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DISSONANCES IN CARILLON CHORDS

G. BUDZYŃSKI and M. SANKIEWICZ

Sound Engineering Department
Faculty of Electronics, Telecommunications and Informatics,
Gdańsk University of Technology
(80-952 Gdańsk, ul. Narutowicza 11/12)

The sound quality of the carillons may pose problems, the more so, as carillon music has often a large, sometimes involuntary, audience. Particular carillon chords, although composed from well tuned bells, produce more or less dissonant sounds. A performed computer analysis of such chords, recorded in the St. Catherine Carillon belfry in Gdańsk, shows origins of dissonances and characterizes their properties. Some possible solutions leading to an enhancement of quality of carillon chords are discussed.

1. Introduction

The problem of dissonant vs. consonant sounding of complex sounds is often considered in musical acoustics. Chords consisted of most simple sounds, even those composed of two sounds, each containing only a few harmonic components may present difficulties at subjective assessment of their sound quality. The more so, when both sounds have rich spectra with many harmonic and inharmonic partials as in the case of bell sounds. Usually, the more rough seems the timbre of a chord, the lower is appreciated the quality of such chord, and it is qualified as sensory dissonance [17].

The phenomenon of dissonance plays an important role in the psychology of music and its theoretical aspects were thoroughly examined in the literature [7]. In the following study, however, authors deal with the practical aspect of the problem, which they encountered during investigations on bells, especially on sounds of bell-chords. Such bell-chords were analyzed on examples recorded from the St. Catherine's carillon, in Gdańsk. The recordings were made inside the belfry-tower of the St. Catherine church (see Figs. 1 and 2).

The carillon, made by the renowned Dutch foundry Koninklijke Klokkengieterij Eisbouts in Asten, was mounted in 1989, in the St. Catherine belfry, as a symbol of reconciliation between German and Polish nations, after all the atrocities of the Nazi regime and the Second World War.

The carillon consists of 37 bells. They are tuned according to the chromatic, equally tempered scale, ranging from c' to c'''' (i.e. from C4 to C7) [5]. Earlier investigation proved the high accuracy of internal and external tuning of the carillon bells [10]. The bells are listed in Appendix A.

The carillon is equipped with a digital steering system which actuates electromagnetically driven hammers (Fig. 3). The four biggest bells besides hammers are equipped with swinging mechanisms and clappers, to produce pealing sounds. The biggest bell also serves as a clock-bell.

The carillon play is executed either automatically, as preprogrammed melodies introduced into the system memory, or manually from a MIDI-keyboard. A traditional keyboard with mechanical system of hammers is to be installed.

Just experimenting with the use of MIDI-keyboard the authors had recently an opportunity to listen to various two-bell chords. Some sounded more-, other less-dissonantly, which became the starting point of an acoustic investigation reported below.

2. Investigation

The continuous progress in understanding the bell properties, in particular those of carillon bells, is due to numerous experimental investigations, the results of which were published in the past [1, 3, 11, 12]. Most of those were obtained in laboratory conditions, i.e. with a bell mounted in a stand, enabling to excite and record bell vibrations in a strictly controlled way. An unusual occasion in such situations was created in Europe after the Second World War, during which forced dismountings of many carillon- and peal-bells were ordered; thereafter some recovered bells were to be checked and measured before their eventual remounting [9].

There is, however, a need for a less precise yet easier and faster investigation of bell instruments, aimed at the general assessment of their sound quality, as well as description of their acoustic properties which have to be introduced into the historical fine-arts documentation of the instrument. Thus, the actual state of knowledge of bell sounds is to be applied to estimate objectively the quality of the bell instruments.

In the particular case of the St. Catherine's carillon, the aim of the investigation, reported here, was to deliver an objective description of the dissonant sounding of particular chords, based mainly on measurements and analyses. Such description may serve as a tool for musicians to prepare scores for carillon music, with frequent use of well sounding chords and omission of dissonant ones.

Measurements, i.e. recordings of carillon sounds were done electroacoustically *in situ* within the belfry. Although a pair of microphones was situated on a gallery at the level of the bells, and sounds were recorded with a two-channel digital recorder, only one channel was used for further processing and analysis.

Recorded sounds were analyzed in the laboratory. Neither direct access to the bells was provided nor any experiments with bell particular excitation or special recording process. This provision is conditioned by the general purpose of the investigation which should be easy to be performed on numerous bells all over the country, with the use of simple technical equipment.

Fig. 1. View of Gdańsk old town with St. Catherine's church belfry-tower in the middle.

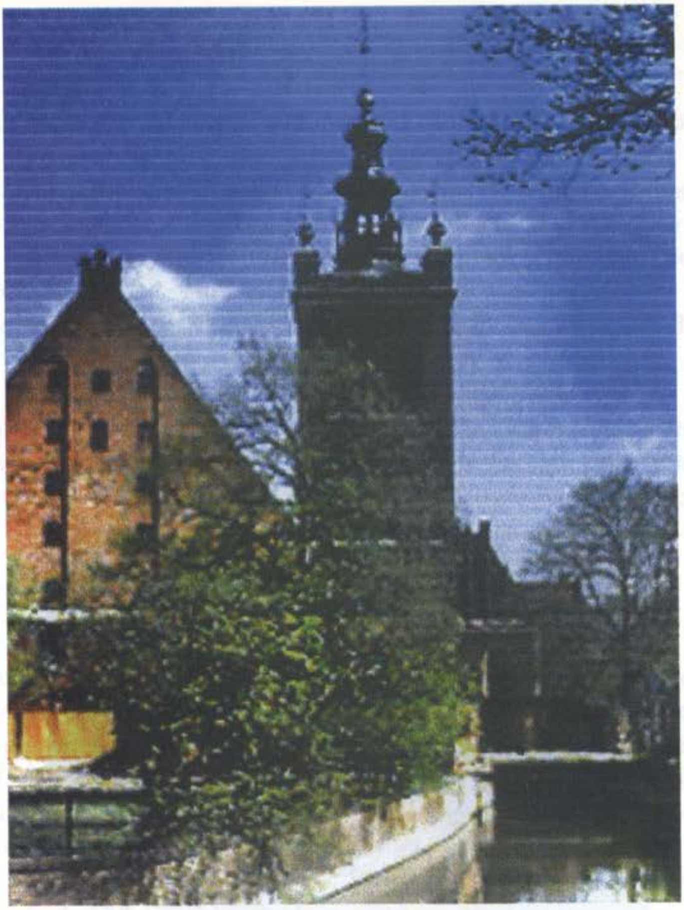

Fig. 2. View of the St. Catherine's tower from the West
(the Big Mill and Radunia river on the foreground).

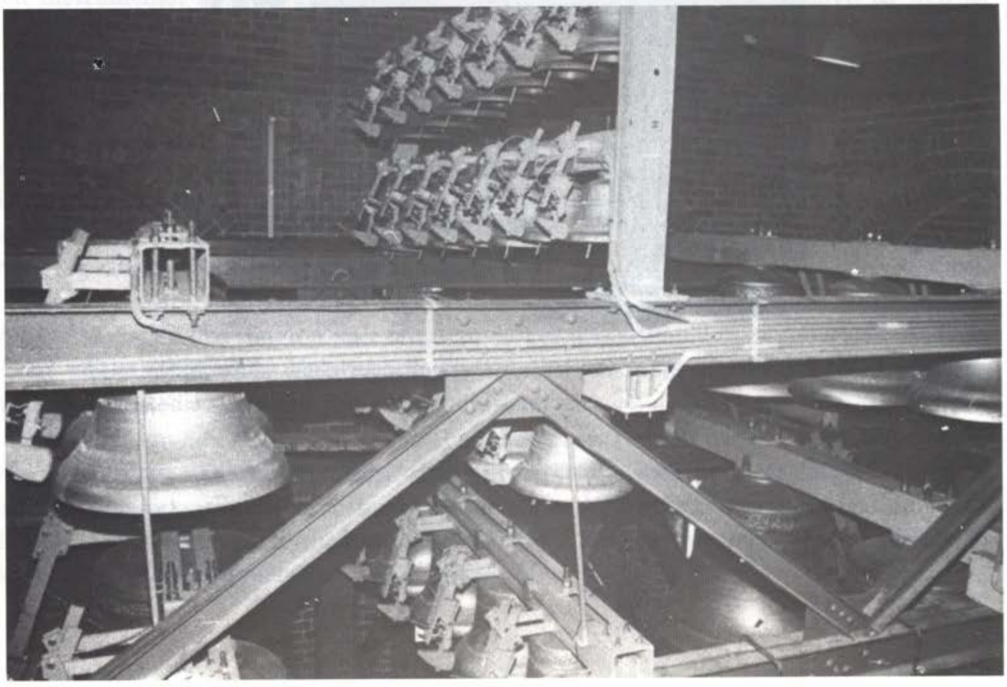

Fig. 3. System of electromagnetic hammers excitating carillon bells.

2.1. Theoretical approach

The general cause of dissonant, i.e. rough or unpleasant sounding of complex sounds, in particular two-bell chords, are, according to Helmholtz's theory [8], fast audible beats, produced by pairs of partials, having comparable levels and differing in frequency by less than three, to three and half semitones. It corresponds, in relation to the middle octave sounds, about 50 to 60 Hz while the lower limit, denoting the beginning of sensation of unpleasantness, may be estimated at about 20 Hz [2]. Further well known physio- and psychological factors influencing the dissonant sounding of musical sounds may be disregarded in this simplified approach.

As compared with other musical instruments, the sound of a bell is extremely rich in well developed partials, especially in its initial period, directly after a clapper- or hammer-stroke. E.g. special laboratory measurements performed on a church bell in 1982 by Perrin, Charnley and DePont showed the existence of 134 distinct, measurable partials [6]. Many of them were so close to each other in frequency that they were producing beats or warble. Yet such beats are mostly inaudible due to their small amplitudes. Some of them can, however, be distinctly heard. On the other hand, measurement of both partial frequencies producing such beats is usually beyond the possibilities of the typical analysis methods due to their limited separating power.

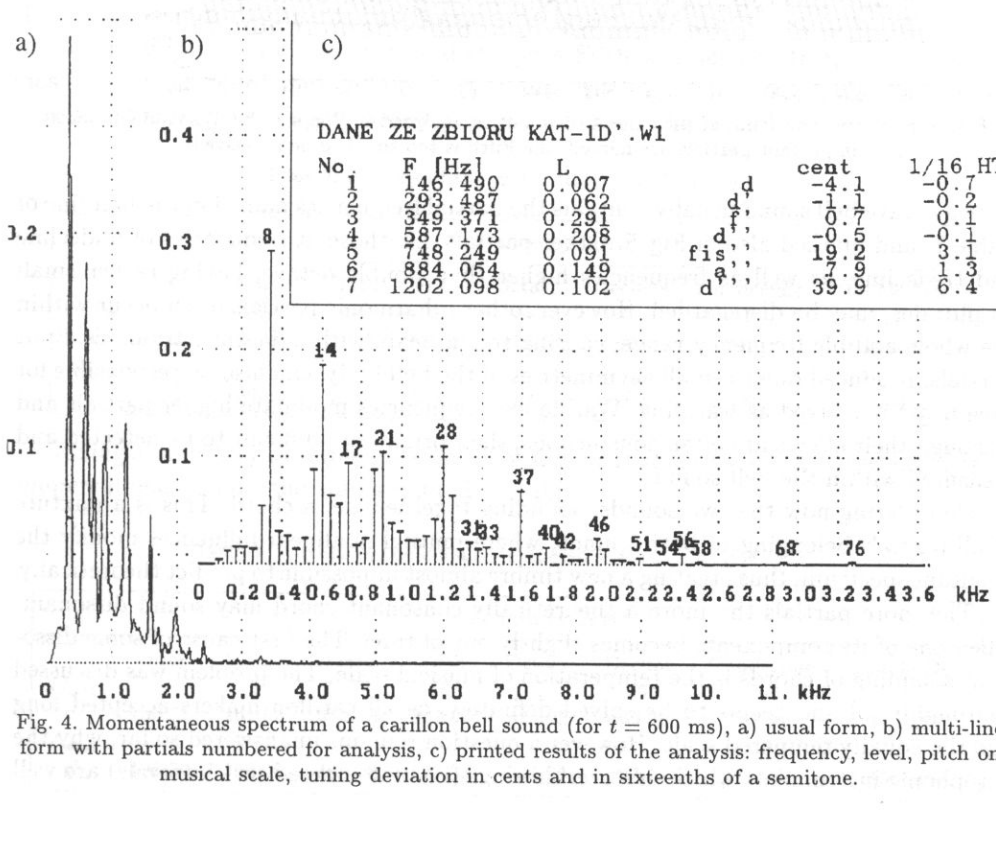

Fig. 4. Momentaneous spectrum of a carillon bell sound (for $t = 600$ ms), a) usual form, b) multi-line form with partials numbered for analysis, c) printed results of the analysis: frequency, level, pitch on musical scale, tuning deviation in cents and in sixteenths of a semitone.

Usual FFT analysis for an octave, good quality carillon bell, measured within this investigation, yields at least 8 distinct partials (see Fig. 4). The evolutive spectrum (Fig. 5) delivers here a better insight into the time-structure of the spectrum and, in particular, shows the amplitude proportions during partial decays.

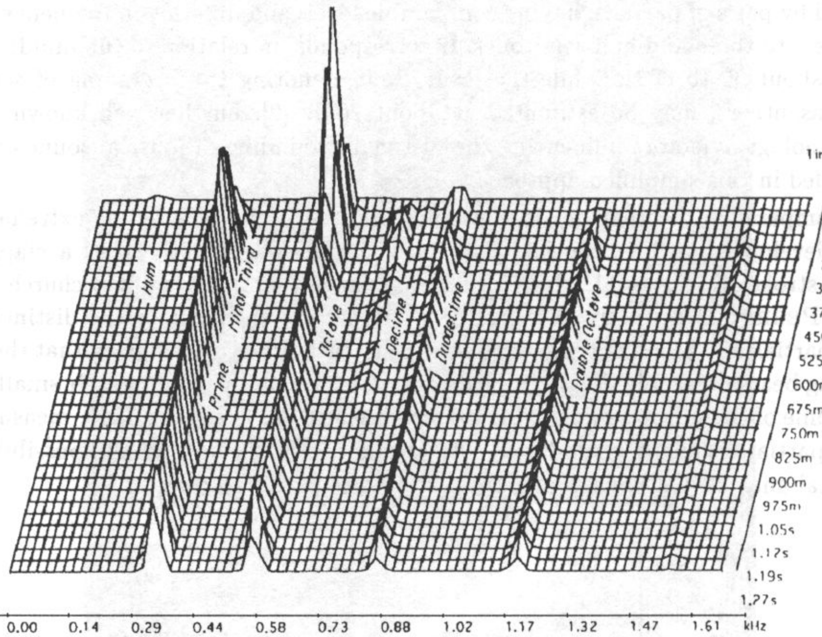

Fig. 5. Evolutive spectrum of the same bell sound as analyzed in Fig. 4 – the d′ sound (its most important partials are named; the Fifth is too small to be perceived).

An octave bell sound usually contains the partial frequencies named in the first line of Table 1, and marked also in Fig. 5. Other partials, i.e. those at frequencies of undecime and tredecime, as well as frequencies higher than double octave, having rather small amplitudes, may be disregarded. However, other inharmonic partials often occur within the whole audible frequency range, causing rough beats with adjacent harmonics. Split partials, produced due to small asymmetries of the bell body, can also be responsible for slow beats perceived as warbling. Warble low frequencies modulate higher partials and although their effects are often conspicuous, their origins are difficult to be detected and measured within the bell sound.

Considering now the two sounds, sounding together, i.e. a chord. This is a mixture of all partials belonging to both sounds, where mutual nonlinear influences modify the resultant spectrum, thus creating a new timbre almost impossible to predict theoretically.

The more partials the more a theoretically consonant chord may sound dissonant, when one of its components becomes slightly out of tune. The first cause of some dissonant sounding of chords is the temperation of musical scale. The problem was discussed thoroughly [3] and seems to be solved definitely, as all carillon makers accepted long ago the equally tempered scale. However a question remains unanswered so far, why the aerophonic instruments which use overblowings (i.e. untempered just intervals) are well sounding together with other equally tempered instruments of the orchestra, while the carillons, never accompanied by other instruments could not remain tuned according to one of the historical systems of the just intonation or of a meantone-temperament. Then some chords would sound better than equally tempered do, first of all the Minor Third where the difference of 16 cents from consonant sounding is easily noticeable.

Taking e.g. a weakly consonant chord of a Minor Third c′ – es′, in equal temperament, one can calculate the following partials (Table 1):

Table 1. Partials of the equally tempered Minor Third c′ – es′ (C4 – E♭4).

	Frequency in Hz							
	Hum	Prime	Third	Fifth	Octave	Decime	Duodecime	Double Oct.
f/f_H	1	2	2.378	2.997	4	5.04	5.99	8
c′	130.81	261.63	311.13	392.00	523.25	659.26	783.99	1046.50
Δ_{aud}	*24.75*		*22.01*	*57.09*		*37.01*		
es′	155.56	311.13	369.99	466.16	622.25	783.99	932.33	1244.51

Higher partials are not considered, as mentioned before, due to their small amplitudes. Checking the differences Δ_{aud} between consecutive partials one finds the four audible beating frequencies (22.01, 24.75, 37.01, 57.09 Hz), falling into the frequency range of audible rough beats. However, a small detuning of a partial may produce further rough beats, resulting in an increase of roughness of timbre.

Contrary to the example presented the pure Fifth without any temperament is the most consonant chord, besides the perfect intervals of prime, octave, duodecime etc. Here (see Table 2) no roughly beating frequencies are to be found.

Table 2. Partials of the just Fifth c′ – g′ (C4 – G4).

	Frequency in Hz							
	Hum	Prime	Third	Fifth	Octave	Decime	Duodecime	Double Oct.
f/f_H	1	2	2.4	3	4	5	6	8
c′	132.0	264.0	316.8	396.0	528.0	660.0	792.0	1056.0
Δ_{aud}								
g′	198.0	396.0	475.2	594.0	792.0	990.0	1188.0	1584.0

Taking as a further example the same chord, namely the Fifth c′ – g′ however, equally tempered, one obtains the values quoted in Table 3:

Table 3. Partials of the equally tempered Fifth c′ – g′ (C4 – G4).

	Frequency in Hz							
	Hum	Prime	Third	Fifth	Octave	Decime	Duodecime	Double Oct.
f/f_H	1	2	2.378	2.997	4	5.04	5.99	8
c′	130.81	261.63	311.13	392.00	523.25	659.26	783.99	1046.50
Δ_{aud}							↙ *58.73* ↗	
g′	196.00	392.00	466.16	587.33	783.99	987.77	1174.66	1567.98

Although the detuning of the natural harmony, caused by the superimposed equal temperament, is small, one audible beats frequency results. Contrary to this, partials entering into a Fourth chord c′ – f′ (C4–F4) yield the five rough beating frequencies: $\Delta_{\mathrm{aud}} = 23.31$; 38.40; 39.20; 42.77; 43.80 Hz. Similar calculations were done for other chords resulting for everyone in several beat frequencies.

A mathematical approach to a part of that problem was recently presented by SLOANE, who in his Correspondences [13, 14, 15, 16] calculated frequencies and amplitudes of beats caused by scale temperament and by small detuning of a partial. He considered not only dyad sounds but also triads, composed of Major and Minor Thirds.

Of course, there are other causes of inharmonicity of carillon bell chords and other beating frequencies. Their occurrence and sources might undergo a broader theoretical investigation. For this study, however, from a practical point of view, their influence may be taken into consideration by analyzing the global dissonance effects occurring at particular chords. Such approach required the sounds of all carillon bells to have been recorded in order to analyze them and subjectively assess in laboratory conditions. For obvious reasons multiple sound experiments *in situ* were impracticable and intolerable for the public in the neighbourhood.

2.2. Sound recording and digital processing

All carillon bells have been recorded by means of a microphone system (Neumann KM83i) and a MiniDisc recorder (Sony MZ1). Microphones were situated inside the belfry, at the height of the bell gallery. Bells were actuated from the MIDI-keyboard, switched into the carillon computer.

Analyses were executed with the aid of a Macintosh Quadra 840 AV computer, equipped with an Audiomedia II Chart. Two software programs were used: the Sound Designer II (DSP/FFT), and the specially designed FFT-anl and FFT-lab programs, which, beside of evaluating sound spectra, permitted to measure the damping rate of a partial in dB/s. This value may be applied as one among several quality criteria for bells.

The investigated chords were synthesized from separate bell sounds by means of the digital processing methods available in the Sound Designer software. Beats were observed using a tunable diapason contained within the same software. Measurements of partial frequencies were executed by means of the flexible and highly accurate FFT-lab program. This program uses a local tracking of phase versus time, to introduce precise frequency corrections to all significant partials within the evaluated sound spectrum. An automated program FFT-anl permitted for quick evaluations and comparisons of the investigated sound spectra.

2.3. Results

The presented results show, first of all, the evolutive spectra of the recorded and analyzed sounds of carillon bells. Among those sounds five examples are selected to illustrate differences between particular spectra, namely c′, cis′, f′, g′, c″ (Figs. 6–10).

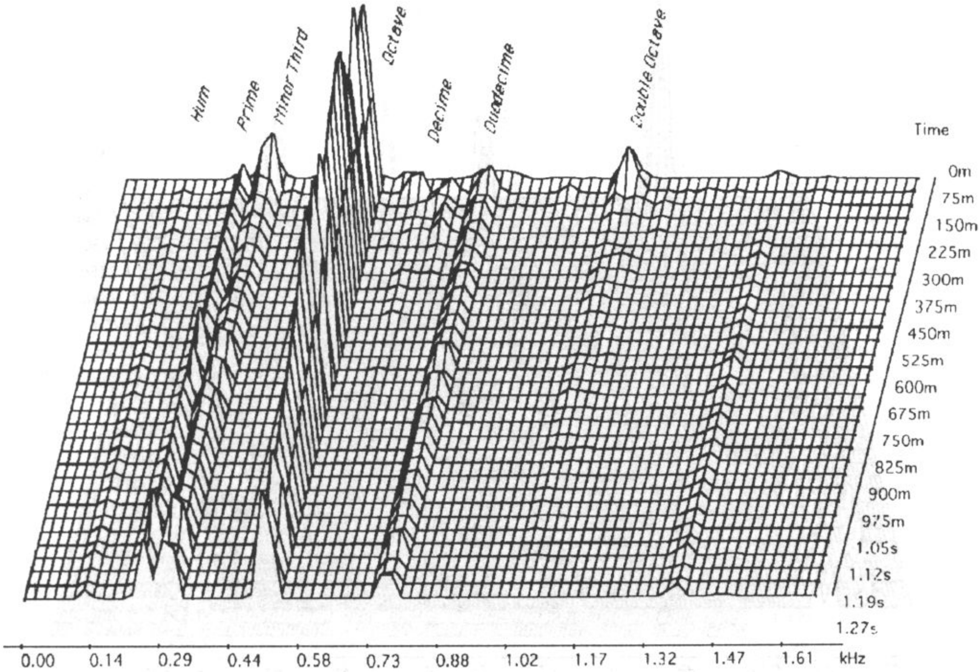

Fig. 6. Evolutive spectrum of the bell sound c′.

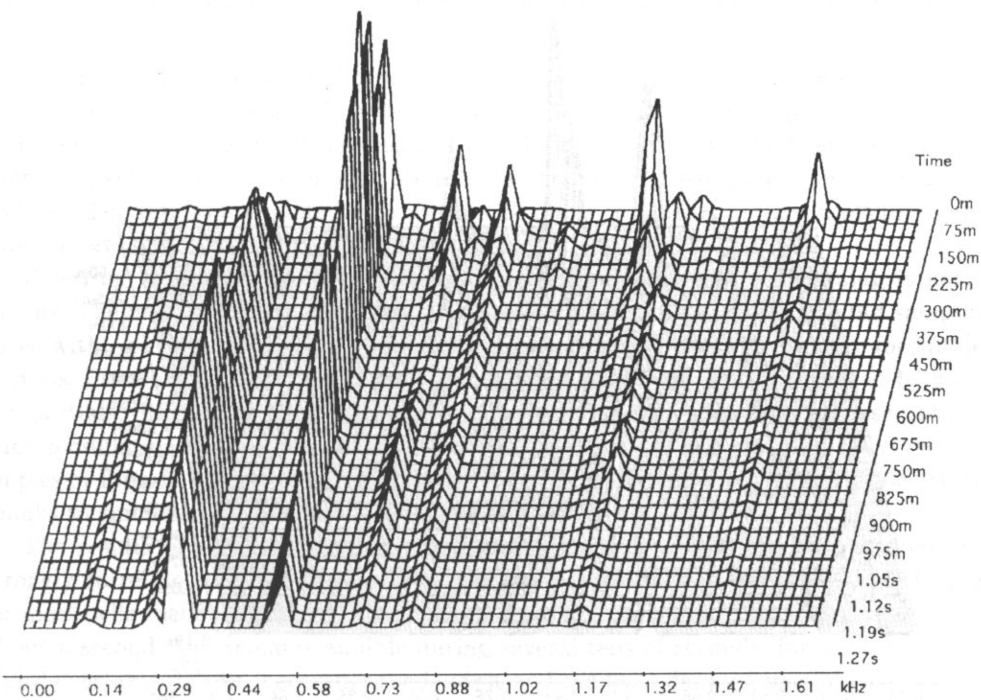

Fig. 7. Evolutive spectrum of the bell sound cis′.

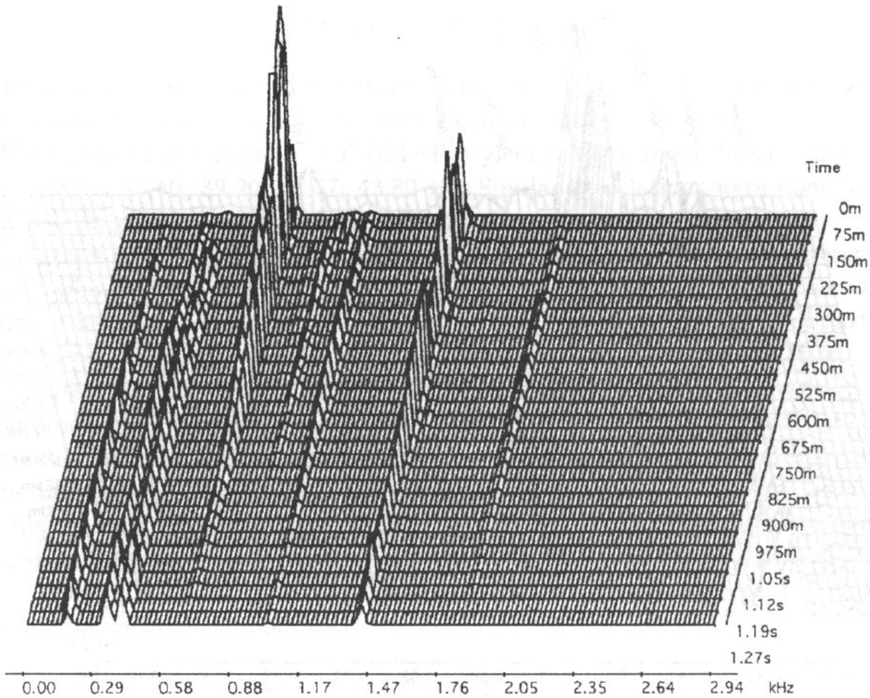

Fig. 8. Evolutive spectrum of the bell sound f′.

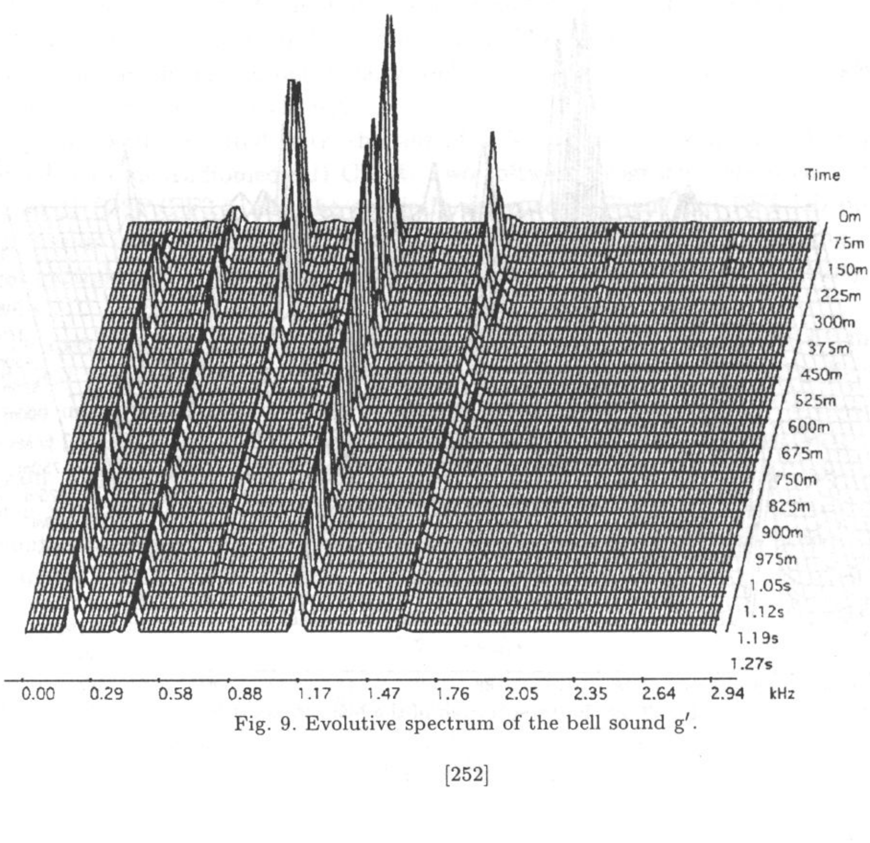

Fig. 9. Evolutive spectrum of the bell sound g′.

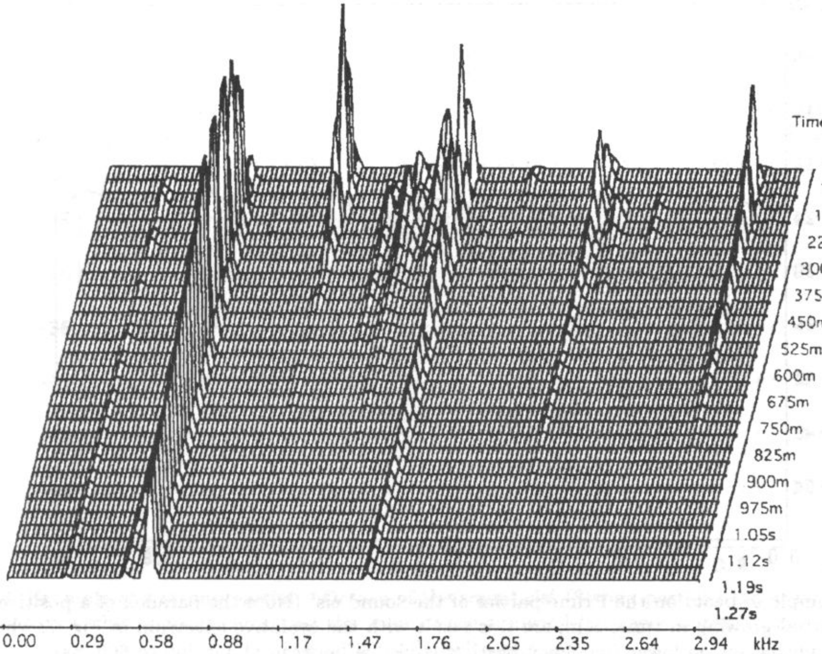

Fig. 10. Evolutive spectrum of the bell sound c″. (Note the prevailing power of the Minor Third).

Moreover, prints of the selected typical examples of computer evaluated wave-forms for the specified partials are joined, in order to visualize their properties, especially the levels and frequencies of the resulting beats (Figs. 11 – 16), as well as amplitude and phase characteristics, serving for precise corrections of measured partial frequency (Figs. 17 and 18). The results are quoted in Appendix B. Some comments are given within figure captions.

Referring to the presented results the precision of frequency determining is noteworthy. The most values obtained from repeated evaluations of recorded sound samples agree with one per mil accuracy (see e.g. Fig. 17). However, this is true only for stable partials. Contrary to these, there are partials modulated with beats, produced by split sub-partials, difficult to be determined within a momentaneous spectral presentation. Such modulations are often conspicuous within evolutive sound spectra (see some examples in Figs. 6 – 10). The above analyzed sound samples may be assessed subjectively thanks to simultaneous multiple sound demonstrations.

The results of spectral amplitude measurements and of damping rate evaluations strongly correlate with the total sound sensations for the investigated bells. Although for most bells the Hum (Lower Octave) prevails over all remaining partials already after about a second, and remains audible during several tens of seconds, for some bells the Third overtakes its role, becoming the loudest and the least damped partial (see e.g. the c″ bell evolutive spectrum shown in Figs. 10 and 14). It proves that amplitude relations participate as much as frequency relations in creating a particular sound sensation of a bell sound.

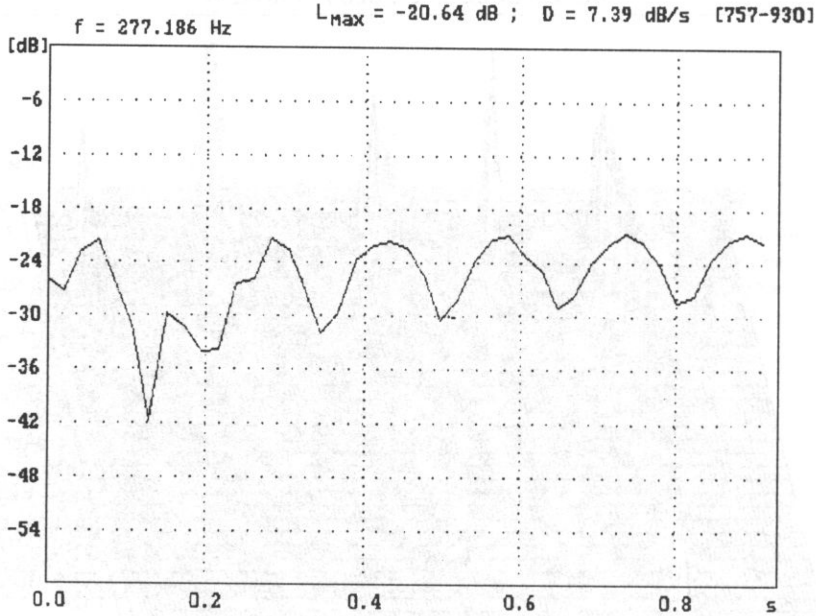

Fig. 11. Example of beats on the Prime partial of the sound cis′ (Note the paradox of a positive value of D – a partial growing in time; compare this result with the evolutive spectrum in Fig. 7, where an unusual pit on lower frequency partials is visible between about 300 to 600 ms).

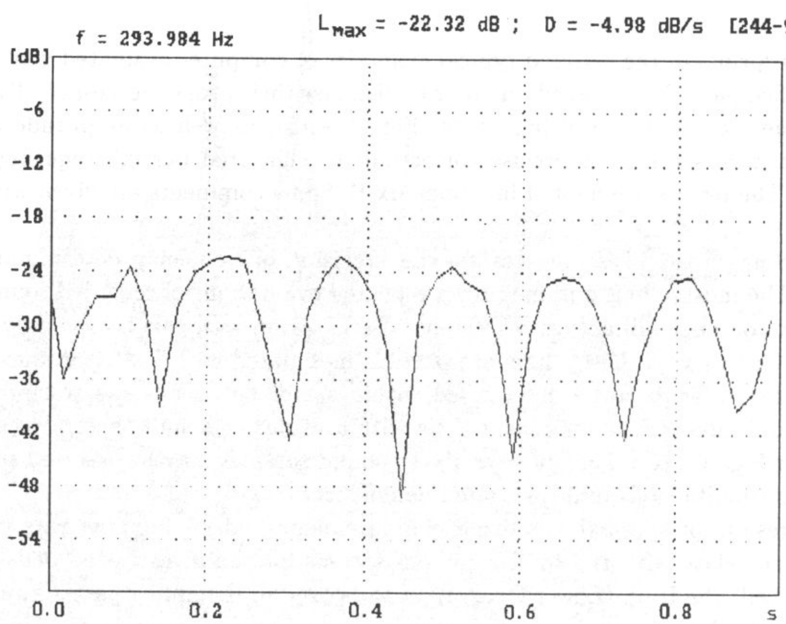

Fig. 12. Example of beats on the Prime partial of the sound d′ (The beats deeply modulate the partial and their waveform is regular; beat frequency is about 7 Hz; despite their depth these beats can not be perceived on the evolutive spectrum in Fig. 5).

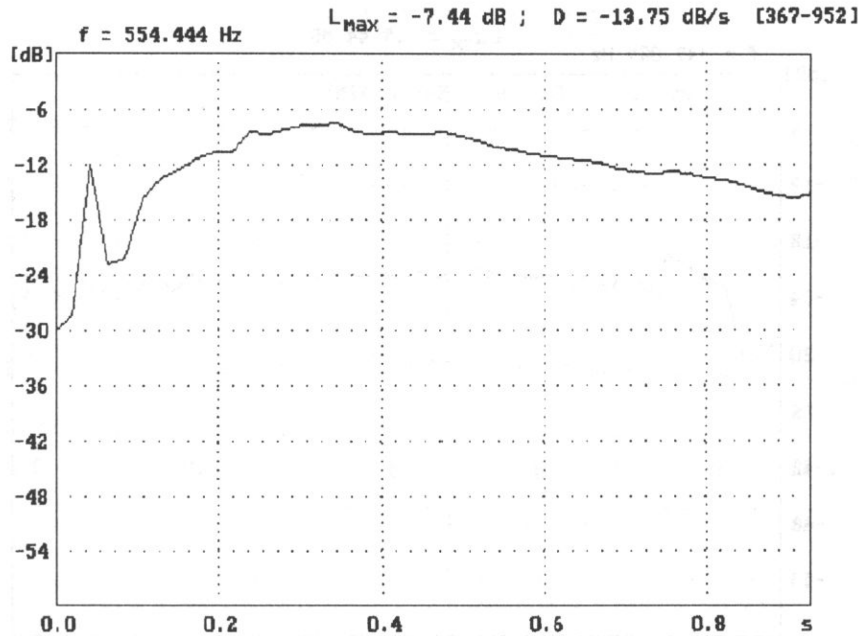

Fig. 13. Example of a strong partial – Octave of the sound cis′ (Strong partial have usually flat phase characteristics which permit correcting the value of the analyzed frequency automatically and evaluate it with six digital precision).

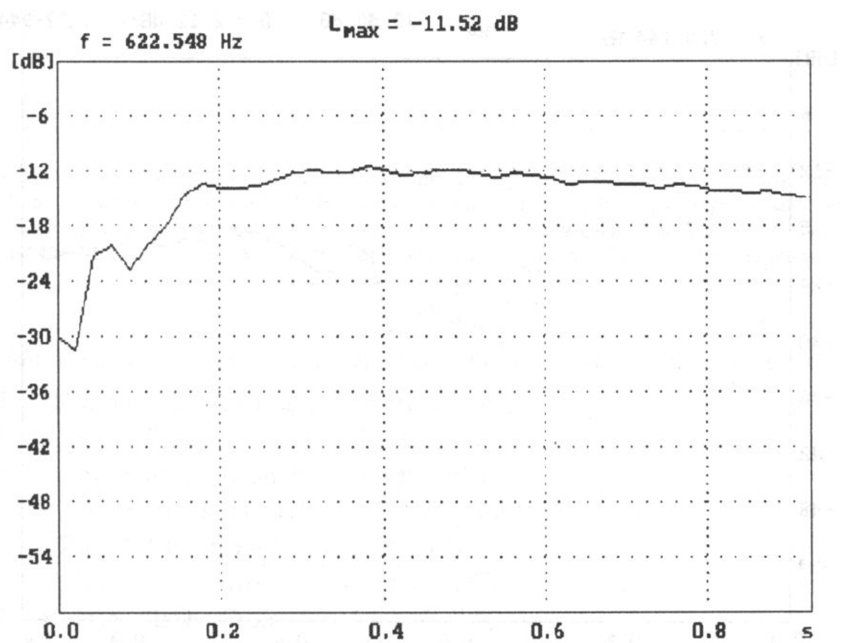

Fig. 14. Example of a strong partial – Minor Third of the sound c″ (The strongest partial, easily noticed in Fig. 10, having extremely low damping rate; such unique property of this bell, basically advantageous, disqualifies it rather as a component of bell chords).

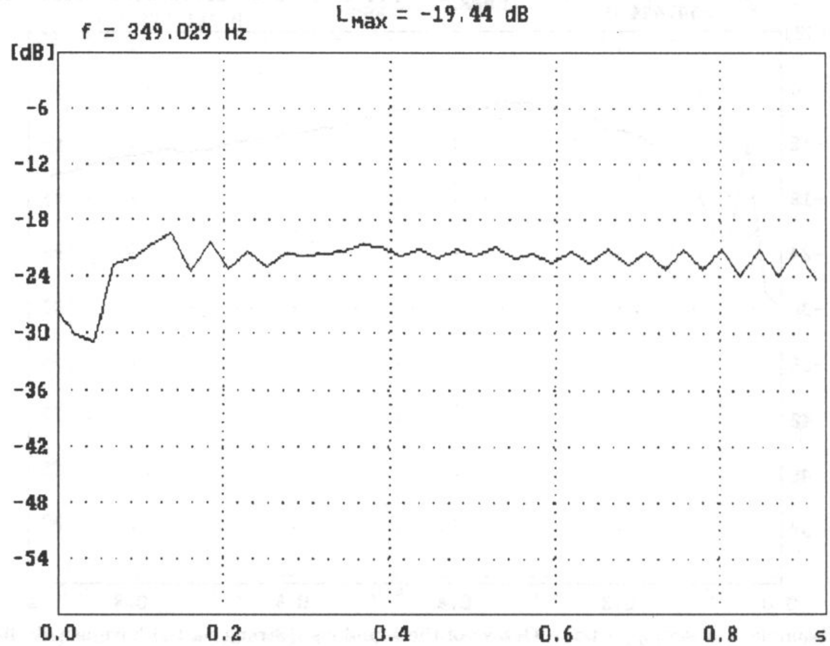

Fig. 15. Example of beats on the Prime partial of the sound f′ (Beats with frequency of about 23 Hz shallowly modulating the partial).

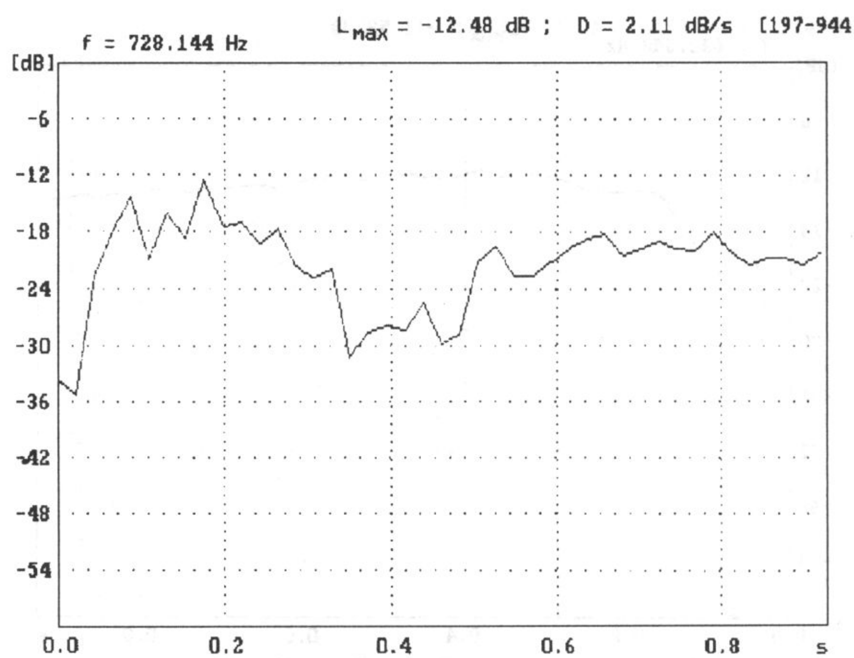

Fig. 16. Example of beats on the Fifth partial of the sound cis′ (The pit mentioned in the comment to Fig. 11 is distinct here, as well as beats with frequency of about 8 Hz).

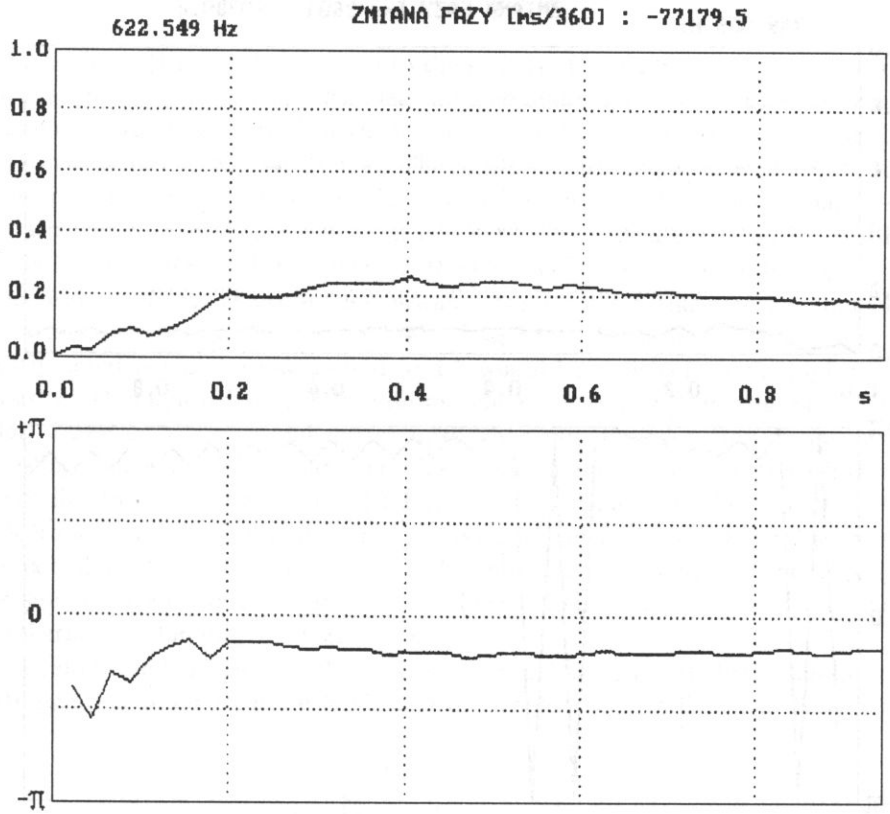

Fig. 17. Example of amplitude and phase versus time characteristics done during the analysis process with FFT-lab program on the Minor Third partial of the sound c″ (Strong amplitude and flat phase characteristics permit attaining a precise correction for frequency evaluation; here, one step before reaching the final value displayed at the completed evaluation, see Fig. 14, the frequency value differs only by 0.001 Hz).

The following chords were synthesized from the recorded sounds in seven categories:

- I 10 Minor Thirds (from c′ – es′ to a′ – c″);
- II 9 Major Thirds (from c′ – e′ to as″ – c″);
- III 8 Pure Fourths (from c′ - f′ to g′ – c″);
- IV 7 Augmented Fourths (from c′ – fis′ to fis′ – c″);
- V 6 Pure Fifths (from c′ – g′ to f′ – c″);
- VI 5 Minor Sixths (from c′ – as′ to e′ – c″);
- VII 4 Major Sixths (from c′ – a′ to es′ – c″).

The remaining intervals as inherently dissonant were abandoned.

The sounds of all the mentioned chords were studied objectively and subjectively. Subjective assessments averaged were carried on by five experienced listeners (authors and their co-workers).

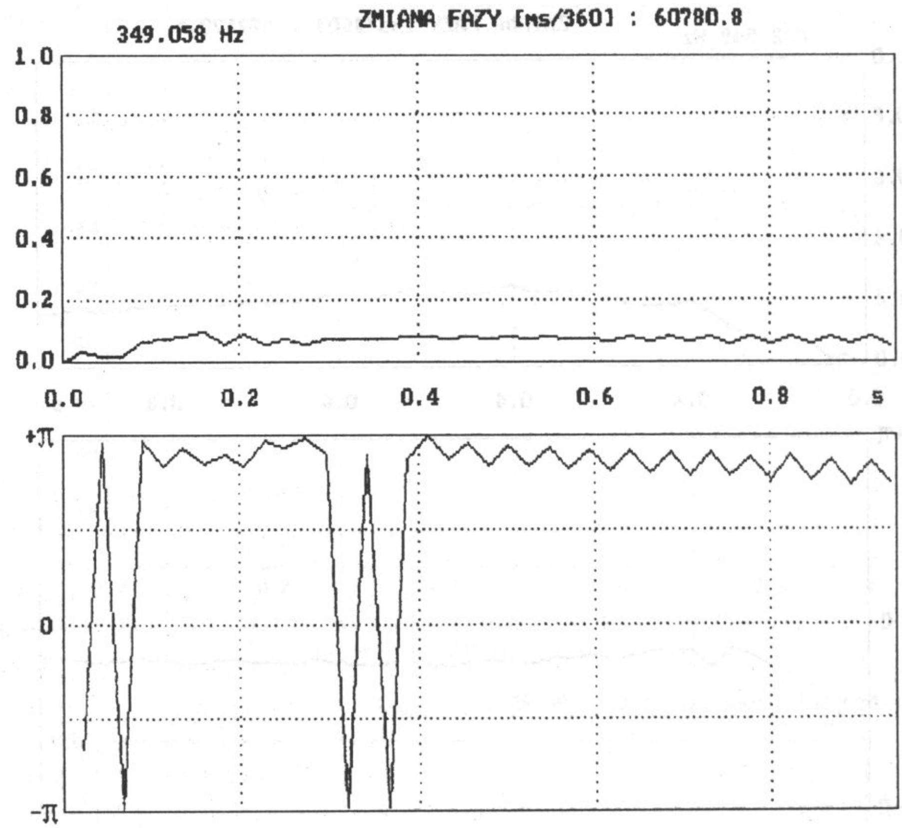

Fig. 18. Amplitude and phase versus time characteristics done during the analysis process with FFT-lab program on the Prime partial of the sound f′ (Same sound as in Fig. 15, however, before the final correction of evaluated frequency; phase fluctuations, with the exception of jumps by 2π, are synchronous with amplitude beats on about 23 Hz).

In each of the category about 50% of the well sounding chords were selected, while others were disqualified. As criteria served: lack of audible beats, good fusion of two sounds, uniform sound decay, long duration of both lowest frequency partials (hums) and a general sensation quality. Then, bells entering into those selected chords were qualified as well fusionable. The three bells, selected as good in that way among thirteen investigated, were: g′, a′, and e′. The fourth one, the c″, perhaps the best sounding otherwise, fell out of fusion due to its too loud Minor Third partial, which masked its other long sounding frequencies. The worst sounding among the bells of the middle octave was the cis′ bell.

The general result of study has showed that chords synthesized exclusively of well sounding bells sounded consonantly, while others less or more dissonantly. It means that the tuning precision as required in the specialized literature [11] does not determine a sufficient criterion of sound quality for contemporary carillons.

3. Sounds of a former carillon

Having devoted so much attention to the sounds of actually existing St. Catherine's carillon, authors cannot give up the idea of disseminate information on an old recording containing several notes played by the former St. Catherine carillon, in the years before the outbreak of the Second World War. The recording, existing as an analogue tape copy (or most probably multiple re-copy) of an acetate disc remained within the content of the sound archives of the Gdańsk Broadcasting Centre and was in the seventies transferred to the Sound Engineering Department of the Gdańsk University of Technology.

The recording contains two single-voice ancient melodies lasting together about 1 min. and played on six bells only: h′, c″, d″, e″, f″, and g″. The duration of particular notes is mostly about one and a half second, which would be sufficient for the high precision analysis, which might deliver precise information about tuning properties of the ancient carillon. However, the vow and flutter introduced into the recording during multiple rerecordings causes the phase characteristics to become instable to a degree, which makes it impossible to use the FFT-anl or FFT-lab programs efficiently. The sound quality of the recording and the tuning of sounds are poor. Nevertheless the general structure of those six bells evolutive spectra were studied by means of the Sound Designer II program. The results are shown in the Figs. 19 to 24. The comparisons to the spectra of the new St. Catherine carillon may be instructive.

It should be added here that the ancient carillon, having had 37 bells, had been built in 1910 by the bellfounder Franz Schilling from Apolda. The carillon was dismounted in

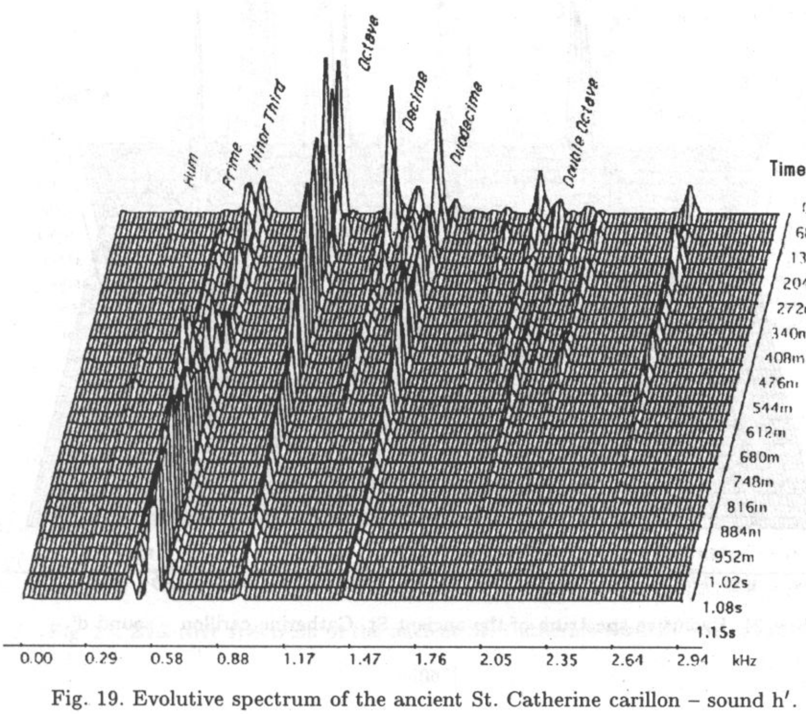

Fig. 19. Evolutive spectrum of the ancient St. Catherine carillon – sound h′.

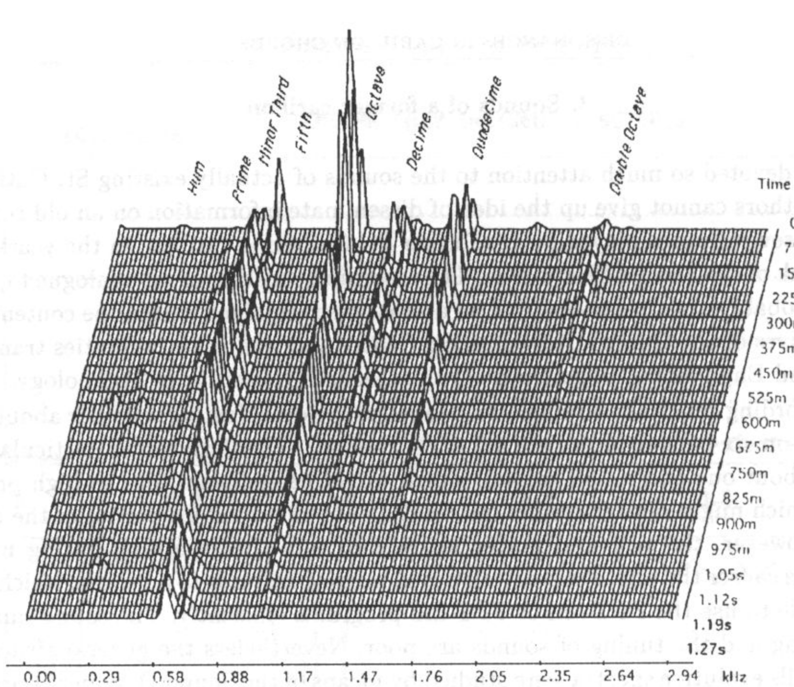

Fig. 20. Evolutive spectrum of the ancient St. Catherine carillon – sound c′.

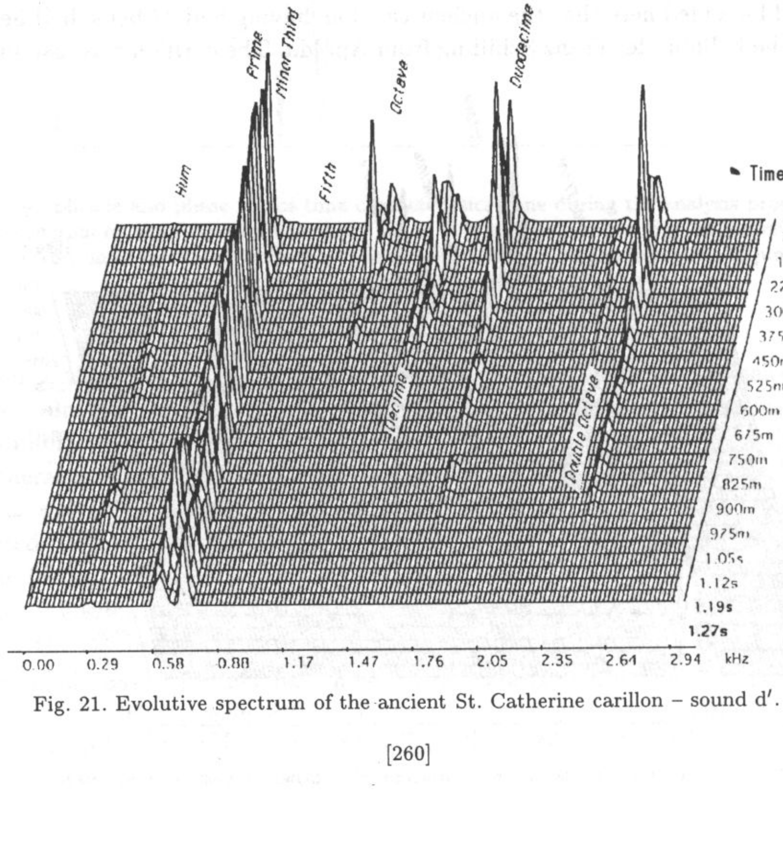

Fig. 21. Evolutive spectrum of the ancient St. Catherine carillon – sound d′.

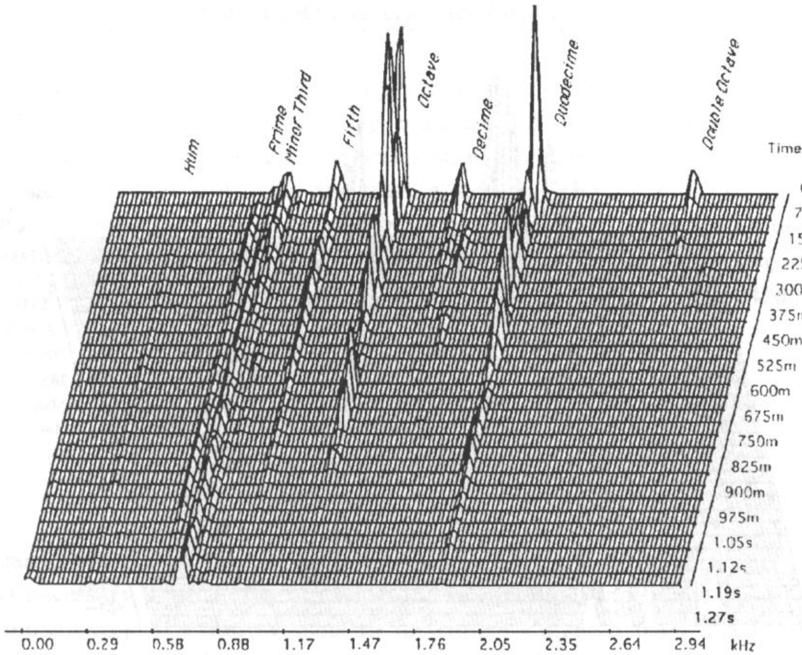

Fig. 22. Evolutive spectrum of the ancient St. Catherine carillon – sound e′.

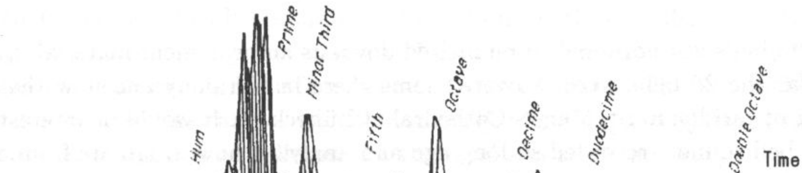

Fig. 23. Evolutive spectrum of the ancient St. Catherine carillon – sound f′.

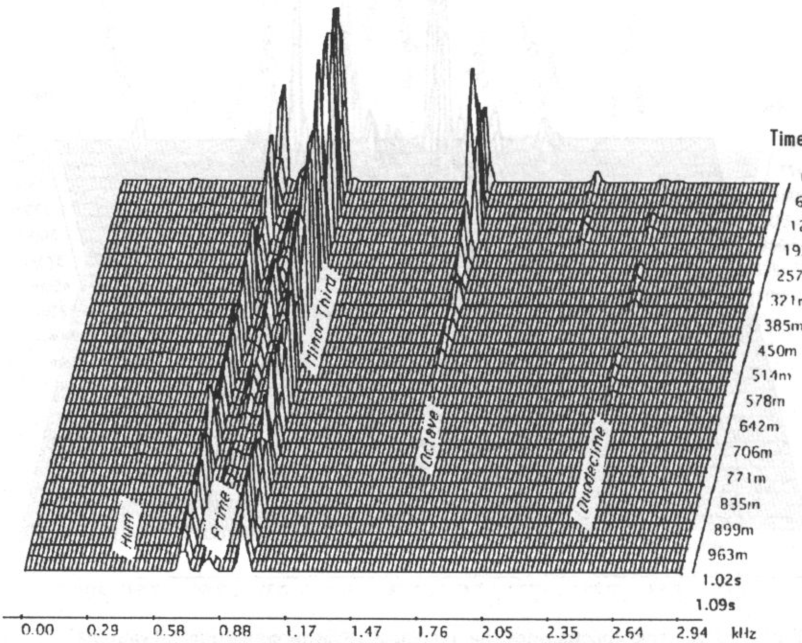

Fig. 24. Evolutive spectrum of the ancient St. Catherine carillon – sound g′.

1942 when its bells were ordered to be melted down as an armament material. Luckily, after the War the 28 bells were recovered somewhere in Germany and now they have become part of carillon in St. Mary's Cathedral at Lübeck [4]. It would be interesting to check if the bell sounds, recorded so long ago and analyzed now, originate from among the recovered, still existing bells, or from those melted down during the War.

4. Critical remarks

The study presented is based entirely on recorded sound samples of the investigated bells. The monophonic recordings, taken for all bells, from the same microphone position inside the belfry, do not represent accurately the true sound images of the particular bell sounds. This is due to different source – microphone distances and moreover – to sound waves reflected from the inner brick walls of the square belfry tower, causing intense standing waves effects. Commenting this imperfection of the method employed one can remark that it might be replaced by another solution, however, at incomparably higher cost of labour and time. Besides, even with multi-channel recordings and elaborate methods of averaging the problem will remain basically the same.

The study was not aimed at affording quantitative results on required tuning precision of consonant carillon sounds. The results presented are given rather as examples of imperfections of the timbre perceived by a common listener of carillon music, as an attempt to define their character, and as hints to their possible removal.

Authors think that the attention of acousticians, working on problems connected with carillon music, was, so far, mostly concentrated on improvement of the tuning methods of carillon bells. An amazing precision was achieved. However, not so much is known on the mechanism of beats and warble, affecting the perceived sound quality of bell sounds, in particular the quality of bell chords. Without suppression of warble or at least reduction of its level by about twenty dB, the use of chords for playing carillon music should be strictly limited, and applied only to cases of a few preselected harmonies of sounds.

5. Conlusions

The carillon music in the past was single-voiced. This was due to carillonists' difficulty and effort necessary to set in motion two or more keys simultaneously. The introduction of electromagnetical traction systems to instruments steered from a MIDI-keyboard, allowed for frequent use of chords in carillon music. This opens a new problem of higher requirements relative to the quality of carillon bell sounds.

The assessment of sound properties of the investigated bells presented ought to be evaluated in relation to all remaining information on those bells, in particular their sound spectra, their tuning precision etc. Part of the information is contained in the Appendices. The more detailed information is without doubt available at the belllfoundry which had produced this carillon i.e. the Koninklijke Klokkengieterij Eijsbouts in Asten. After a proper evaluation and discussion, the results of this study may turn out to be a useful suggestion for further improvements in carillon design and tuning.

The result of this study may also serve as a hint for musicians playing carillons with the use of MIDI keyboards. It may turn their attention to the necessity of checking the fusion-ability of particular bell sounds, as well as sound quality of particular chords, before decisions on arrangement and mode of performed music composition are undertaken.

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Appendix A. St. Catherine's carillon – list of the bells [5]

No.	Note	Diameter [m]	Mass [kg]
1	c′	1.564	2200
2	cis′	1.474	1840
3	d′	1.390	1535
4	dis′	1.310	1285
5	e′	1.235	1075
6	f′	1.166	905
7	fis′	1.100	760
8	g′	1.038	640
9	gis′	0.980	535
10	a′	0.925	450
11	ais′	0.873	375
12	h′	0.824	320
13	c″	0.774	263
14	cis″	0.727	220
15	d″	0.686	186
16	dis″	0.646	162
17	e″	0.611	141
18	f″	0.580	123
19	fis″	0.554	108
20	g″	0.532	95
21	gis″	0.510	84
22	a″	0.487	74
23	ais″	0.468	66
24	h″	0.448	59
25	c‴	0.431	53
26	cis‴	0.413	48
27	d‴	0.396	44
28	dis‴	0.382	40
29	e‴	0.366	35
30	f‴	0.352	32
31	fis‴	0.341	30
32	g‴	0.328	27
33	gis‴	0.317	25
34	a‴	0.306	23
35	ais‴	0.296	21
36	h‴	0.287	19
37	c⁗	0.270	18

Appendix B. The measurement results

BELL		Hum		Prime		Third		Fifth	
No.	Note	f [Hz]	Δf [ct]	f [Hz]	Δf [ct]	f [Hz]	Δf [ct]	f [Hz]	Δf [ct]
1	c′	130.814	0	261.505	−0.8	311.097	−0.1	399.313	31.2
2	cis′	138.554	−0.5	277.186	0	329.752	0.6	*	*
3	d′	146.517	−3.8	293.984	1.8	349.367	0.6	*	*
4	dis′	155.586	0.2	311.273	0.8	369.863	−0.7	*	*
5	e′	164.853	0.4	329.784	0.4	392.784	3.4	*	*
6	f′	174.431	−1.9	349.029	−0.9	414.817	−2.1	*	*
7	fis′	184.556	−4.2	369.756	−1.2	440.308	1.2	*	*
8	g′	195.856	−1.3	*	*	466.422	0.9	*	*
9	gis′	207.301	−2.9	415.006	−1.3	494.718	2.9	626.461	11.6
10	a′	219.987	−0.1	439.591	−1.7	523.191	−0.2	*	*
11	ais′	232.949	−0.9	466.328	0.6	554.973	1.9	703.457	12.3
12	h′	247.124	1.2	494.288	1.4	587.822	1.4	724.017	−37.8
13	c″	261.651	0.1	523.339	0.2	622.548	0.8	790.233	13.7

BELL		Octave		Decime		Duodecime		Double Oct.	
No.	Note	f [Hz]	Δf [ct]	f [Hz]	Δf [ct]	f [Hz]	Δf [ct]	f [Hz]	Δf [ct]
1	c′	523.031	−0.6	651.256	−21.2	786.254	4.9	1074.064	45.1
2	cis′	554.444	0.2	728.144	−28.3	833.965	6.9	1153.421	68.4
3	d′	587.161	−0.5	748.261	19.2	884.049	7.9	1224.119	71.4
4	dis′	621.685	−1.7	790.487	14.2	934.989	4.9	1293.464	66.8
5	e′	659.629	0.9	*	*	994.508	11.7	1377.721	75.9
6	f′	698.139	−0.8	906.558	51.4	1054.207	12.7	1461.087	77.7
7	fis′	739.086	−2.2	952.904	37.8	1112.509	5.8	1539.701	68.4
8	g′	784.011	0	985.706	−3.7	1183.991	13.7	1643.025	80.9
9	gis′	829.934	−1.5	1065.238	30.7	1227.781	−23.5	1727.311	67.5
10	a′	880.308	0.6	1143.772	53.8	1325.814	9.5	1837.195	74.8
11	ais′	932.059	−0.5	1204.972	44.1	1408.349	14.1	1954.346	−18.7
12	h′	988.183	0.7	1294.448	68.1	1489.629	11.2	2062.688	74.7
13	c″	1046.810	0.5	1351.806	43.1	1575.208	7.9	2177.602	68.6

* denotes uncertain result due to insufficient partial level or excessive beats.

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