 = kS_2\,, \tag{3.9}$$

where k is the factor of proportionality of the receiving ultrasonic transducer.

The sum of the surfaces S_1 and S_2, on which the ultrasonic beams falls, is equal to a^2

$$S_1 + S_2 = a^2. \tag{3.10}$$

The surfaces S_1 and S_2 can be calculated from

$$S_1 = a^2/2 - xa \tag{3.11}$$

and

$$S_2 = a^2/2 + xa. \tag{3.12}$$

From (3.8)–(3.12) we obtain

$$\Delta A = A_2 - A_1 = 2xak. \tag{3.13}$$

From (3.7) and (3.13) it follows that

$$v_D = \frac{c}{2akD}\Delta A. \tag{3.14}$$

The volume flow-rate can be calculated from (3.14) and (3.5) bacause $v_l = v_D$ (the ultrasonic beam is in the pipe axis).

The scheme of the principle of operation of a Doppler flowmeter primary device [2, 3, 15, 16] is given in Fig. 3 c. The frequency difference between the frequency of the received signal and that of the emitted one is equal to [16 p. 401]:

$$\Delta f = \left(\frac{c - v\cos\alpha}{c + v\cos\beta} - 1\right)f, \tag{3.15}$$

where f is the frequency of the emitted signal.

For $v \ll c$ and for $\beta = \alpha$ the fluid velocity can be calculated from:

$$v = \frac{c}{2f\cos\alpha}\Delta f. \tag{3.16}$$

In this paper, the author proposes a classification based on the method of taking information about measured value from the flow phenomenon; this classification is shown in Fig. 4 as an example of flow-rate measurements in open channels.

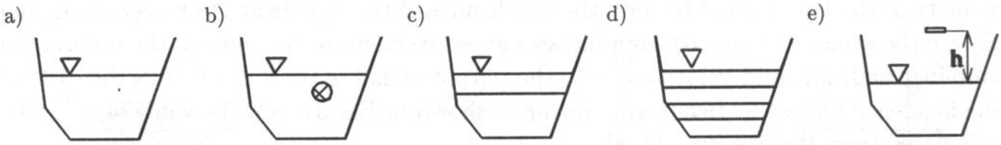

Fig. 4. Classification of flowmeter primary devices: a) the flow measurement is based on the point velocity measurement, b) the flow measurement is based on the average velocity measurement over some surface in the flow area, c) the flow measurement is based on the average velocity measurement along a segment in the flow area, d) the flow measurement is based on the average velocity measurements along some segments in the flow area, e) the flow measurement is based on the level measurement h and the model of the flow in the channel.

From the point of view of the measurement user (in automatic control or data processing), the operation principle and the construction of the flowmeter are less important, because the main metrological parameter is the measurement error, which depends mostly on the flowmeter sensor.

Ultrasonic flowmeter sensors with clamp-on heads are shown in Fig. 5.

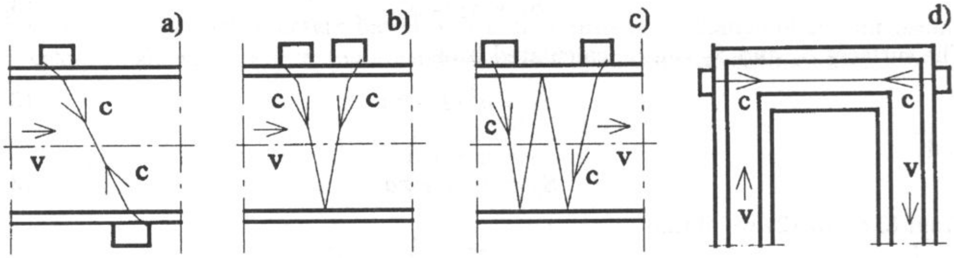

Fig. 5. Configurations of the ultrasonic flowmeter primary devices with clamp-on heads: a) with a single path in the pipe diameter, b) with a single path in the pipe diameter and a single reflection from the internal wall surface, c) with a single path in the pipe diameter and multiple internal reflections, d) with a path along the pipe axis.

4. A mathematical model of an ultrasonic flowmeter primary device with a point velocity measurement

The mathematical model of the primary device is based on the sensitivity factor which depends on the shape of the velocity distribution and on the construction of the primary device (the segment of the pipe, ultrasonic transducers and their layout). The point velocity is only a mathematical idea because each real sensor has some dimensions, even a laser beam sensor. In every situation we must take into account the ratio of the sensor dimension to that which characterizes the flow phenomena. For example, in a large shallow river, an ultrasonic transducer with a diameter of a few centimetres can be accepted as a point velocity measurement device, but it could not be accepted for measurements in a pipe with a diameter 5 times greater than that of the ultrasonic transducer. The scheme of the primary device is shown in Fig. 6, but the diameters R_s of the ultrasonic emitter and receiver are negligible in comparison with the pipe radius R.

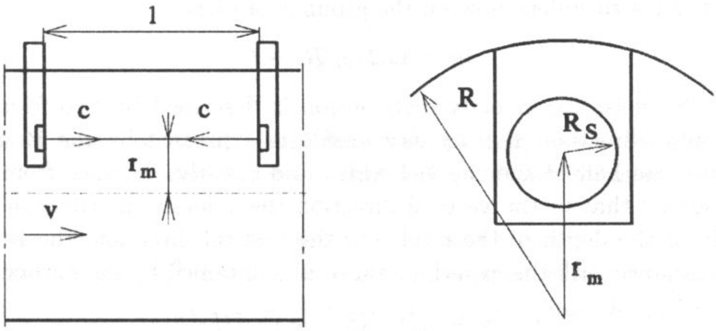

Fig. 6. Insertion meter: R_S – ultrasonic transducer radius, r_m – distance from the pipe axis.

In most practical cases, the goal of velocity measurements is to calculate the volume flow-rate in a conduit or in an open channel. There are two main solutions of this problem:

a) to measure the velocity at one point and calculate then the volume flow-rate on the grounds of the mathematical model of the primary device, or

b) to measure the velocities at many points of the flow area and obtaining the flow-rate by averaging single measurements or using some integration methods.

The first method needs a mathematical model of the velocity distribution in the conduit or in the open channel. The real velocity profiles are so different [5] that it is difficult to estimate the true value of the sensitivity factor.

The velocity distribution in laminar regime in a pipe with a circular cross-section is expressed by

$$v = v_0 \left[1 - (r/R)^2\right], \tag{4.1}$$

where: R – pipe radius, r – current radius, v_0 – velocity along the pipe axis.

For a turbulent flow, the Prandtl formula (power-low velocity profile) is most frequently used:

$$v = v_0(1 - r/R)^{1/n}, \tag{4.2}$$

where n depends on the Reynolds number and the roughness of the pipe wall.

Miller [15] proposes that the value of n for smooth pipes can be calculated as a function of the Reynolds number,

$$n = 1.66 \log Re. \tag{4.3}$$

The author proposed [22] the following formula for the laminar ($m = 2$) and for the turbulent flow:

$$v = v_0 \left[1 - (r/R)^m\right]. \tag{4.4}$$

The last equation enables the description of the velocity distribution also for the intermediate regime (between a laminar and a turbulent flow). Assuming that the velocity distribution shape coefficients K, defined by (3.6) for the velocity distributions described by (4.2) and (4.3) and expressed by (6.2) and (6.3), are equal, it is possible to calculate the value of m for a turbulent flow on the grounds of (4.3):

$$m = 3.32 \log Re - 1. \tag{4.5}$$

For open channels, the velocity distribution is described by two formulae [5, 25]. In the horizontal direction, the velocity profile is expressed by the Prandtl formula (4.2); R means one half of the channel width and r is the distance from the channel axis. It is assumed that in the vertical direction the velocity distribution is the same, independently of the depth of the fluid. For the vertical direction, the Bazin formula, which is in acordance with the experimental results obtained by the author [22], is used:

$$v = v_0 - mH^{-2}(h - h_0)^2 (JH)^{0.5}, \tag{4.6}$$

where h – depth of the liquid, v_0 – maximum velocity for the depth h_0, H – liquid level, m – experimental coefficient which depends on the roughness of the channel, J – hydraulic gradient.

For these equations it is possible to calculate the sensitivity factor K_s as the ratio of the average velocity at the cross-section v_s to the measured velocity v_m:

$$K_s = v_s / v_m \, , \tag{4.7}$$

where v_m is the velocity for the radius r_m or, in an open channel, the velocity at a point of distance r_m from the channel axis and h_m from the liquid level.

For velocity distributions described by the Eqs. (4.1), (4.2) and (4.4) at a distance r_m from the pipe axis, the sensitivity factors will be as follows [22]:

$$K_s = \frac{0.5}{1 - (r_m/R)^2}, \tag{4.8}$$

$$K_s = \frac{2n^2}{(n+1)(2n+1)(1 - r_m/R)^{1/n}}, \tag{4.9}$$

$$K_s = \frac{m}{(m+2)\left[1 - (r_m/R)^m\right]}. \tag{4.10}$$

The mathematical model of the ultrasonic flowmeter primary device is a relation between the output value (time difference Δt) and the measured value (volume flow-rate q_v):

$$\Delta t = f(q_v). \tag{4.11}$$

The time difference for a sensor like that in Fig. 6, but for $R_s \ll R$, can be calculated from the following equation:

$$\Delta t = \frac{l}{c - v_m} - \frac{l}{c + v_m}. \tag{4.12}$$

For $v_m \ll c$,

$$\Delta t = \frac{2lv_m}{c^2}. \tag{4.13}$$

The measured value q_v is a product of the pipe cross-section and the average velocity v_S:

$$q_v = \frac{\pi D^2}{4} v_S. \tag{4.14}$$

From Eqs. (4.7), (4.13) and (4.14), the mathematical model of the primary device can be found,

$$\Delta t = \frac{8l}{\pi D^2 c^2 K_S} q_v. \tag{4.15}$$

5. Mathematical model of an ultrasonic flowmeter primary device for average velocity measurements over some surface of the flow area

In Fig. 6 the scheme of a flowmeter of the ultrasonic insertion type is shown. The output signal of this primary device depends on the average velocity on the surface of the ultrasonic transducer, and in virtue of this signal the total volume flow-rate is calculated.

For the velocity distribution described by the formula (3.1) (laminar flow), the measured average velocity can be calculated from

$$v_m = v_0 \left(1 - \frac{r_m^2}{R^2} - \frac{R_S^2}{2R^2}\right). \tag{5.1}$$

For a turbulent flow and the velocity distribution model (3.3) for $m = 6$,

$$v_m = v_0 \left(1 - \frac{r_m^6}{R^6} - \frac{9r_m^4 R_S^2}{2R^6} - \frac{3r_m^2 R_S^4}{R^6} - \frac{R_S^6}{4R^6}\right). \tag{5.2}$$

For a velocity distribution described by (4.4), the averaged velocity over the cross-sectional area is as follows:

$$v_S = v_0 \frac{m}{m+2}. \tag{5.3}$$

From (5.1) or (5.2), (4.14), (5.3) and (4.13) we obtain the model of the primary device. For example, for a laminar flow:

$$\Delta t = \frac{16l}{\pi D^2 c^2} \left(1 - \frac{r_m^2}{R^2} - \frac{R_s^2}{2R^2}\right) q_v. \tag{5.4}$$

6. Mathematical models of flowmeter primary devices for average velocity measurements over some segments of the flow area

In Fig. 7 typical primary devices of ultrasonic flowmeters are shown.

In the case of a time of flight ultrasonic flowmeter (which is most popular in practice [11, 27] and with primary devices like that shown in Fig. 7 a, the mathematical model of primary device is expressed from (3.3), (3.5) and (3.6) by the formula

$$\Delta t = \frac{8l \cos\alpha}{\pi D^2 c^2 K_S} q_v. \tag{6.1}$$

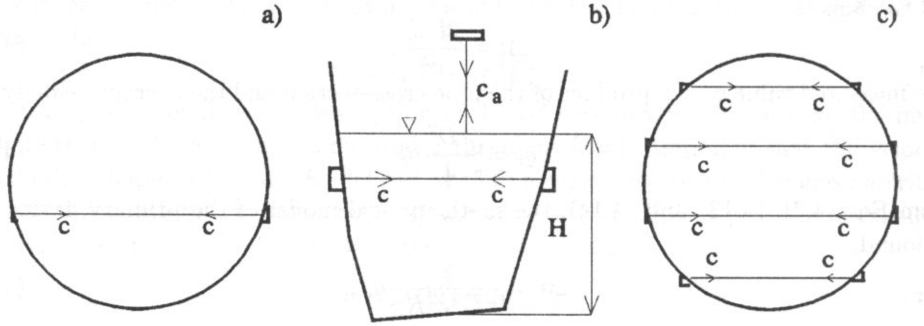

Fig. 7. Primary devices of ultrasonic flowmeters: a) one-path with heads mounted in the pipe wall, b) one-path for an open channel with level measurement, c) multi-path for a closed conduit, c_a – sound velocity in air being at rest.

When the ultrasonic beam is located at the pipe diameter, as in Fig. 2, the sensitivity factor value K_S is equal to that of the velocity distribution shape coefficient K. The value of K for a laminar flow is 0.75, and for a turbulent one we have, according to equations (4.2) and (4.4),

$$K = 2n/(2n+1), \tag{6.2}$$

$$K = (m+1)/(m+2). \tag{6.3}$$

On the grounds of the sensor model in secondary device, the measured value q_v is calculated. For example as shown in Fig. 7 b, the model of the primary device can be expressed in the following way:

$$\Delta t = \frac{2l \cos\alpha}{c^2 K_s(H) S(H)} q_v , \tag{6.4}$$

where $K_s(H)$ is the sensitivity factor, $S(H)$ is the flow area.

In [25] a one-path ultrasonic flowmeter was analysed. It is possible to decrease the measurement error in the multi-path primary device (which is recommended for a distorted velocity distribution). The mathematical model of the multi-path primary device is expressed by $N+1$ equations:

$$\Delta t_i = \frac{2l_i \cos\alpha_i}{c^2 K_{si} S_i} q_{vi} , \tag{6.5}$$

$$\sum_{i=1}^{N} q_{vi} = q_v , \tag{6.6}$$

where N is the number of ultrasonic paths (in Fig. 7 c $N = 4$), l_i is the length of the path i of the ultrasonic beam in the fluid, α_i is the angle between l_i and the pipe axis, K_{si} is the sensitivity factor in the part i of the cross-section, S_i is the surface of the part i of the cross-section, q_{vi} is the volume flow-rate through S_i.

In [11, 22, 27] recommendations for spacing of ultrasonic paths in the conduit with a circular section are given. Multi-path ultrasonic flowmeters are used in the case when the velocity profile is distorted [13, 24] and the accuracy demands are high. There are two reasons for the primary device sensitivity changes:

1) velocity distribution shape coefficient changes with: a) Reynolds number changes accompanying the flow-rate and viscosity changes, b) roughness of the pipe wall changes;

2) velocity profiles distortion caused by disturbing flow elements before or behind the ultrasonic flowmeter primary device. In such situations multi-path primary devices shown in Fig. 8 are recommended.

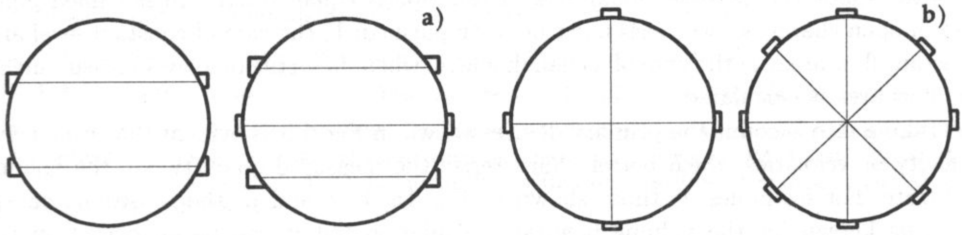

Fig. 8. Multi-path primary devices: a) with paraller paths, b) with crossed paths.

Reynolds number changes or wall roughness changes make the sensitivity of the primary devices shown in Fig. 8 b change as in the case of a one-path primary device. The primary devices presented in Fig. 8 b decrease the influence of the distorted flow profile, while those shown in Fig. 8 a decrease the sensitivity error in both situations.

For an ultrasonic flowmeter primary device with clamp-on heads (Fig. 5 a) [26], the model of the primary device is expressed as follows:

$$\Delta t = \frac{8l \cos \alpha}{\pi D^2 c^2 K} q_v + t_r \,, \tag{6.7}$$

where: l – length of the path of the ultrasonic beam in the fluid, t_r – time being the following sum:

$$t_r = \sum_{i=1}^{4} l_i / c_i \,. \tag{6.8}$$

Here l_i – paths of the ultrasonic beams in the pipe wall and in the heads, c_i – sound velocities in the pipe wall and in wedges of the heads.

7. Discussion of the results obtained

In the article, theoretical fundamentals of mathematical modelling of ultrasonic flowmeter primary devices are given. The classification of the principles of measurements used in ultrasonic flowmeters is commonly known. The mathematical descriptions of principles shown in Fig. 2 (change of the ultrasonic wave velocity) and in Fig. 3 c (Doppler effect) were taken from the literature. The author derived a simple model of the physical phenomena for the beam deflection principle (Fig. 3 a) on the grounds of equation (3.7) given in literature [2, 8] and made a sugestion concerning the construction of the primary device (Fig. 3 b). This model (Eq. (3.14)) combines the difference of the amplitudes of the received electric signals with the average velocity over the pipe diameter. When the projection of the ultrasonic path on the cross-section of the pipe is on line with the pipe diameter, the velocity v_l is equal to v_D and the measured value can be calculated from (3.5) and (3.6). The methods of estimation of the velocity distribution shape coefficient K were given in [22].

The classification of the ultrasonic flowmeter primary devices proposed by the author in Fig. 4 covers all practical cases of pipes and open channels. In pipes (filled with gas or with liquid) the cross-sectional area of the pipe is equal to πR^2. In not filled pipes and in open channels, two cases must be distinguished: 1) the case of constant level and constant flow area, 2) the case of a changing level when, in virtue of a level measurement, the flow area is calculated.

Taking into account the primary devices shown in Fig. 4 it is evident that we receive velocity or velocities which better characterize the measured quantity, i.e. the volume flow-rate. For examples as those shown in Fig. 4a, b, c and d, the sensitivity factor must be known for the volume flow-rate calculation but it can be estimated on the grounds of the shape of the velocity distribution in the channel. The situation shown in Fig. 4e demands prior calibration in order to estimate the swelling curve, i.e. the function $q_v = f(h)$.

The classification of primary devices with clamp-on transducers shown in Fig. 5 is based on a review of the literature and on the author's papers (the first is mentioned in [26]). The author took part in the joint scientific work which resulted in models of ultrasonic flowmeters with primary devices introduced in Fig. 5a and 5b. The author worked out the primary device shown in Fig. 5d and the ultrasonic flowmeter with this primary device has worked in the student laboratory for several years.

The velocity distribution shape coefficient K defined by the equation (3.6) depends on the mathematical model of the velocity distribution (Eqs. (4.2) and (4.4) for turbulent flow), and the functions of K are given in (6.2) and (6.3). In literature many equations for n in the Prandtl formulae are reported (Eq. (4.3) is only an example) and the author derived the formulae for m in Eq. (4.4), i.e. Eq. (4.5).

8. Conclusions

1. The classification of the principles of measurements with ultrasonic flowmeters and the classification of the primary devices with clamp-on heads and with heads mounted in the pipe wall are presented.

2. An original equation for m (power in Eq. (4.4) for describing the velocity profile) is derived, and also an original model for primary device based on the beam deflection principle is presented.

3. For various (Fig. 2, 3, 4, 5) ways of taking a measured quantity, mathematical models are given; they enable the calculation of the relation between the volume flow-rate q_v and the quantity measured in the secondary device (Fig. 6, 7, 8).

4. The derived mathematical models enable the calculation of errors connected with the installation of the ultrasonic sensors in the primary device, with the fluid properties

and the flow conditions (Reynolds number which depends on the volume flow-rate and fluid viscosity).

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