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PROPAGATION OF SOUND WAVES OF FINITE AMPLITUDE IN A HORN AT FREQUENCIES BELOW THE CUT-OFF FREQUENCY

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An investigation of the wave of finite amplitude in hyperbolic horn with annular cross-section is described. The fluid in the horn is assumed to be nondissipative. The equation of the sound wave propagation in the horn is solved for the case of frequencies below the cut-off frequency. The analysis is qiven in Lagrangian coordinates.

1. Introduction

The problem of propagation of sound waves with finite amplitude in horns at frequencies above the cut-off frequency was described in the papers [4, 10]. In this work the case where the input wave has a frequency below the cut-off frequency but her harmonics have frequencies above that of cut-off is discussed. Harmonics waves are then favoured in propagation with respect to the fundamental and they can be amplified. This problem is considered for hyperbolic horns with annular cross-section which are frequently applied [2, 5–8, 10].

2. Analysis of the propagation equation of wave with finite amplitude for excitation frequencies below the cut-off frequency

The equation of propagation of a wave with finite amplitude in a horn with arbitrary shape has the following form [4]:

$$\ddot{\xi} = \frac{c^2}{\left[\dfrac{S(x)}{S(a)}\right]^{\gamma-1}(1+\xi')^{\gamma}} \left\{ \frac{\dfrac{\partial}{\partial a}\left[\dfrac{S(x)}{S(a)}\right]}{\dfrac{S(x)}{S(a)}} + \frac{\xi''}{1+\xi'} \right\}, \tag{2.1}$$

where ξ is the displacement of the acoustic particle, S is the cross-sectional area of the horn, c is the sound velocity for small amplitudes, a is Lagrangian coordinate, γ is the adiabatic exponent, $x = a + \xi$ is Eulerian coordinate. Dots and commas in equation

(2.1) denote differentiation with respect to time and to the coordinate a, respectively. Equation (2.1) was formulated under the assumption that the horn is filled by a lossless gaseous medium. In the derivation of equation (2.1) the nonlinearity of the equation of continuity, Euler's equation and adiabate equation was taken into account [4].

The following dependence between the cross-sectional area and position of the horn's axis determines the family of hyperbolic horns with annular cross-section [8]:

$$S = \frac{S_0}{\cosh \varepsilon} \cdot \cosh(mx + \varepsilon)\,, \tag{2.2}$$

where $S_0 = \pi d_0 h_0$ (Fig. 1) is the area at the throat, m is the coefficient of flare of the horn and ε is the coefficient of shape of the walls; $\varepsilon \in [0, \infty)$.

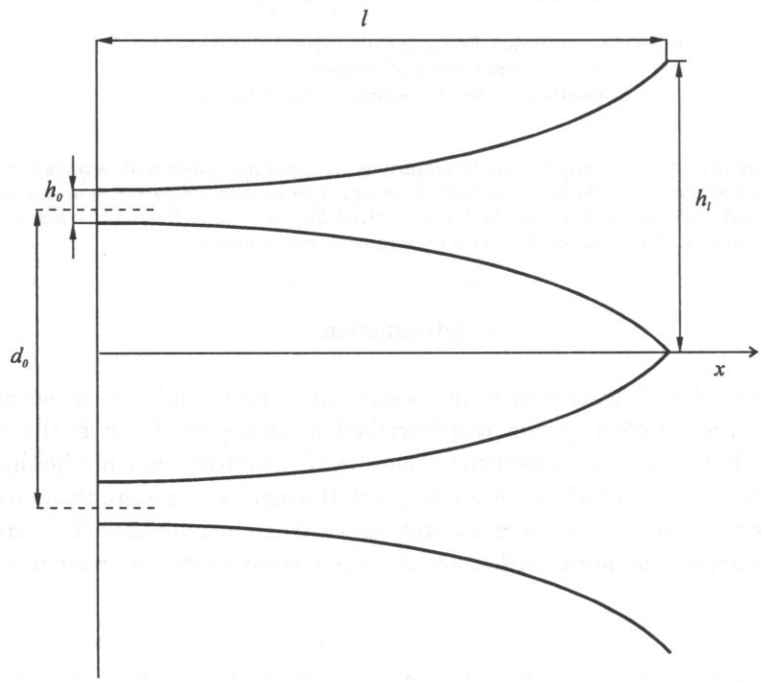

Fig. 1. The longitudinal section of a horn defined by (2.2).

Let us assume that there are no refflection at the mouth of a horn and that a hypothetical annular piston vibrating with harmonical motion is the source of waves at the throat $a = 0$:

$$\xi(0, t) = k^{-1} A \cos \omega t\,, \tag{2.3}$$

where k is the wave number, ω is the pulsation and t is time. The dimensionless amplitude $A = 2\pi M$, where M is the Mach acoustic number [11].

It is known [10, 11] that even for relatively high intensity of the sound we have $A \ll 1$. Therefore the displacement ξ of the acoustic particle has the form of a power series of the amplitude A:

$$\xi(a, t) = k^{-1} \left[A \cdot \varphi_1(a, t) + A^2 \cdot \varphi_2(a, t) + \ldots\right], \tag{2.4}$$

where $\varphi_1(a,t)$, $\varphi_2(a,t), \dots$ corresponds to succesive harmonics. The functions $\varphi_1(a,t)$, $\varphi_2(a,t), \dots$ must reduce for $a = 0$ to

$$\begin{aligned} \varphi_1(0,t) &= \text{const}, \\ \varphi_2(0,t) &= \varphi_3(0,t) = \ldots = 0. \end{aligned} \tag{2.5}$$

In the case of hyperbolic horns with annular cross-section the functions $\varphi_1(a,t)$, $\varphi_2(a,t)$ fulfil the following equations [10]:

$$\varphi_1'' + m\left[\operatorname{tgh}(ma+\varepsilon)\right]\varphi_1' + m^2\left[1 - \operatorname{tgh}^2(ma+\varepsilon)\right]\varphi_1 - \frac{1}{c^2}\ddot{\varphi}_1 = 0, \tag{2.6}$$

$$\varphi_2'' + m\left[\operatorname{tgh}(ma+\varepsilon)\right]\varphi_2' + m^2\left[1 - \operatorname{tgh}^2(ma+\varepsilon)\right]\varphi_2 - \frac{1}{c^2}\ddot{\varphi}_2 = \psi(a,t), \tag{2.7}$$

where

$$\begin{aligned} \psi(a,t) = \varphi_1\ddot{\varphi}_1\frac{m(\gamma-1)}{kc^2}\operatorname{tgh}(ma+\varepsilon) + \varphi_1'\ddot{\varphi}_1\frac{\gamma}{kc^2} + \frac{\varphi_1'\varphi_1''}{k} \\ + \varphi_1^2\frac{m^3\operatorname{tgh}(ma+\varepsilon)\left[1-\operatorname{tgh}^2(ma+\varepsilon)\right]}{k} \\ - \varphi_1\varphi_1'\frac{m^2\left[1-\operatorname{tgh}^2(ma+\varepsilon)\right]}{k}. \end{aligned} \tag{2.8}$$

Substituting to equation (2.6)

$$\varphi_1(a,t) = \phi_1(a)\cdot e^{i\omega t} \tag{2.9}$$

we obtain the equation of the first harmonic wave:

$$\phi_1'' + m\left[\operatorname{tgh}(ma+\varepsilon)\right]\phi_1' + \left[\frac{m^2}{\cosh^2(ma+\varepsilon)} + k^2\right]\phi_1 = 0. \tag{2.10}$$

For frequencies above the cut-off frequency the solution of this equation is [10]:

$$\phi_1(a)\frac{B_1}{\sqrt{\cosh(ma+\varepsilon)}}e^{i\overline{K}(ma+\varepsilon)} + \frac{B_2}{\sqrt{\cosh(ma+\varepsilon)}}e^{-i\overline{K}(ma+\varepsilon)}, \tag{2.11}$$

where B_1 and B_2 are constants. $\overline{K}$ can be presented in the following form:

$$\overline{K} = \left\{k^2m^{-2} - \frac{1}{4} + \frac{3}{ml}\left[\operatorname{tgh}\varepsilon - \operatorname{tgh}(ml+\varepsilon)\right]\right\}^{1/2}. \tag{2.12}$$

Here l is the length of the horn (Fig. 1).

In the case of frequencies below the cut-off frequency $\overline{K} = -i\bar{\chi}$ [9] and the solution of equation (2.10) can be presented in the form

$$\phi_1(a) = \frac{B_1}{\sqrt{\cosh(ma+\varepsilon)}}e^{\bar{\chi}(ma+\varepsilon)} + \frac{B_2}{\sqrt{\cosh(ma+\varepsilon)}}e^{-\bar{\chi}(ma+\varepsilon)}, \tag{2.13}$$

where

$$\bar{\chi} = \left\{\frac{1}{4} - k^2m^{-2} - \frac{3}{4ml}\left[\operatorname{tgh}\varepsilon - \operatorname{tgh}(ml+\varepsilon)\right]\right\}^{1/2}. \tag{2.14}$$

Subsequently, taking into account the formula (2.9) and considering only the real part of the solution, the following equation can be obtained for the wave propagating from the inlet to the outlet:

$$\varphi_1(a,t) = \frac{B_3 e^{-\bar{\chi}(ma+\varepsilon)}}{\sqrt{\cosh(ma+\varepsilon)}} \cos\omega t - \frac{B_4 e^{-\bar{\chi}(ma+\varepsilon)}}{\sqrt{\cosh(ma+\varepsilon)}} \sin\omega t . \tag{2.15}$$

The constants B_3 and B_4 can be determined from the boundary condition (2.5):

$$B_3 = \sqrt{\cosh\varepsilon}\, e^{\bar{\chi}\varepsilon} , \qquad B_4 = 0 \tag{2.16}$$

and the formula (2.15) may now be written

$$\varphi_1(a,t) = \sqrt{\frac{\cosh\varepsilon}{\cosh(ma+\varepsilon)}} \cdot e^{-\bar{\chi}ma} \cdot \cos\omega t . \tag{2.17}$$

With the help of (2.17), the right-hand side of (2.7) can be presented as follows:

$$\psi = \sigma(a)\left[1+\cos 2\omega t\right] , \tag{2.18}$$

where

$$\sigma(a) = \frac{e^{-2\bar{\chi}ma}\cosh\varepsilon}{4\cosh(ma+\varepsilon)}\Big[D_1 \operatorname{tgh}^3(ma+\varepsilon) + D_2 \operatorname{tgh}^2(ma+\varepsilon) + D_3 \operatorname{tgh}(ma+\varepsilon) + D_4\Big] \tag{2.19}$$

while

$$\begin{aligned} D_1 &= -\frac{15m^3}{4k}, \qquad D_2 = -\frac{9\bar{\chi}m^3}{2k}, \\ D_3 &= \frac{m^3(7-6\bar{\chi}^2)}{2k} - mk(\gamma-2), \\ D_4 &= 2m\gamma k\bar{\chi} + \frac{m^3(3\bar{\chi}-2\bar{\chi}^3)}{k} . \end{aligned} \tag{2.20}$$

The term $\psi(a,t)$ is a periodical function of time with pulsation 2ω. Therefore the function $\varphi_2(a,t)$, as an integral of equation (2.7), has also pulsation 2ω. The function $\varphi_2(a,t)$, which corresponds to second harmonic is a sum [3]:

$$\varphi_2(a,t) = \varphi_{21}(a,t) + \varphi_{22}(a,t). \tag{2.21}$$

The component $\varphi_{21}(a,t)$ is the general solution of a homogeneous equation coupled with equation (2.7). In the case where the frequency of the second-order harmonic is above the cut-off frequency, the function $\varphi_{21}(a,t)$ has a form similar to (2.11):

$$\varphi_{21} = \frac{C_1}{\sqrt{\cosh(ma+\varepsilon)}} \cos\left[2\omega t - \overline{K}_1(ma+\varepsilon)\right] - \frac{C_2}{\sqrt{\cosh(ma+\varepsilon)}} \sin\left[2\omega t - \overline{K}_1(ma+\varepsilon)\right] , \tag{2.22}$$

where

$$\overline{K}_1 = \left\{4k^2 m^{-2} - \frac{1}{4} + \frac{3}{4ml}\left[\mathrm{tgh}\,\varepsilon - \mathrm{tgh}(ml+\varepsilon)\right]\right\}^{1/2}. \tag{2.23}$$

The component $\varphi_{22}(a,t)$ is the particular solution of the equation (2.7) and has a form similar to the term $\psi(a,t)$ (2.18):

$$\varphi_{22}(a,t) = g(a) + f(a)\cos 2\omega t. \tag{2.24}$$

Introducing (2.24) into equation (2.7) we obtain equations for the functions $g(a)$ and $f(a)$:

$$g''(a) + m\left[\mathrm{tgh}(ma+\varepsilon)\right] g'(a) + \frac{m^2}{\cosh^2(ma+\varepsilon)} g(a) = \sigma(a), \tag{2.25}$$

$$f''(a) + m\left[\mathrm{tgh}(ma+\varepsilon)\right] f'(a) + \left[\frac{m^2}{\cosh^2(ma+\varepsilon)} + 4k^2\right] f(a) = \sigma(a). \tag{2.26}$$

The solution of the equation (2.25) can be presented in the following form [3]:

$$g(a) = g_2(a)\int \sigma(a)\,da - \frac{1}{m} g_1(a)\int \sigma(a)\cdot \sinh(ma+\varepsilon)\,da + C_1 g_1(a) + C_2 g_2(a), \tag{2.27}$$

where

$$g_1(a) = \frac{1}{\cosh(ma+\varepsilon)}, \tag{2.28}$$

$$g_2(a) = \frac{1}{m}\,\mathrm{tgh}(ma+\varepsilon). \tag{2.29}$$

The solution of the equation (2.26) is expressed by

$$f(a) = f_2(a)\int \frac{f_1(a)\sigma(a)}{W(a)}\,da - f_1(a)\int \frac{f_2(a)\sigma(a)}{W(a)}\,da + C_1 f_1(a) + C_2 f_2(a), \tag{2.30}$$

where

$$f_1(a) = \frac{\cos\left[\overline{K}_1(ma+\varepsilon)\right]}{\sqrt{\cosh(ma+\varepsilon)}}, \tag{2.31}$$

$$f_2(a) = \frac{\sin\left[\overline{K}_1(ma+\varepsilon)\right]}{\sqrt{\cosh(ma+\varepsilon)}}, \tag{2.32}$$

$$W(a) = \frac{m\overline{K}_1}{\cosh(ma+\varepsilon)}. \tag{2.33}$$

Finally the displacement of the acoustic particle in the hyperbolic horn with annular cross-section, for frequencies below the cut-off frequency, can be presented as follows:

$$\xi(a,t) = k^{-1}A\varphi_1(a,t) + k^{-1}A^2\left[\varphi_{21}(a,t) + g(a) + f(a)\cos 2\omega t\right], \tag{2.34}$$

where $\varphi_1(a,t)$, $\varphi_{21}(a,t)$, $g(a)$ and $f(a)$ are expressed by formulas (2.17), (2.22), (2.27) and (2.30), respectively. The constants C_1, C_2 can be determined from the condition (2.5).

3. The exponential horn with annular cross section

In this important particular case $\varepsilon \to \infty$ and the formula (2.2) has the following form:

$$S = S_0 e^{mx}. \tag{3.1}$$

The cut-off frequency for the exponential horn is [4]

$$f_c = \frac{mc}{4\pi}. \tag{3.2}$$

The solution (2.17) of equation (2.6) for the frequencies below the cut-off frequency is simplified to the form

$$\varphi_1(a,t) = e^{-\left(\frac{m}{2}+\sqrt{\frac{m^2}{4}-k^2}\right)a} \cos\omega t. \tag{3.3}$$

The equation (2.7) for the second harmonic is

$$\varphi_2'' + m\varphi_2' - \frac{1}{c^2}\ddot{\varphi}_2 = \psi(a,t), \tag{3.4}$$

where the right-hand side of this equation has the following form:

$$\psi(a,t) = \left\{N\left[(\gamma+1)k - \frac{m^2}{k}\right] + \frac{m}{2}\left[(5-\gamma)k - \frac{m^2}{k}\right]\right\} e^{-(m+2N)a}\frac{1+\cos 2\omega t}{2}. \tag{3.5}$$

Here

$$N = m \lim_{\varepsilon\to\infty} \bar{\chi} = \sqrt{\frac{m^2}{4} - k^2}. \tag{3.6}$$

The particular solution of equation (3.4) can be presented in the following form:

$$\varphi_{22}(a,t) = [C + D\cos 2\omega t]\, e^{-(m+2N)a}. \tag{3.7}$$

The coefficients C and D can be found by introducing $\varphi_{22}(a,t)$ into equation (3.4):

$$C = \frac{\frac{1}{2}N\left[(\gamma+1)\omega^2 c - m^2c^3\right] + mc\left(\frac{5-\gamma}{4}\omega^2 - \frac{m^2c^2}{4}\right)}{2m\omega c^2 N + m^2\omega c^2 - 4\omega^3}, \tag{3.8}$$

$$D = \frac{\frac{1}{2}N\left[(\gamma+1)\omega^2 c - m^2c^3\right] + mc\left(\frac{5-\gamma}{4}\omega^2 - \frac{m^2c^2}{4}\right)}{2m\omega c^2 N + m^2\omega c^2}. \tag{3.9}$$

Note that for a frequency $f \ll f_c$

$$C \simeq D \simeq -\frac{m}{4k}. \tag{3.10}$$

In the case where the frequency of the second harmonic is above the cut-off frequency, the general solution of a homogeneous equation coupled with equation (3.4) has the following form:

$$\varphi_{21}(a,t) = Pe^{-ma} + Qe^{-ma/2}\cos(2\omega t - N_1 a), \tag{3.11}$$

where

$$N_1 = m \lim_{\varepsilon\to\infty} \overline{K}_1 = \sqrt{4k^2 - \frac{m^2}{4}}\,. \tag{3.12}$$

The sum of (3.7) and (3.11) represents a solution of equation (3.4). Next, from the boundary condition (2.5), the constants P and Q can be found. Thus at $a = 0$

$$\varphi_{21}(0,t) + \varphi_{22}(0,t) = P + Q\cos 2\omega t + C + D\cos 2\omega t = 0. \tag{3.13}$$

This condition is fulfiled at all times when

$$P = -C, \qquad Q = -D. \tag{3.14}$$

Finally, for excitation frequencies below the cut-off frequency, the displacement of the acoustic particle in the exponential horn takes in the second approximation following form:

$$\xi(a,t) = k^{-1}Ae^{-\left(\frac{m}{2}+N\right)a}\cos\omega t + k^{-1}A^2\Big\{Ce^{-ma}\left(e^{-2Na}-1\right) - De^{-\frac{m}{2}a}\left[\cos\left(2\omega t - N_1 a\right) - e^{-\left(\frac{m}{2}+2N\right)a}\cos 2\omega t\right]\Big\}. \tag{3.15}$$

It can be noticed that in above formula the term

$$k^{-1}A^2C\,e^{-ma}\left(e^{-2Na}-1\right) \tag{3.16}$$

occurs. This term is independent of time and signifies that during the acoustic motion, a layer of air inside the horn oscillates about a mean position which is not its position of rest but is displaced in the direction of propagation of the wave.

By differentiating equation (3.15) with respect to time the vibration velocity of a particle can be obtained. The vibration velocity is a sum of two components: the first one is the term with pulsation ω (first harmonic):

$$v_1 = -Ace^{\left(\frac{m}{2}+N\right)a}\sin\omega t\,. \tag{3.17}$$

The second component with pulsation 2ω (second harmonic) can be presented as follows:

$$v_2 = 2cA^2De^{-\frac{m}{2}a}\left[\sin(2\omega t - N_1 a) - e^{-\left(\frac{m}{2}+2N\right)a}\sin 2\omega t\right]. \tag{3.18}$$

When the distance from the source at the throat is big enough that

$$e^{-\left(\frac{m}{2}+2N\right)a} \ll 1 \tag{3.19}$$

and the only important part of the second harmonic wave in formula (3.18) is

$$v_2 \simeq 2cA^2De^{-\frac{m}{2}a}\sin(2\omega t - N_1 a). \tag{3.20}$$

In this case the ratio of the amplitudes of vibration velocities of both harmonics is equal to

$$\eta = \frac{2cA^2|D|e^{-\frac{m}{2}a}}{Ace^{-\left(\frac{m}{2}+N\right)a}} = 2A|D|e^{Na}\,. \tag{3.21}$$

The square of this ratio gives the ratio of the radiated powers.

At the end the exponential horns with the same dimensions of the throat and of the mouth but with different lengths are taken into account in the numerical example:

— width of the channel at the inlet $h_0 = 1.5 \cdot 10^{-3}$ m,

— diameter of the annular channel $d_0 = 10^{-1}$ m,

— width of the channel at the outlet $h_l = 10^{-1}$ m,

— lengths (coefficients of flare, cut-off frequencies):

$$l_1 = 60 \cdot 10^{-2}\text{m} \quad \left(m_1 = 7\frac{1}{\text{m}}, \quad f_{c_1} = 190\,\text{Hz}\right),$$

$$l_2 = 42 \cdot 10^{-2}\text{m} \quad \left(m_2 = 10\frac{1}{\text{m}}, \quad f_{c_2} = 270\,\text{Hz}\right),$$

$$l_3 = 28 \cdot 10^{-2}\text{m} \quad \left(m_3 = 15\frac{1}{\text{m}}, \quad f_{c_3} = 406\,\text{Hz}\right),$$

$$l_4 = 21 \cdot 10^{-2}\text{m} \quad \left(m_4 = 20\frac{1}{\text{m}}, \quad f_{c_4} = 540\,\text{Hz}\right),$$

$$l_5 = 15 \cdot 10^{-2}\text{m} \quad \left(m_5 = 28\frac{1}{\text{m}}, \quad f_{c_5} = 760\,\text{Hz}\right).$$

For the acoustic particles at the mouth of the horn the termin the formula (3.19) can be written as follows:

$$\delta = e^{-\left(\frac{m}{2}+2N\right)l}. \tag{3.22}$$

In Fig. 2 δ as a function of frequency for expotential horns with the above dimensions is shown. We can see that practically $\delta \ll 1$. Thus, the ratio of the amplitudes of vibration velocities of both harmonics can be calculated from formula (3.21) which can now be presented in the form

$$\eta = 4\pi M|D|e^{Nl}. \tag{3.23}$$

The relation between the ratio η and the vibration frequency of the piston at the throat, in the range of frequency $\frac{1}{2}f_c < f < f_c$, for the horn with length $l = 0.6$ m is presented in Fig. 3. The five curves in Fig. 3 correspond to five values of the Mach number, from $M = 0.002$ (intensity of sound level at the throat $J_0 = 156$ dB) to $M = 0.015$ ($J_0 = 173$ dB). The relation $\eta = \eta(f)$ for the horn with length $l = 0.15$ m is presented in Fig. 4. The graphs of the functions $\eta = \eta(f)$ for horns with another lengths resemble the graphs in Fig. 3 and Fig. 4, and therefore they are not presented here.

It is shown in Fig. 3 and Fig. 4 that, for frequencies below the cut-off frequency, the amplitude of vibration velocity of the second harmonic increases in comparison with that of the first one when the frequency decreases. This increase is faster when the amplitude of the piston which initiates the wave is greater.

It can be seen comparing these results with the results of the [10], that for frequencies below the cut-off frequency the second harmonic is more amplified than for the frequencies above the cut-off. Assume e.q. that the length of the horn is 0.15 m ($f_c = 760$ Hz) and Mach acoustic number is 0.01 ($J_0 = 170$ dB). In this case for frequency $f = \frac{1}{2}f_c$ from the (Fig. 4) we obtain $\eta \simeq 60\%$, but for frequency $f = 2f_c$ we have $\eta \simeq 5\%$ (see [10]).

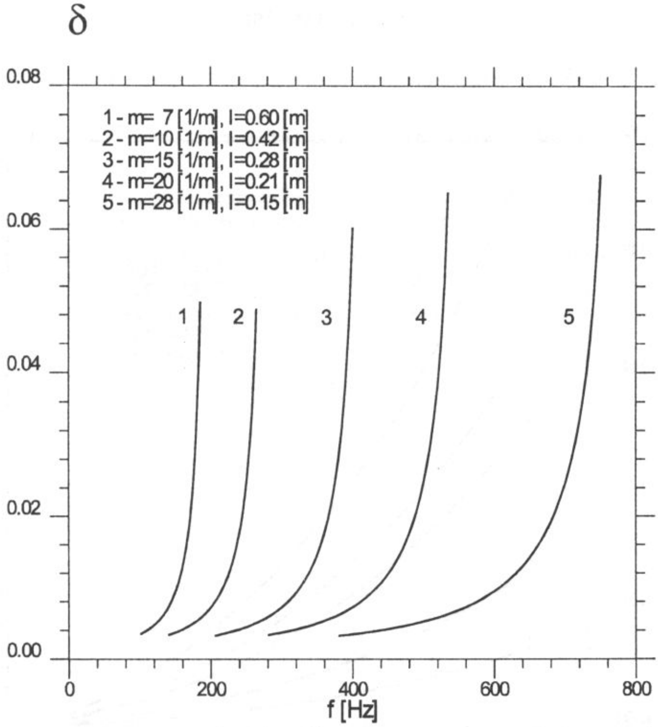

Fig. 2. The coefficient δ [see (3.22)] for the horns with different lengths.

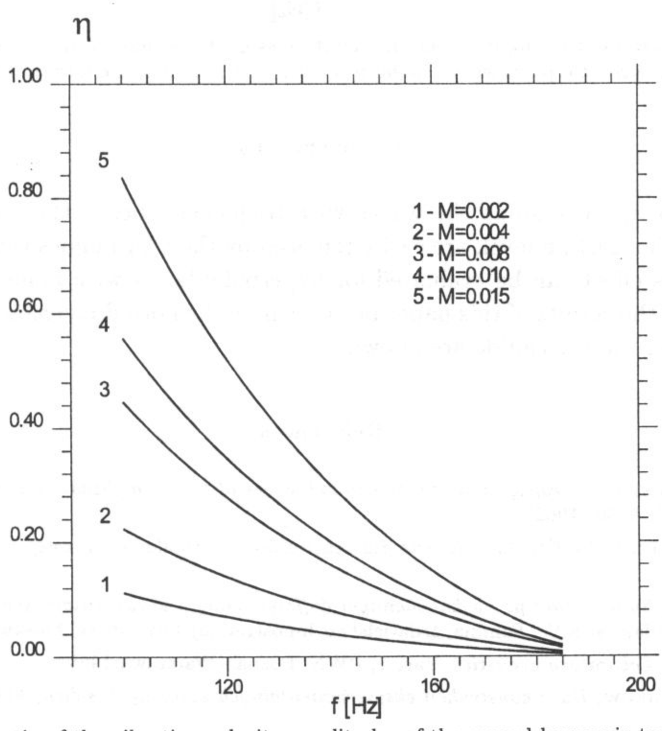

Fig. 3. The ratio of the vibration velocity amplitudes of the second harmonic to the first one for the acoustic particles at the horn mouth. The length of the horn $l = 0.6\,\mathrm{m}$.

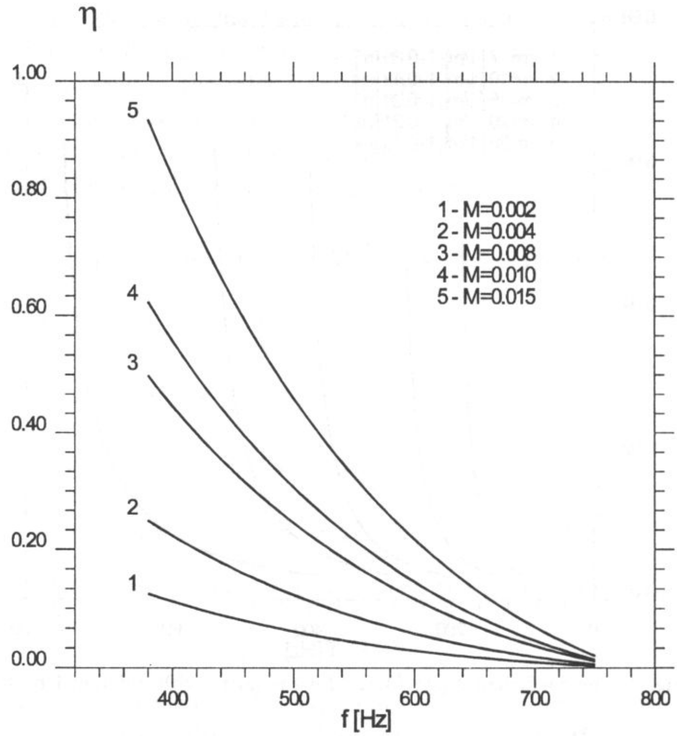

Fig. 4. The ratio of the vibration velocity amplitudes of the second harmonic to the first one for the acoustic particles at the horn mouth. The length of the horn $l = 0.15$ m.

4. Conclusions

In the acoustic waveguide the waves with frequencies below the cut-off frequency, once they reach a certain level, are easily replaced by their harmonics which are strongly amplified. This effect can be evaluated for hyperbolic horns with annular cross-section on the basis of the results of this paper in case where the horn dimensions and amplitude at the throat of the waveguide are known.

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