EXPERIMENTAL MODAL ANALYSIS OF THE LOUDSPEAKER SYSTEM

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This paper is concerned with an experimental modal analysis of a loudspeaker system. Based on Frequency Response Functions (FRFs) measured in about 500 points uniformly spaced on the front panel of the loudspeaker system several vibrations modes, mainly of the loudspeaker membranes, have been found. Then, the vibrations of the loudspeaker membranes were compared with vibrations of the cabinet of the loudspeaker system. It has been shown that the vibrations of the loudspeaker system cabinet were very small relative to the vibration of the loudspeaker membranes. Thus, the loudspeaker system enclosure, which may be considered as a set of vibrating plates is a source of rather week sound and may be neglected.

1. Introduction

1.1. What is modal analysis?

A term *modal analysis* refers to a process of characterising dynamic response of a structure by describing its vibrational motion by means of a set of mathematical relationships, generally referred to as modal properties [1, 2, 13, 5]. Vibrational modes of a structure can be obtained by means of one of two different approaches: mathematical models or experimental analysis [4].

Mathematical models generally discretize a structure by dividing it into a number of masses and springs, either by assuming a lumped mass and spring approach or by using the Finite Element Method (FEM) where each element is considered as a mass-spring system. A computer program, then, solves the eigenvalue extraction problem to obtain the mass, frequency, damping and mode shapes of each eigenmode of the system. Once the modal properties are determined the system is entirely described.

Experimental modal analysis on the other hand starts from a set of measured Frequency Response Functions (FRFs) of a structure. The FRF is defined as a ratio of a response signal spectrum and an excitation signal spectrum. Based on these functions the modal properties (i.e. modal frequencies, modal damping and mode shapes) are calculated without any specific assumptions concerned with the distribution of mass and stiffness of the structure. The modal model, that consists of the modal properties, is obtained by analytical curve fitting of one or more FRFs. The experimental modal analysis,

based on FRFs measurements, may be used to describe a dynamic behaviour of any vibrating object including a loudspeaker system.

In terms of modal analysis, the FRF for the response X_i, at a point i, due to an excitation force F_k applied at a point k [4, 13, 10, 11], has a form:

$$\frac{X_i}{F_k} = H_{ik}(s) = \sum_{r=1}^{n} \left(\frac{r_{ikr}}{s - p_r} + \frac{r_{ikr}^*}{s - p_r^*} \right), \tag{1}$$

where r_{ikr} – residue for r-th mode, s – Laplace variable, $p_r = \sigma_r + j\omega_r$, for $k = 1, ..., r$ – eigenvalue or the pole of the FRF, σ_r – modal damping of the r-th mode, ω_r – modal frequency of the r-th mode.

If all points i and k are taken into consideration, then Eq. (1) takes form:

$$H(\omega) = \sum_{r=1}^{n} \left(\frac{\{u_r\}\{u_r\}^t}{j\omega - p_r} + \frac{\{u_r^*\}\{u_r^*\}^t}{j\omega - p_r^*} \right). \tag{2}$$

The Laplace variable s in Eq. (1) has been replaced by the frequency ω along the frequency axis; p_r and $\{u_r\}$ are complex quantities. Thus, each mode of vibration is defined by a pair of complex conjugate poles (p_r, p_r^*) and a pair of complex conjugate mode shapes $(\{u_r^*\}, \{u_r^*\})$; $\{u_r\}^t$ and $\{u_r^*\}^t$ are transpositions of $\{u_r\}$ and $\{u_r^*\}$ respectively. A set of FRFs with indices i and k is usually arranged in a matrix called a modal matrix.

1.2. Assumptions

Modal analysis imposes following assumptions on the investigated object:

⇒ The system is linear. It means that a response of the system is proportional to an excitation. This assumptions has three following implications for FRFs measurements:

• Superposition. A measured FRF is not dependent on the type of excitation waveform used.

• Homogeneity. A measured FRF is independent of the excitation level.

• Reciprocity. The FRF measured between any two points of the structure is independent of which of them is used for excitation or response. Therefore, only one row or column of the modal matrix needs to be measured in order to identify the modal parameters of the structure. The only exceptions to the above statement are rows and columns corresponding to components of the modal vector which are zero. These are known as the nodal points of the mode shape.

⇒ The structure is causal. It does not start to vibrate before it is excited.

⇒ The structure is stable. The vibrations dies out when the excitation is removed.

⇒ The structure is time-invariant. The dynamic characteristics do not change during the measurements.

⇒ Modes are defined in a global sense. Mode shapes are defined for all degrees of freedom of the model. Frequency and damping of the mode do not vary significantly from one part of the structure to another.

⇒ Only one mode exists at each modal frequency.

These assumptions can be satisfied in a large number of test situations. However, accuracy of the resulting modal parameters depends largely on how closely the analysed system follows them.

In many cases, to simplify calculations, it is necessary to assume that a loudspeaker and a loudspeaker system are approximately linear systems. However, this assumption is only reasonable true for small displacements of the structure. In this case loudspeaker systems can be treated as linear systems in the whole explored frequency range. The other conditions which modal analysis imposes on the investigated system, as causality, stability and time-invariantness, are undoubtedly met by the loudspeaker system.

1.3. The role of modal analysis with respect to acoustics. Aim of the experiment

In classic experimental modal testing a structure is excited at one or more points and the response is determined at one or more points. It was the reason that at the beginning the modal analysis was used for dynamic investigation of large objects [3], like cars, ships, aircrafts, buildings, mill elements, vibrating machines, etc. It was relatively easy to excite the structure by a shaker or by an impact hammer with a force transducer and to measure the response signal by means of an accelerometer attached at some key points [7]. In this classic form experimental modal analysis is successfully used in acoustics in the investigations of the dynamic behaviour of music instruments like violins, pianos, guitars, handbells, etc. [3].

However, application of modal analysis to investigation of objects whose mass was comparable to mass of accelerometer, like commonly encountered in electroacoustics loudspeaker membranes, was almost impossible. It is possible to attach the accelerometer to the loudspeaker membrane, but the response signal determined by the accelerometer will not be the response of the membrane itself, but the response of the membrane loaded by the accelerometer. Such a signal is rather useless in modal investigations. This serious drawback has disappeared when a non-contact method of detection and measuring the structure response signal has appeared. A laser velocimeter enables successful measurements of the loudspeaker membrane response. It is not necessary to cover the membrane by a reflective material, so the problem of unwanted mass attached to the membrane does not exist.

Then, the use of the laser velocimeter is the only chance of successful modal analysis of loudspeaker membranes and complete loudspeaker systems. However, some more modifications of the classic experimental modal analysis technique have to be made too (see next paragraph for details).

In this paper the modal analysis was used to analyse vibrations of a complete loudspeaker system. The main purpose was to find, visualise and compare all vibrating areas of the loudspeaker system including both the speakers and the cabinet of the system. It is well known that the loudspeaker system cabinet vibration can influence the reproduced sound by a loudspeaker system giving so-called "colouration" effect. The effect may be simply due to the interference of reproduced sound from loudspeakers with unwanted sound produced by thrilling surfaces or due to resonances of the cabinet. If areas with unwanted significant vibrations are well known, it will be possible to eliminate them or,

at least, to change their resonant frequency by attaching an additional mass, a damping element or a stiffening element.

2. Measuring setup and measuring conditions

A block diagram of a measuring system used in the investigation is presented in Fig. 1. The object under investigations was a typical ZgP-40-8-85 "Tonsil" loudspeaker system taken by chance from the manufacture. The system consisted of one woofer (GDN 20/40/12) and two tweeters (GDW 9/15/6 and GDW 5/40/9). The dimensions of the system were: $256 \times 456 \times 210$ mm. The loudspeaker system was mounted on a motor-step table controlled by a computer, enabling its positioning.
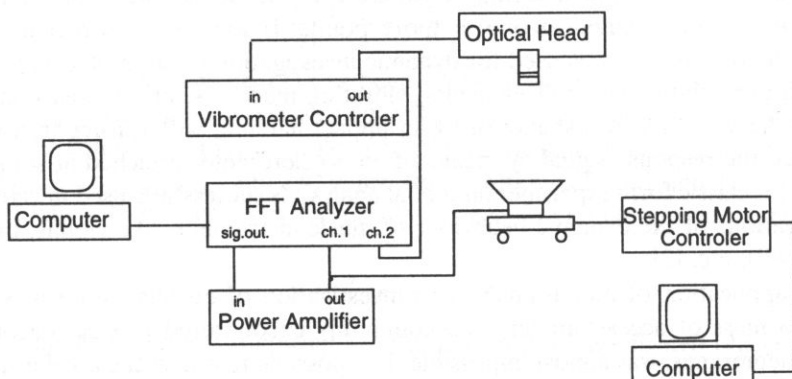
Fig. 1. Measuring setup for modal analysis used in the investigations.

The system was excited by a constant voltage (RMS), flat spectrum, pseudo-random noise. The frequency range of the signal was $0 - 4$ kHz. An example of the exciting signal spectrum is shown in Fig. 2. To preserve natural working condition of the system the signal was delivered to the loudspeaker's voice-coils. The signal was also delivered to one channel of the dual channel spectrum analyser (ONO SOKKI CF 6400) which was the main part of the measuring equipment.

The response signal from the vibrating body, i.e. the changes of the velocity of a vibrating point of the system, was measured in a non-contact way by means of a laser velocimeter based on the Doppler effect. Velocity was measured in the parallel direction to the main axes of symmetry of the loudspeakers. The velocimeter was fixed on a frame and its position did not change during experiment. The signal from the velocimeter was fed to the second channel of the analyser.

The spectrum of the excitation evoked by the stimulating pseudo-random noise was broadband and contained discrete components whose frequencies corresponded to the frequencies of analysis. A repeatable nature of the stimulating waveform enabled successful application of time averaging procedure for FRFs. So the signal-to-noise ratio was significantly improved.
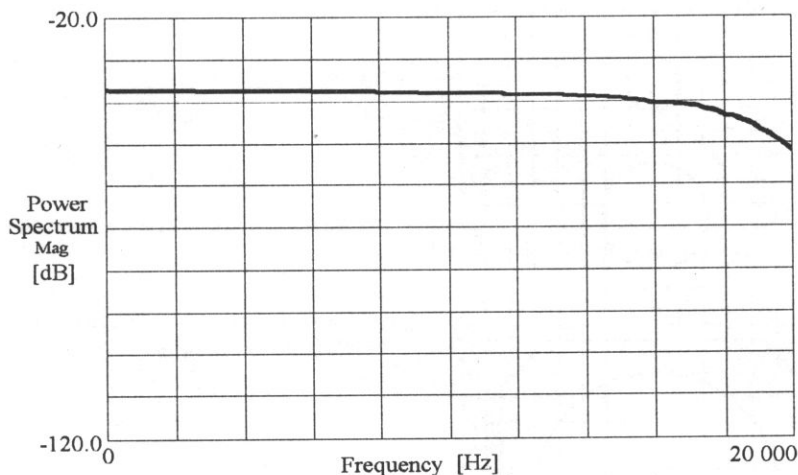
Fig. 2. The spectrum of the excitation signal used in the investigation.

The rectangular window, as the most appropriate for pseudorandom noise and transient signals, was used in both measuring channels. The analyser calculated FRFs based on the Eq. (1). Then, FRFs were transferred via GPIB card to a computer and stored for further analyse by means of the SMS STAR*Modal*® software.

Experimental modal analysis is based on a set of measured Frequency Response Functions (FRFs). A distribution and a number of the measuring points applied to the object are very important for the accuracy and the resolution of the analyse. They must reflect the geometry of the investigated object in an optimal and the best possible way. The maximum number of the data points that the STAR*Modal*® software could cope with was 500. A measuring "net" consisting of 17 points placed along x-axis and 29 points placed along y-axis was constructed. Thus, the loudspeaker system was characterised by means of 493 points uniformly spaced on the front panel of the system. The distance between two adjacent measuring points was approximately equal 1.5 cm. The distribution of the measuring points, plotted in Cartesian coordinates is given in Fig. 3.

It would be the best if the majority of measuring points were placed in areas of the most significant possible vibrations. However, as before the experiment positions of nodal points were unknown we decided to distribute all available measuring points uniformly. If some modes of a single loudspeaker were of the main interest then all points should cover it.

In the carried out experiments it was necessary to adopt the classic modal analysis to measurements performed directly on a loudspeaker [8, 9]. The classic modal analysis assumes that the excitation signal is a force signal. However, in the presented experiments the membrane was driven by the voice-coil. Thus, it was impossible to fulfil this requirement because the value of exciting force was unknown. The voice-coil voltage was taken as the input variable instead of the force.

For the carried out analysis a point excitation was assumed. The central point of the woofer's membrane was chosen as the stimulation point in all calculation. The only way
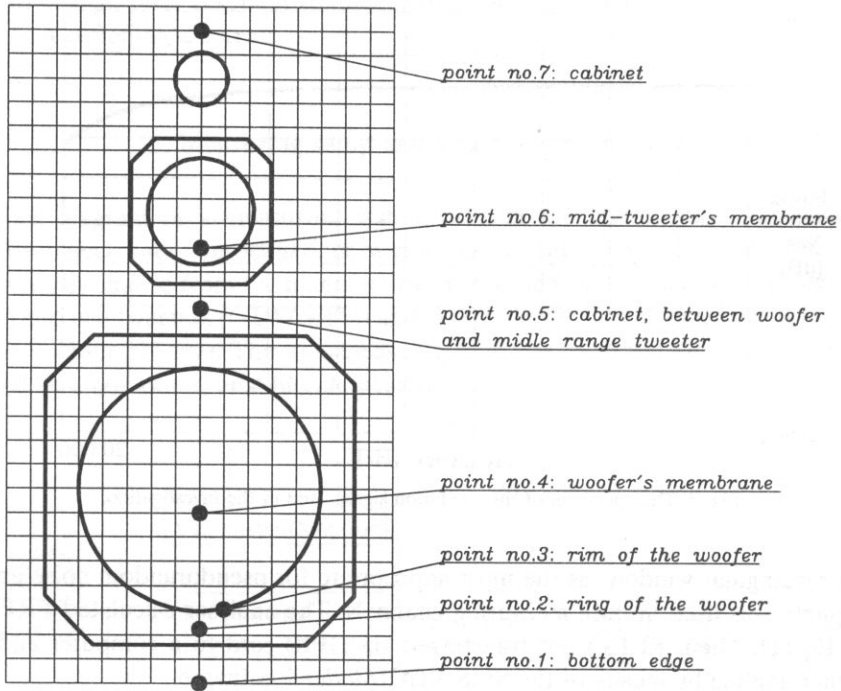
Fig. 3. Measurement geometry of the front panel of the "Tonsil" loudspeaker system.

of measuring vibrations of the loudspeaker, that reflects a normal condition of its action, is to excite the loudspeaker by means of its voice-coil. This necessity results from the nature and the structure of the loudspeaker. On the other hand this assumption might be a source of a certain error. This error can be estimated by means of the multi-point excitation version of the modal analysis and is under investigation.

3. Results of modal analysis of the loudspeaker system
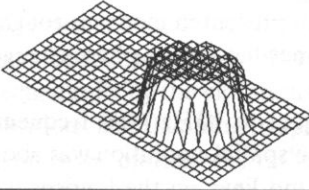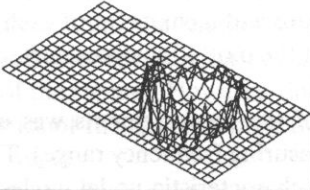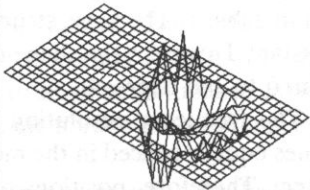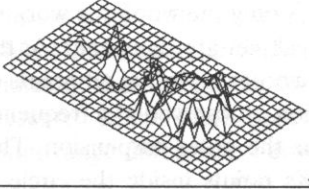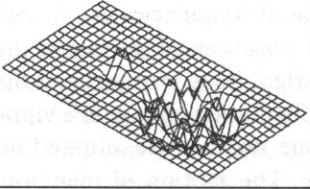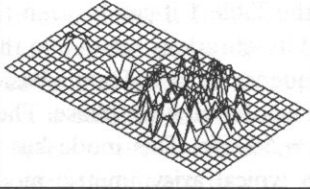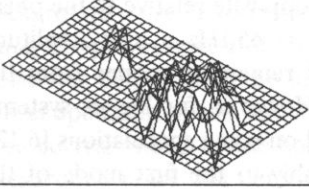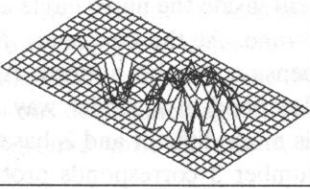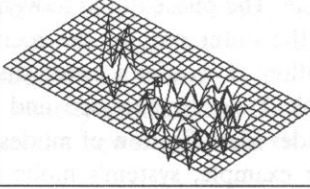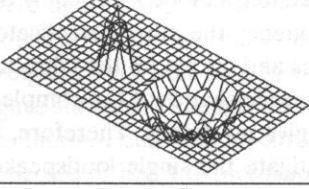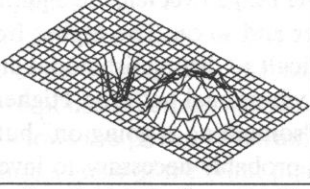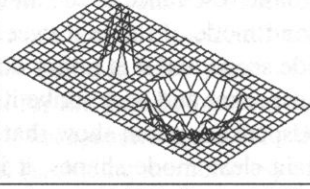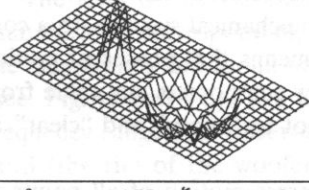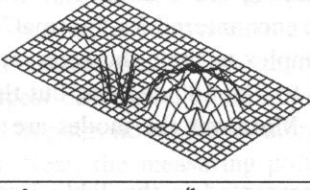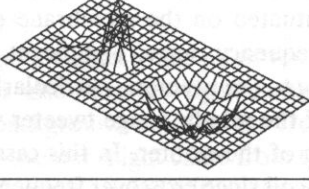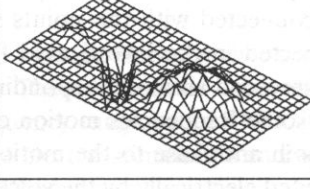
Based on the frequency response functions measured in all 493 points the modal parameters (i.e. modal frequencies, modal damping and mode shapes) were calculated. The results of modal analysis are listed in Table 1. Each cell of the table shows modal frequency, the mode shape and the percentage of critical damping. The percentage of critical damping is defined as [10]:
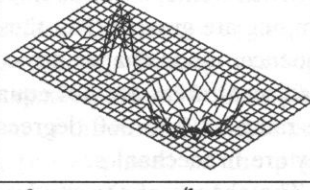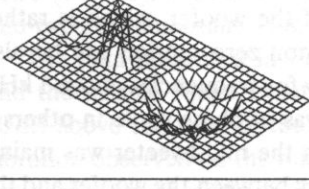
$$\xi_r = \frac{\sigma_r}{\sqrt{(\sigma_r^2 + \omega_r^2)}} \cdot 100\% , \qquad (3)$$

where σ_r – damping value in s^{-1} for r-th mode, ω_r – damped natural angular frequency in s^{-1} for r-th mode.

The shape of the mode is simply a plot of motion in z direction expressed in a linear scale, determined in each measuring point of the loudspeaker system for one

Table 1. Results of modal analysis for the complete loudspeaker system.

$f_0 = 90$ Hz $\xi_0 = 39.0\%$	$f_1 = 580$ Hz $\xi_1 = 6.0\%$	$f_2 = 650$ Hz $\xi_2 = 5.4\%$
$f_3 = 1.16$ kHz $\xi_3 = 7.0\%$	$f_4 = 1.44$ kHz $\xi_4 = 4.9\%$	$f_5 = 1.64$ kHz $\xi_5 = 4.3\%$
$f_6 = 1.80$ kHz $\xi_6 = 3.7\%$	$f_7 = 1.99$ kHz $\xi_7 = 3.8\%$	$f_8 = 2.30$ kHz $\xi_8 = 1.9\%$
$f_9 = 2.55$ kHz $\xi_9 = 3.3\%$	$f_{10} = 2.68$ kHz $\xi_{10} = 2.7\%$	$f_{11} = 2.85$ kHz $\xi_{11} = 3.6\%$
$f_{12} = 2.97$ kHz $\xi_{12} = 4.7\%$	$f_{13} = 3.20$ kHz $\xi_{13} = 5.0\%$	$f_{14} = 3.43$ kHz $\xi_{14} = 4.6\%$
$f_{15} = 3.55$ kHz $\xi_{15} = 5.3\%$	$f_{16} = 3.74$ kHz $\xi_{16} = 1.4\%$	$f_{17} = 3.92$ kHz $\xi_{17} = 3.8\%$

particular phase of the vibrating movement. The units along z-axis are the same in each cell in Table 1. The basic structure and a character of each of presented modes is roughly constant. However, the shape of the plotted surfaces may somewhat vary as phase changes from 0 to 360 degrees.

The frequency resolution of the measurements was equal to 10 Hz (400 frequency values equally spaced in the measuring frequency range). The spatial resolution was about 1.5 cm. Therefore, positions of characteristic nodal circles and lines on the loudspeaker membranes are difficult to identify. However, from the sequence of mode shapes shown in the Table 1 it can be seen that at frequencies f_0, f_1 and f_2 only the woofer is working and its vibrations relative to the vibrations of the rest of the cabinet are significant. At the frequency $f_0 = 90$ Hz the basic rigid-body resonance of the woofer is visible; all points of the woofer move in phase. The first mode of woofer's vibration is observed at a frequency $f_1 = 580$ Hz. This mode has one nodal circle situated near the outer suspension. This is a typical axisymmetric mode. The motion of membrane's points inside the circle is characterised by a smaller amplitude than the motion of the points outside of the nodal circle. The phase of the movement inside the nodal circle is opposite relative to the phase of the outer part of the membrane. At the frequency $f_2 = 650$ Hz a high-amplitude motion of woofer's outer suspension can be noticed. All remaining modes from the Table 1 can by analysed and described in a similar way. However, for higher system's modes identification of modes is more difficult and is based on some speculations [6,12]. For example, system's mode number 3 corresponds probably to the first mode of the mid-tweeter; the fifth system's mode probably fits in the second mode of the woofer and the basic resonance of the middle range tweeter; the eighth system's mode is probably the second mode of the mid-tweeter and so on. The higher frequency the more complicated mode shape and it is rather difficult to distinguish nodal lines and circles on the vibrating loudspeaker and to describe its vibrational patterns. Higher mode shapes of the complete loudspeaker system show that "something is going on" but give no details. Therefore, to obtain clear mode shapes, it is probably necessary to investigate the single loudspeaker placed in the system.

When values of modal damping are examined it can be stated that all values of damping are greater than these encountered for "normal" mechanical systems. In a consequence, mode shapes are complex in their character. It means that phases of motion of adjacent areas are not equal to 0 or 180 degrees, but they can possess any value from the range of 0 to 360 degrees. Moreover the modes are not so "sharp" and "clear" as they are in mechanics.

The shapes of the modes presented in the Table 1 express motion of all points in the parallel direction to the main axes of loudspeakers. Generally, the greatest motion is connected with the points situated on the membrane of the woofer, which is rather expected in the investigated frequency range. Significant, non zero motion can be also observed at areas corresponding to the tweeters, particularly for frequencies above 1 kHz. At some frequencies motion of the middle range tweeter was in phase while in others it was in antiphase to the motion of the woofer. In this cases the mid-tweeter was mainly excited electrically by the voice-coil since crossover frequency between the woofer and the middle range tweeter was equal 3.5 kHz and the slope of the crossover filter characteristic

was about 6 dB/octave. The crossover frequency between the mid-tweeter and the tweeter was 10.5 kHz and the slope of the filter was 6 dB/octave. The amplitude of the tweeter's movement was only slightly higher than the motion of the cabinet. Due to frequency range of measurement this result was not unexpected. Both tweeters were also excited mechanically by the vibrations of the cabinet. However, due to small amplitudes of cabinet vibrations and damping properties of the suspension of the membrane their influence on membranes motion was rather small.
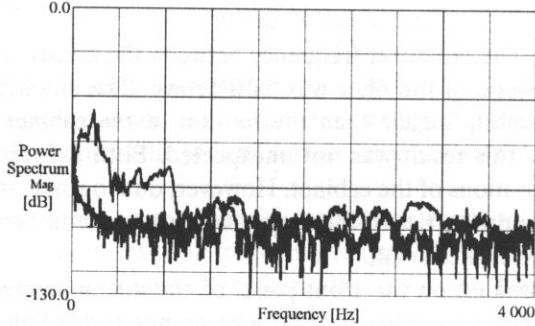
All active points situated on the front panel of the system correspond to the vibration of the speakers. The points beyond the membranes (excluding some points in the adjacent area to the woofer) may be characterised by a near zero velocity. It means that cabinet must be relatively rigid. It may transmit some vibrations (to excite the membranes of the tweeters in some way) but it behaves quite calm. However, mode shapes presented in the Table 1 are expressed in a linear scale. So it is difficult to asses the dynamic range differences between vibrations of the cabinet relative to the vibrations of the membranes. To asses this differences some additional measurements were carried out. These measurement were concerned with determination of the power spectrum of the background noise, and the power spectrum of the response signal in all measuring points. First, the power spectrum of the velocity was determined while exciting pseudo-random noise was delivered to the loudspeaker system. Then, the signal was turned off and the power spectrum of the velocity was determined again. The velocity measured in the later case corresponded to the background noise vibrations, thus the power spectrum of the background noise was determined.

Examples of the power spectra are presented in Figs. 4 a – g (the data presented in these figures were collected in points which are shown in Fig. 3 by means of black dots). Both curves are plotted in a logarithmic scale (in dB) and are scaled on a 20 dB units. Frequency is given in a linear scale on abscissa. The upper curve in each panel of the figure shows the magnitude of response spectrum measured for the pseudo-random noise excitation. Each bottom curve shows the power spectrum of background noise. The data collected in Fig. 4 depict response signals of the front panel (including membranes) only, since significant vibrations were observed on this plate only.
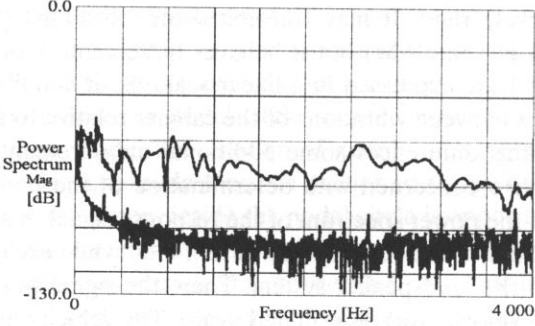
The level of the vibration connected with the stimulating signal is generally higher than the level corresponding to the background noise particularly for the points placed on the membranes and for all points situated no further than 3 cm from vibrating membrane (see. Fig. 4 c and 4 d). The largest difference, equal approximately 60 − 75 dB in whole frequency range, was observed in point no. 4 (the membrane of the woofer) and in point no. 3 (the rim of the woofer). Next, the measuring point (no. 2) placed on the plastic ring of the woofer showed also significant vibrations, about 30 dB above the background noise (Fig. 4 b). Significant vibrations were also observed in point no. 6 situated on the mid-tweeter membrane (Fig. 4 f).

The data presented in Fig. 4 e suggest that point no. 5 situated between the woofer and the middle range tweeter also vibrated significantly. These vibrations were about 20 dB above the level of the background noise in the frequency range up to 3 kHz. The vibration observed on the mid-tweeter might came from the electric stimulation and were not caused by vibrating air-volume bounded within the cabinet.
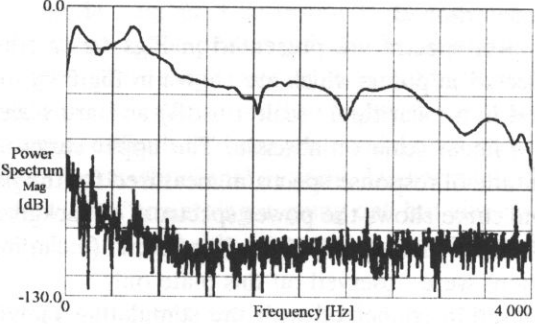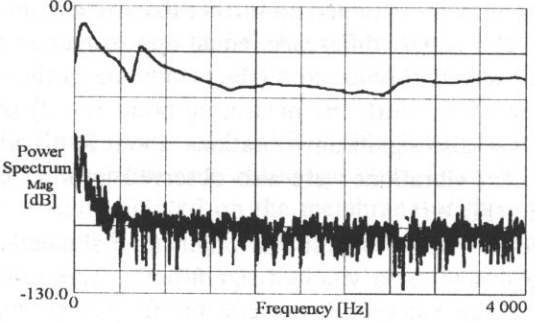
a) p. no. 1

Power
Spectrum
Mag
[dB]

0.0

-130.0

0   Frequency [Hz]   4 000

b) p. no. 2

Power
Spectrum
Mag
[dB]

0.0

-130.0

0   Frequency [Hz]   4 000

c) p. no. 3

Power
Spectrum
Mag
[dB]

0.0

-130.0

0   Frequency [Hz]   4 000

d) p. no. 4

Power
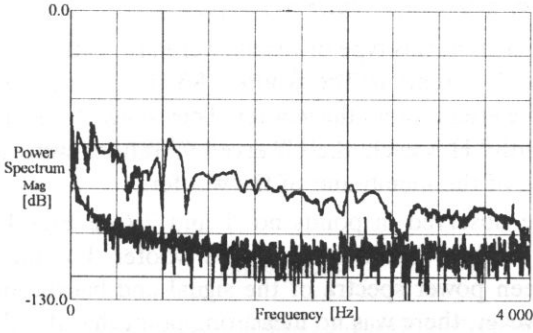Spectrum
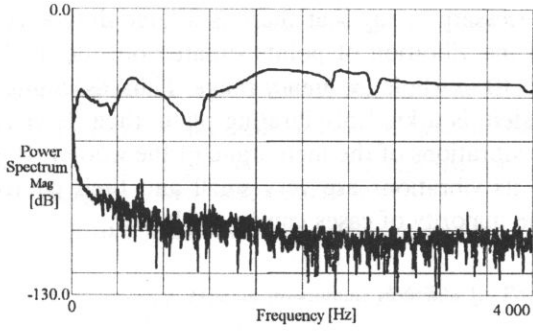Mag
[dB]

0.0

-130.0

0   Frequency [Hz]   4 000

[124]

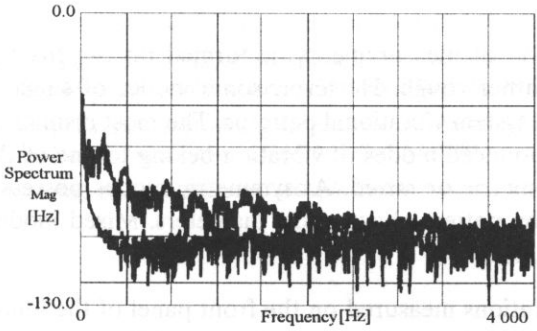e) p. no. 5

f) p. no. 6

g) p. no. 7

Fig. 4. Response signal – upper curve; noise signal – bottom curve for the loudspeaker system in the different points placed on front panel of the system (see also Fig. 3).

It is worthwhile to say that this type of results were also gathered for all points placed close (no further than 2 – 3 cm) to the woofer. So the data presented in Fig. 4 e are representative for whole area around the woofer. They suggest that in real conditions this area vibrates significantly. However, the observed vibrations were approximately 40-50 dB below the vibration of the membrane of the woofer.

No vibrations were observed in points no. 1 and 7 (see Figs. 4 a and g) and in all other points situated further than 3 cm from the woofer. For these points there is a small difference between power spectra of the signal and background noise in the low frequency region. However, there was no measuring point that this difference was higher than 5 dB in the whole frequency range. Similar measurements have been carried out on the back and side plates of the cabinet and very similar results have been obtained. On the other hand it is necessary to say that there is a huge difference between vibrations of the membrane and the vibration of points situated outside of the membranes. This difference is at least 60 dB in whole frequency range. Thus, assuming that the front panel of the loudspeaker system is a kind of vibrating plate, then its vibrations are 1 000 000 times smaller than the vibrations of the membrane of the woofer. It means that the front panel is almost calm, its vibrations are very small and their contribution to the total vibration pattern in the majority of cases can be neglected.

4. Conclusions

1) Spatial resolution of measurements performed on the front panel of the loud-speaker system was rather rough. However, some modes of single loudspeakers were found in the set of the system vibrational patterns. The most distinct, the most character-istic and the best pronounced modes of vibration belong to one of the three categories: axisymmetric, antisymmetric or mixed. Axisymmetric modes possess only nodal circles. Antisymmetric vibration patterns have nodal diameters. Mixed modes exhibit nodal cir-cles and nodal diameters.

2) The biggest vibrations measured on the front panel of the loudspeaker system are related mainly to the vibrations of the membranes of the speakers. Vibrations of the woofer and the middle range tweeter are at least 60 dB higher than vibrations observed in the most points of the front panel of the cabinet.

3) Vibrations of the front panel of the loudspeaker (excluding the areas of the mem-branes) are very week relative to the vibrations of the membranes. They are no higher than 5 dB above the background noise. The only exception is the area close to the woofer where the vibrations were about 20 dB above the level of the background noise.

4) The contribution of the vibrations of the front panel of the investigated loudspeaker system (excluding the membranes) to the sound field radiated by this system is rather small and may be neglected.

Acknowledgments

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