ARCHIVES OF ACOUSTICS
22, 3, 297–306 (1997)

TOTAL SCATTERING CROSS-SECTION OF A NONDEFORMABLE SPHERE PARTIALLY INSONIFIED AT A CIRCULAR ANNULAR SPOT

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The axisymmetric problem of scattering of a finite sound beam with an annular cross-section by an acoustically soft or immovable rigid sphere immersed in a fluid medium is considered. The pressure in the incident quasi-plane wave is represented in terms of the partial impact parameters by the superposition of the spherical harmonics with the Lorentz multipole resonance distribution. The analysis of the total scattering cross-section vs frequency and wave beam parameters is performed. It is shown that a high-resolution of the total scattering cross-section resonances corresponding to the Franz creeping waves is achieved when the base rays of the incident wave are grazed on the sphere and the beam is narrowing. The particularities of these resonances are considered for the case of soft and rigid spheres.

1. Introduction

The problem of interaction of the sound waves with spherical objects in a liquid has been considered by many investigators for a long time. Although the formal solution of the corresponding mathematical problem is relatively simple, its interpretation meets with some difficulties caused by the complexity of the physical phenomena of the acoustic scattering process (see, e.g. [3, 4]). It is known that the structure of the echo-signal from a deformable solid sphere is formed by the geometrical waves of the reflection and transmission as well as by the reradiated diffractive (surface and creeping) waves excited when the sound rays incident on the obstacle's surface at critical angles or grazely. The isolation from echoes or others contributions can be made using, for example, the resolution property of the angular spectra of the incident wave that is peculiar for finite wave beams [9, 15]. Thus, insonifying nondeformable spheres or circular cylinders by well collimated and centrally incidented bounded acoustic beams, the differential cross-sections vs circular frequency and the beam width were studied [6, 7, 13]. In particular, differences between the sonar cross-sections for the acoustically soft and rigid objects were found [13]. It was shown that the structure of those cross-sections are formed by a superposition of specularly reflected waves and of the waves reradiated by the edge points of the sound spot on the scattering surface as well as of the Franz creeping waves. In order to investigate the spectral properties of the diffracted waves of the any type in a "clear" form, it is necessary to concentrate the incident sound beam in the neighbourhood of the critical point of wave excitation on the scatterer's surface. Then the contribution of this wave in the total echo will be most intensive. Hence, in several experiments an elective reradiation of the surface acoustic waves of Rayleigh and Lamb types was observed when a finite sound beam incidents on solid and hollow elastic circular cylinders [2, 9, 18], or of circumferential waves of the Franz and Stoneley type when a solid cylinder is insonified almost grazely by a narrow beam [9, 17]. Analogously, in optics rainbow and glory were observed by oblique illumination of a water droplet by a laser beam [5], however the explanation of these phenomena presented most extensively by H. NUSSENZWEIG [10, 11] was carried out utilizing the plane wave theory neglecting the finiteness of the wave beam dimensions.

In this paper we propose a method based on the selection of the Franz creeping wave by means of a sound beam with a ring cross-section. Note that using of an annular acoustic beam makes it possible to obtain a non-specular reflection from an elastic sphere [12, 14] that was first established experimentally by S. SASAKI [6] when an entire ultrasonic beam was directed on a plane water-metal interface. Note also that in [12, 14] the pressure distribution across the beam section was assumed step-wise that did not permit to describe the scattering of the sound beam incidented tangentially on the sphere. This demerit is removed by the introduction of an annular wave beam with modal Lorentz cross-section distribution of the acoustic pressure amplitudes.

2. Solution of the problem

Let us assume in an ideal (nonviscous and nonheat-conducting) compressible fluid an acoustically impenetrable (soft or immovable rigid) sphere of radius a, the centre of which is simultaneously the origin of the spherical coordinates r, θ, φ. Suppose that a harmonic sound wave (with time dependence of $e^{-i\omega t}$, where ω is the circular frequency and t is the time) excited by a keen-directive transducer into the direction $\theta = 0$ incidents on the sphere. Then the acoustic pressure in the incident wave beam may be described as follows:

$$p_{\text{inc}}(R, \psi, \omega) = A(\psi, k) R^{-1} e^{ikR} + O\left[(\xi_0/R)^2\right] \qquad (R \gg \xi_0), \tag{1}$$

where $A(\psi, k)$ is the axisymmetric directivity with acoustic axis passing across the center of the target, $k = \omega/c$ is the wavenumber, c is the sound velocity in the fluid; R, ψ, φ are spherical coordinates with origin in the center of the transducer; the active surface characteristic linear dimension of the latter is ξ_0. Furthermore

$$\begin{aligned} R &= \sqrt{\xi^2 + x_3^2} = \sqrt{r^2 + 2rx_3^0 \cos\theta + (x_3^0)^2}\,, \\ \xi &= |\boldsymbol{\xi}|, \qquad \boldsymbol{\xi} = (x_1, x_2), \qquad x_3 = R\cos\psi = x_3^0 + r\cos\theta, \end{aligned} \tag{2}$$

where x_3^0 is the distance between the centre of the transducer and that the scatterer, ξ and x_3 are the radial and axis coordinates of the beam cross-section, respectively.

If the sound beam is sufficiently narrow, then from Eq. (1) we get the following expressions for the quasi-plane wave [15]:

$$p_{\text{inc}}(r,\theta,\omega) = A(k)R(\xi/x_3^0,k)e^{ikr\cos\theta} + O\left[1/(kx_3^0)^2\right] \qquad (kx_3^0 \gg 1) \tag{3}$$

or

$$p_{\text{inc}}(r,\theta,\omega) = A(k)\sum_{l=0}^{\infty} i^l(2l+1)R_l(k)j_l(kr)P_l(\cos\theta). \tag{4}$$

Here the function $A(k)$ is defined from the condition:

$$\begin{gathered} A(\psi,k)(x_3^0)^{-1}\exp(ikx_3^0) = A(k)R(\xi/x_3^0,k) \\ (\psi \approx \operatorname{tg}\psi = \xi/x_3^0, \quad \psi \ll 1) \end{gathered} \tag{5}$$

and corresponds to the Fourier-spectrum of the incident pulse modulation, $R(\xi/x_3^0,k)$ is the directive pattern of the quasi-plane wave,

$$R_l(k) \equiv R_l(\varrho/x_3^0,k) \tag{6}$$

is the coefficient determining the directivity of the partial wave with the impact parameter $\varrho_l = \sqrt{l(l+1)}\,/k$ and the angular moment l, that satisfies the inequality $\varrho_l \ll x_3^0$; $j_l(kr)$ is the Bessel spherical function of order l, $P_l(\cos\theta)$ is the Legendre polynomial.

Thus, the acoustic scattering pressure is determined by the formula

$$p_{\text{sc}}(r,\theta,\omega) = f(\theta,k)r^{-1}e^{ikr} + O\left[(a/r)^2\right] \qquad (r \gg a) \tag{7}$$

as $r \to \infty$, where

$$f(\theta,k) = k^{-1}\sum_{l=0}^{\infty}(2l+1)f_l(k)R_l(k)P_l(\cos\theta) \tag{8}$$

is the scattering amplitude (or form function) and $f_l(k)$ is the partial scattering amplitude

$$f_l(k) = i\Omega j_l(x)/\Omega h_l^{(1)}(x) \qquad (x = ka, \quad l = 0,1,2,...), \tag{9}$$

where $h_l^{(1)}(x)$ is the Hankel spherical function of order l of the first kind, $\Omega \equiv 1$ and $\Omega \equiv d/dx$ for the soft and rigid spheres, respectively.

In the experiments, the total (effective) scattering cross-section $\sigma_t(k)$ is an important characteristic [1] which, in the case of a sphere insonified by a finite wave beam, is written as follow [15]:

$$\sigma_t(k) = Z_0^{-1}\sum_{l=0}^{\infty}|R_l(k)|^2\sigma_l(k); \tag{10}$$

here $\sigma_l(k)$ is the partial cross-section corresponding to the plane wave scattering:

$$\sigma_l(k) = (4\pi/k^2)(2l+1)|f_l(k)|^2 \qquad (l = 0,1,2,...) \tag{11}$$

and

$$Z_0 = \sigma_0^{-1}\int_{\sigma_0}|R(\xi/x_3^0,k)|^2\,d\boldsymbol{\xi}, \tag{12}$$

where σ_0 is a square of the beam cross-section over which the averaging of the incident Poynting vector is carried out.

Let suppose that the directivity pattern of the quasi-plane wave near the scatterer have the the Lorentz form

$$R(\xi/x_3^0, k) = \frac{i\nu/2}{(\xi_0 - \xi)/x_3^0 + i\nu/2}, \tag{13}$$

where $x_3^0\nu$ is an effective beam width.

Because

$$R_l(k) = \frac{i\nu/2}{(\xi_0 - \varrho_l)/x_3^0 + i\nu/2}, \tag{14}$$

in accoudance with Eq. (8), the essential contribution to the scattering amplitude $f(\theta, k)$ will be due to the partial waves with impact parameters satisfying the equality

$$\varrho_l = \xi_0 \,. \tag{15}$$

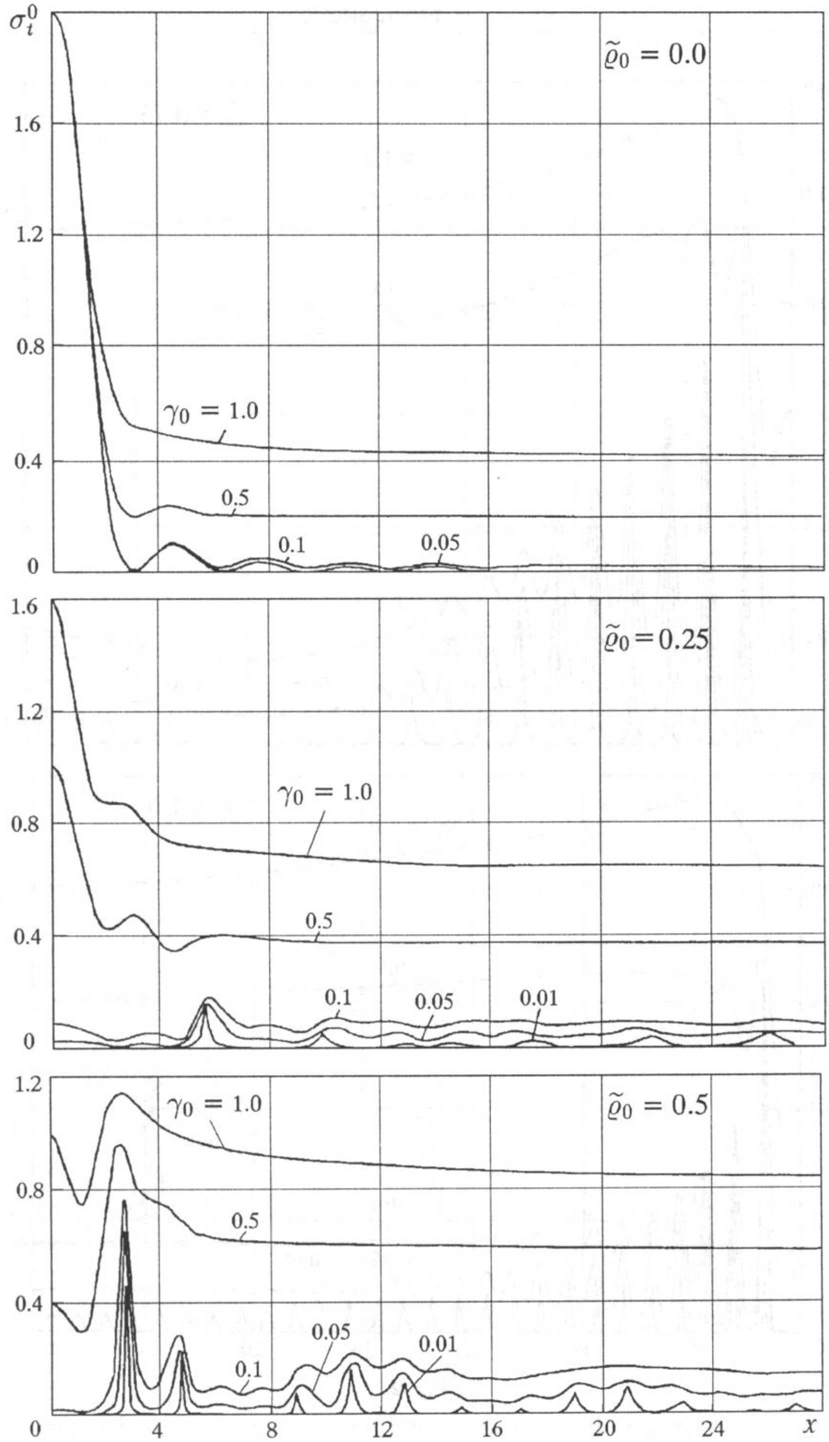
Equation (14) corresponds to a single-level representation of the angular spectra of the incident wave amplitude.

3. Analysis of the results

On the base of the Eqs. (10) – (15), the calculation of the effective cross-section $\sigma_t^0(k) = Z_0\sigma_t(k)/a^2$ were performed as a function of the nondimensional frequency x for the case of the acoustically soft sphere (Figs. 1, 2) and the acoustically rigid immovable sphere (Figs. 3, 4).

The plots show that when a solid cylindrical beam (nondimensional impact parameter $\widetilde{\varrho}_0 = \xi_0/a$ is equal to zero, $Z_0 = \ln 2$) or a ring cylindrical beam ($Z_0 = \pi/4$) with $\widetilde{\varrho}_0 < 0.5$ incidents on the spherical surface then the cross-sections for wide beams ($0.5 < \gamma_0 \leq 1$, $\gamma_0 = x_3^0\nu/a$) qualitative unimportant are distinguished from the corresponding cross-sections of the plane wave incidence case [1]. However with the increasing sound spot dimension the frequency dependence of $\sigma_t^0(k)$ is getting an oscillatory character with increasing an amplitude more and more because of the narrowing Fresnel zones of the different phases that are formed on the insonified sphere side [6, 7, 13]. Then also the minimal levels of $\sigma_t^0(k)$ approach the zero value. The increasing beam impact parameter ($\widetilde{\varrho}_0 \to 1$) and the decreasing effective width γ_0 yield amplitude fluctuations of $\sigma_t^0(k)$ displayed in the almost whole frequency range that is considerable. The values of $\sigma_t^0(k)$ for the soft and rigid spheres are clearly distinguished between earth by the phase modulation. Note that in the case of the plane wave scattering similar differences are not observed because the cross-section $\sigma_t^0(k)$ approaches the asymptote $2\pi a^2$ in both cases of the soft and rigid obstacles.

A specific scattering of the narrow sound beam occurs when base rays incident grazely on the spherical object ($\widetilde{\varrho}_0 = 1$). Then the general amplitude level of $\sigma_t^0(k)$ is unimportant because only one half of the whole energy transported by the beam is lost in the interaction process. Furthermore, with narrowing beam the role of the specular reflection sharply decreases. Then, the amplitude oscillations, which are observed leading off with $\widetilde{\varrho}_0 \geq 0.5$, become specially expressive and periodic and disappear slowly for



σ_t^0
$\tilde{\varrho}_0 = 0.0$
1.6
1.2
0.8
0.4
0
$\gamma_0 = 1.0$
0.5
0.1
0.05
1.6
$\tilde{\varrho}_0 = 0.25$
1.2
0.8
0.4
0
$\gamma_0 = 1.0$
0.5
0.1
0.05
0.01
1.2
$\gamma_0 = 1.0$
$\tilde{\varrho}_0 = 0.5$
0.8
0.5
0.4
0.05
0.01
0.1
0
4
8
12
16
20
24
x

Fig. 1.

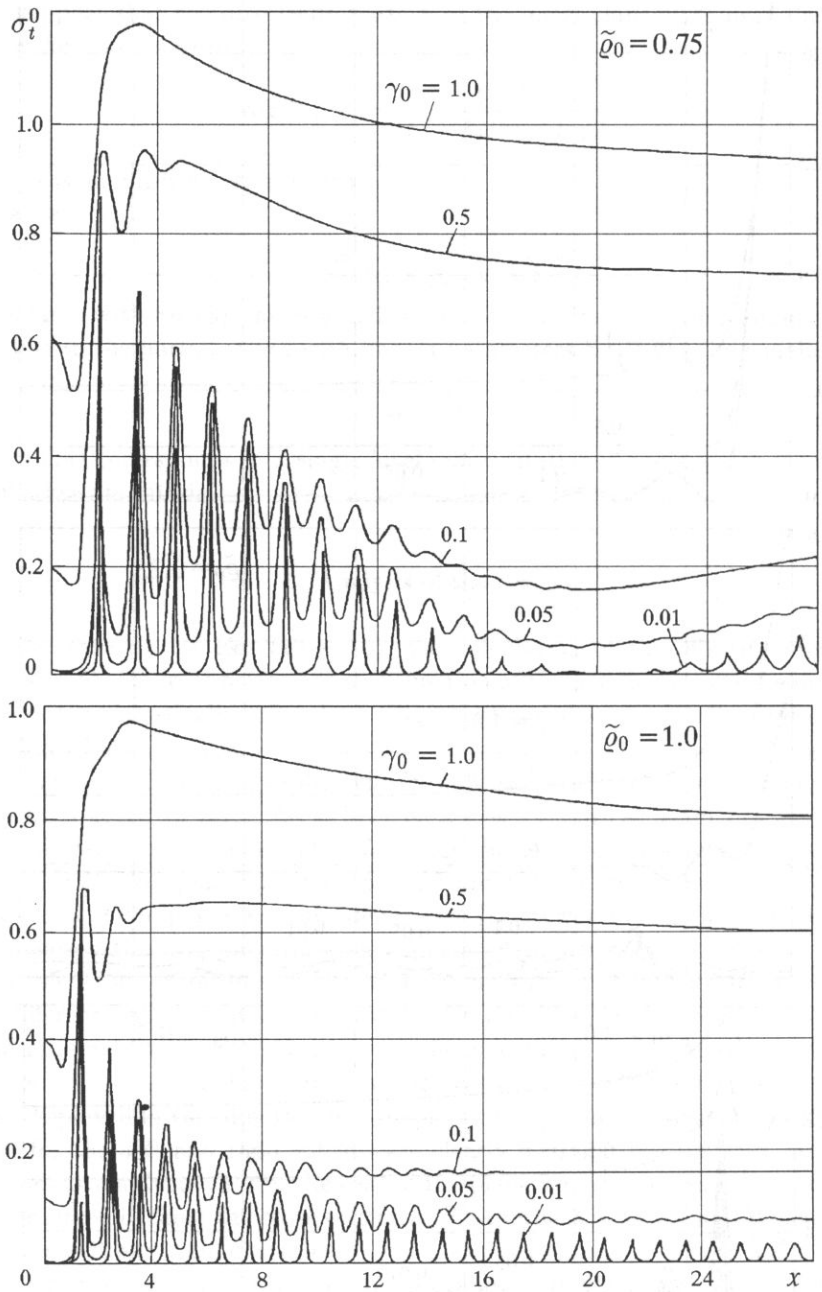


Fig. 2.

$\widetilde{\varrho} \approx 1$ (Figs. 2, 4). The resonance positions on the frequency scale with a step of $\Delta x \approx 1$ determining the phase and group circumferential wave velocities indicate resonances cor-

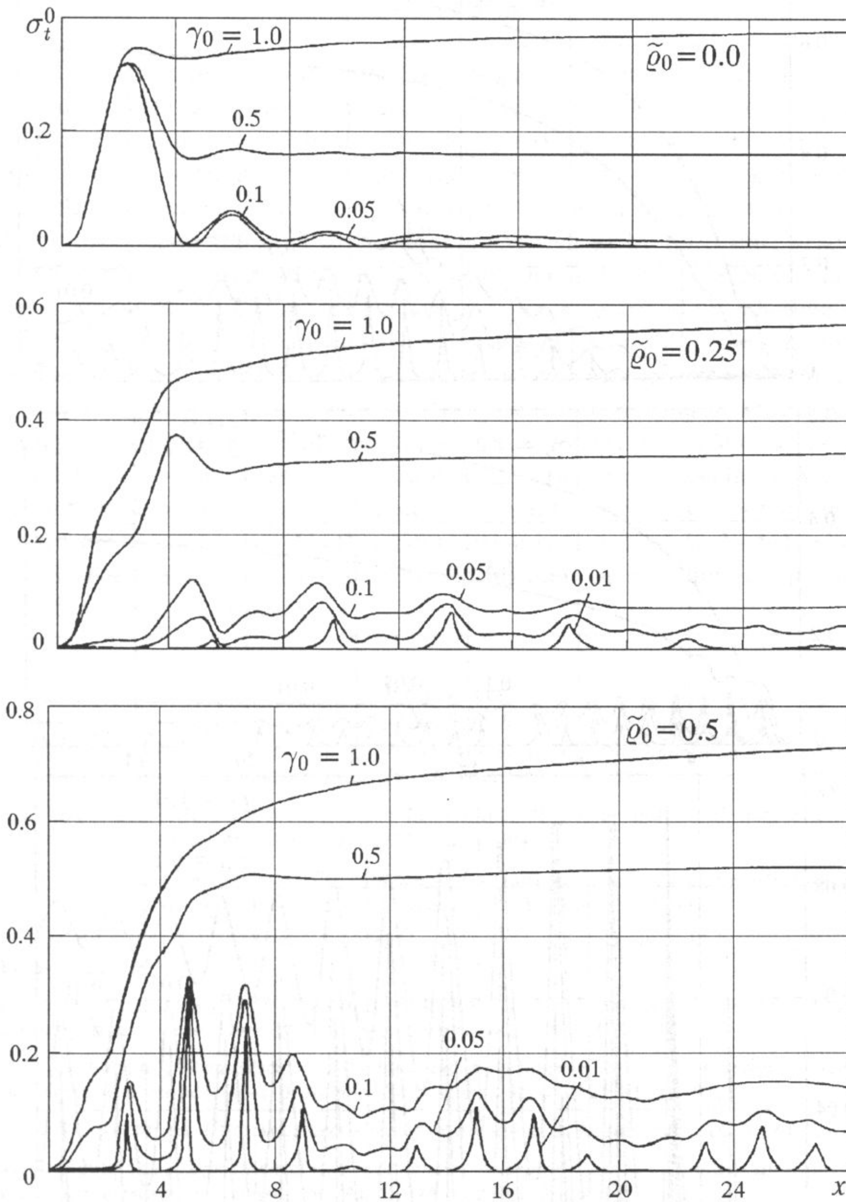
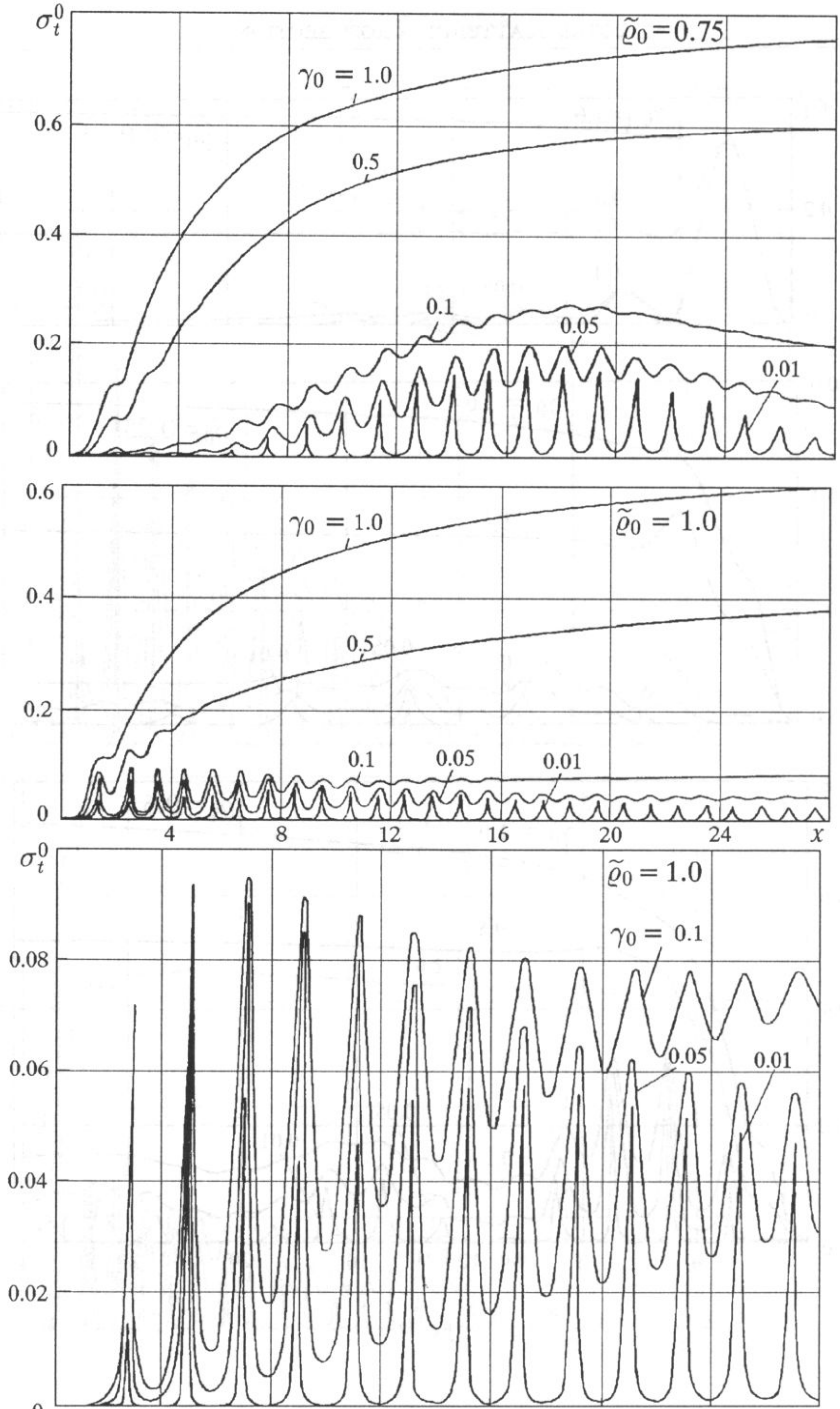


Fig. 3.

responding to the Franz creeping waves. Here these waves are separated very explicitly when $\widetilde{\varrho}_0 = 1$ and $\gamma_0 \ll 1$. It has also shown that within a considerable frequency range the resonance amplitude of $\sigma_t^0(k)$ for the soft sphere is almost twice as much than for the rigid one.



σ_t^0
$\widetilde{\varrho}_0 = 0.75$
$\gamma_0 = 1.0$
0.5
0.1
0.05
0.01
0
0.2
0.4
0.6
$\widetilde{\varrho}_0 = 1.0$
$\gamma_0 = 1.0$
0.5
0.1
0.05
0.01
4
8
12
16
20
24
x
σ_t^0
$\widetilde{\varrho}_0 = 1.0$
$\gamma_0 = 0.1$
0.05
0.01
0.02
0.04
0.06
0.08
2
4
6
8
10
12
x

Fig. 4.

4. Conclusions

One of the effective methods of selection of the different contributions to the echo-signal from obstacles immersed in a fluid (gas) includes the control over the parameters of an incident bounded sound beam. For the spherical scatterer case by directing the acoustical axis of the transducer to the target center, this beam can be chosen axisymmetric with a ring cross-section. By scanning of the beam based rays on the obstacle spherical surface from the pole to the equator as well as by narrowing gradually the beam ring width an analysis of the generative mechanisms of the reflective and diffractive waves can be made. The acoustic beam pressure is described by spherical harmonic superpositions with the Lorentz partial amplitude distribution. The computations of the total scattering cross-section σ_t as a function of the frequency show the resonance character of this value under grazed insonifying (with succesive narrowing of the hollow beam ring) acoustically soft and immovable rigid spheres. The distances between the resonances indicate that these resonances belong to the Franz creeping waves. The resonance amplitude level of this waves scattered from the soft sphere is two times larger than that from the rigid one. Another information is contained in the behaviour of the envelope of these resonances. This is the oscillating function with different particularities for the soft and rigid scatterers.

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