ACOUSTIC POWER OF FLUID-LOADED CIRCULAR PLATE LOCATED IN FINITE BAFFLE

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The oblate spheroidal coordinate system was used for calculation of the acoustic power radiated by a thin circular plate located in a finite baffle. It was assumed that the plate was clamped at the circumference of the planar limited baffle and radiated into lossless homogeneous liquid medium. The vibrations of the plate were forced by time harmonic external pressure. The damping effects caused by internal friction in the plate material as well as dynamic influence of the waves emitted by the plate were taken into consideration. The formula for the acoustic power was derived by the application of properties of eigenfunctions of plate equation of motion.

Notations

a plate radius,
A_l expansion coefficients,
B bending stiffness,
b baffle radius,
c propagation velocity of a wave in fluid,
c_m, c_n expansion coefficients,
E Young's modulus,
f surface density of the force exciting vibrations,
f_n expansion coefficients,
F time dependent surface density of the force exciting vibrations,
F_0 time independent constant,
h_η, h_ξ, h_φ components of measurement tensor,
h acoustic parameter, $h = k_0 a(b/a)$,
H plate thickness,
J_m m-order Bessel functions,
I_m m-order modified Bessel functions,
k_0 wave number,
k_p structural wavenumber,
m mass of the plate per surface unit,
n normal component,
N acoustic power radiated by the plate,
N' normalised acoustic power radiated by the plate,
N^∞ norm factor,
N_l Flammer norm factor,
p sound presure,
r radial variable in polar coordinates,
R coefficient of internal damping,

$R_{0l}^{(3)}(-ih,,i\xi)$ radial spheroidal function of the third kind, l-order,
$S_{0l}^{(1)}(-ih,\eta)$ angular spheroidal function of the first kind, l-order,
ν Poisson's ratio,
v normal component of the vibration velocity of points on the surface of the plate,
v_n vibration velocity of points on the surface of the plate for mode $(0,n)$,
v_{0n} normalised coefficient of the vibration velocity,
w transverse dislocation of points on the surface of the plate,
W time dependent transverse dislocation of points on the surface of the plate,
W_{nl} characteristic function,
ε_1 fluid-loading parameter,
ε_2 parameter of the plate damping,
γ_n $= k_n a$ is solution of the homogeneous plate equation of motion,
χ_l mutual impedance,
η spheroidal coordinate,
λ length of acoustic wave in fluid,
σ area of the plate with baffle,
ξ spheroidal coordinate,
ζ_{mn} mutual impedance,
ψ acoustic potential,
Ψ time dependent acoustic potential,
ω angular frequency of the force exciting vibrations,
ϱ density of the plate material,
ϱ_0 density of the fluid.

1. Introduction

The problem of radiation of acoustic waves by circular planar sources located in a limited baffle caught the attention of acoustic researches in the thirties [3]. It is well known that for waves longer than the dimensions of the considered sources the obtained results do not fully tally with characteristics calculated for the sources with infinite baffle and Huygens – Rayleigh integral applied [1, 2, 4]. Detailed analytical investigations have been made for the piston with uniform and parabolic velocity distribution [1,2,11] and also for a freely vibrating membrane [12].

Most papers dealing with the influence of a finite baffle apply properties of the oblate spheroidal coordinates system [1 – 4, 11]. Interest in acoustic radiation from sources on oblate spheroidal baffles results primarily from the reparability of the scalar wave equation in coordinates in question and the wide variety of useful shapes that are natural to this systems. For a circular plate supplied with a finite rigid baffle the oblate spheroid is also particularly suited to the study of sound radiation, so the basic quantities that characterise an acoustic field were calculated by the author in a similar way [6, 7, 8]. Spheroidal geometry offers a convenient system in which the curvature of the radiating surface may be varied and the relative size of the vibrating surface to the baffle surface may be changed. Of particular interest is the case in which the spheroid reduces to a flat circular source (plate) in the xy plane. In this way the sound field around the plate in question can be obtained by solving the separable Helmholtz wave equation in the oblate spheroidal coordinates with Neuman's boundary condition.

Properties of the oblate spheroidal coordinate system have been used to calculate the acoustic pressure for the freely vibrating plate [8] and for the fluid-loaded plate excited harmonically at low frequencies [7]. This paper gives formulae for the acoustic power

of the plate clamped at a finite baffle and excited to vibrate by an external force. The mathematical model of the plate includes internal dissipation and interaction with fluid.

2. Assumption of the analysis

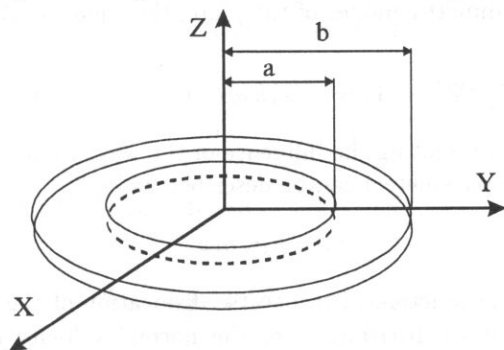Consider the fluid-plate configuration as illustrated in Fig. 1.

Fig. 1. A circular plate in a rigid baffle with radius b.

A circular thin plate with radius a and thickness H is surrounded by an ideal liquid medium with the static density ϱ_0. It is assumed that the plate is made of a homogeneous isotropic material with density ϱ, Poisson's ratio ν, Young's modulus E. The plate is clamped in a flat, rigid and finite baffle with radius b and is excited to vibration by an external time-harmonic force:

$$F(r, \phi, t) = f(r, \phi)e^{-i\omega t} = F_0 e^{-i\omega t}, \tag{2.1}$$

where $F_0 = $ const for $0 < r < a$.

Taking into account only linear, harmonic and axially-symmetric vibrations of the plate in a steady state, as well as the influence of a radiated wave on vibrations of the plate and an internal damping inside the plate's material, the plate differential equation of motion can be described as follows [5, 9, 10]:

$$BV^4 W(r, \phi, t) + m\frac{\partial^2 W(r, \phi, t)}{\partial t^2} + R\frac{\partial}{\partial t}\left[\nabla^4 W(r, \phi, t)\right]$$
$$= R(r, \phi, t) - \varrho_0 \frac{\partial \Psi(r, \phi, 0, t)}{\partial t}, \tag{2.2}$$

where $B = EH^3/12(1-\nu^2)$ is the bending stiffness, $W(r, \phi, t) = w(r, \phi)e^{-i\omega t}$ – transverse dislocation of points on the surface of the plate, m – mass of the plate per surface unit, R – coefficient of internal damping, $\Psi(r, \phi, 0, t)$ – acoustic potential on the surface of the plate, related to the acoustic pressure p in the fluid by the equation [15]

$$p = -\varrho_0 i\omega\Psi(r, \phi, z) \tag{2.3}$$

and satisfies the Helmholtz equation

$$\left(\nabla^2 + k_0^2\right)\Psi = 0, \tag{2.4}$$

with the condition

$$\frac{\partial \Psi}{\partial n}\bigg|_{z=0} = -v(r,\phi) = i\omega w(r,\phi), \tag{2.5}$$

k_0 denotes the acoustic wavenumber at the frequency ω, $v(r,\phi)$ – amplitude of surface velocity distribution on the plate.

Using well-known formulae appropriate for harmonic phenomena and taking into account only axially-symmetric modes of the plate, the equation (2.2) can be expressed as [5, 9, 10]:

$$\left(k_p^{-4}\nabla^4 - 1\right)v(r) - \varepsilon_1 k_0 \psi(r,0) = -\frac{i}{\omega m}f(r). \tag{2.6}$$

The parameter ε_1 representing the influence of the wave radiated by the plate on its vibration (fluid-loading parameter) can be described as [5, 7]:

$$\varepsilon_1 = \varrho_0 c / m\omega. \tag{2.7}$$

In the equation (2.6) the function of transverse dislocation of the plate $w(r)$ has been replaced by wanted surface distribution of the normal velocity $v(r)$. The structural wavenumber k_p in the vacuum at frequency ω is defined by

$$k_p^4 = m\omega^2/\overline{B}, \tag{2.8}$$

where $\overline{B}$ is the complex rigidity

$$\overline{B} = B - i\omega R = B(1 - i\varepsilon_2) \tag{2.9}$$

and parameter ε_2

$$\varepsilon_2 = \omega R / B \tag{2.10}$$

is a measure of the plate damping.

3. Solution of the Helmholtz equation

For the plate located in a finite baffle, the problem of determining the far-field acoustic pressure cannot be treated with the well known Rayleigh's formula. In this paper the solution of Eq. (2.4) in conjunction with (2.5) has been obtain by the use of the method of separation of variables in the oblate spheroidal coordinate system (OSCS) [6]. Due to symmetry of radiated waves with respect to z axis, the following equation for outgoing waves has been obtained [14]:

$$\psi(\eta,\xi) = \sum_{l}^{\infty} A_l S_{0l}^{(1)}(-ih,\eta) R_{0l}^{(3)}(-ih,i\xi), \tag{3.1}$$

where $S_{0l}^{(1)}(-ih,\eta)$ denotes angular spheroidal function of the first kind, $R_{0l}^{(3)}(-ih,i\xi)$ – radial spheroidal function of the third kind, $h = k_0 b$ and A_l – the expansion coefficients.

The coefficients can be derived from Neuman's boundary condition (2.5) which in oblate spheroidal system has the form

$$\frac{\partial \Psi}{\partial n} = \frac{1}{h_\xi} \frac{\partial \Psi}{\partial \xi}\bigg|_{\xi=\xi_0} = \begin{cases} -v(\xi_0, \eta), & \eta_0 \leq \eta \leq 1, \\ 0, & \eta_0 \geq \eta \geq -\eta_0, \\ v(\xi_0, \eta), & -1 \leq \eta \leq -\eta_0. \end{cases} \tag{3.2}$$

Applying the orthogonal property of angular spheroidal functions [14], we finally obtain [8]:

$$A_l = -\frac{bW_{nl}}{\dfrac{\partial R_{0l}^{(3)}(-ih, i0)}{\partial \xi} N_l}, \tag{3.3}$$

where N_l denotes the norm factor [14] and

$$W_{nl} = \int_{\eta_0}^{1} v(\eta) S_{0l}(-ih, \eta)\eta \, d\eta \tag{3.4}$$

is the characteristic function in OSCS. The vibration velocity distribution in the oblate spheroidal coordinate system $v(\eta)$ is an unknown function. It will be determined by applying the orthogonal series method.

4. Solution of the plate equation of motion

By applying the well known eigenfunction expansion theorem, one can derive the solution of Eq. (2.7). In order to do it the vibration velocity distribution $v(r)$ and the external force $f(r)$ will be expressed as the infinite series of eigenfunctions of the homogeneous plate equation

$$v(r) = \sum_{n}^{\infty} c_n v_n(r), \tag{4.1}$$

$$f(r) = \sum_{n}^{\infty} f_n v_n(r). \tag{4.2}$$

The quantity c_n denotes unknown expansion coefficients, while f_n can be determined by means of the orthonormal property

$$f(r) = \int_{0}^{a} f(r)v_n^*(r)r \, dr. \tag{4.3}$$

For the clamped circular plate the eigenfunctions $v_n(r)$ take the form [5, 9]:

$$v_n(r) = v_{0n}\left[J_0(\gamma_n r/a) - \frac{J_0(\gamma_n)}{I_0(\gamma_n)} I_0(\gamma_n r/a)\right], \tag{4.4}$$

where $\gamma_n = k_n a$ is solution of the equation $J_0(\gamma_n)I_1(\gamma_n) + J_1(\gamma_n)I_0(\gamma_n) = 0$, $n = 1, 2, \ldots$ and have an orthonormal property if

$$v_{0n} = 1/\left(aJ_0(k_n a)\right). \tag{4.5}$$

Regarding the following equation

$$\nabla^4 v_n(r) = k_n^4 v_n(r) \tag{4.6}$$

as a result we obtain

$$\sum_n c_n \left(k_p^{-4} k_n^4 - 1\right) v_n(r) - \varepsilon_1 k_0 \psi(r, 0) = -\frac{i}{\omega m} \sum_m f_n v_n(r). \tag{4.7}$$

The equation (4.7) is now expressed in the oblate spheroidal coordinate system (OSCS) with the use of the following transformation [14]

$$r = b\left[(1 - \eta^2)(\xi_0^2 + 1)\right]^{1/2}. \tag{4.8}$$

Using properties of OSCS and assuming $\xi_0 = 0$, $r = b(1 - \eta^2)$, the obtained expressions become appropriate for the plate in the finite baffle. The eigenfunctions are

$$v_n(\eta) = v_{0n} \left[J_0\left(\gamma_n \frac{b}{a}\sqrt{1 - \eta^2}\right) - \frac{J_0(\gamma_n)}{I_0(\gamma_n)} I_0\left(\gamma_n \frac{b}{a}\sqrt{1 - \eta^2}\right)\right] \tag{4.9}$$

and they remain orthonormal if

$$v_{0n} = b/(aJ_0(k_n a)). \tag{4.10}$$

In turn, Eq. (4.7) is multiplied by the orthonormal function $v_m^*(\xi_0, \eta)$ and integrated on the surface of the spheroid. Denoting the components of the left side of Eq. (4.7) as L_1 and L_2 and its right side as L_3, the following integrals are obtained:

$$L_1 = \iint_\sigma \sum_n c_n \left(k_p^{-4} k_n^4 - 1\right) v_n(\xi_0, \eta) v_m^*(\xi_0, \eta) \, d\sigma,$$

$$L_2 = -\varepsilon_1 k_0 \iint_\sigma \psi(\xi_0, \eta) v_m^*(\xi_0, \eta) \, d\sigma, \tag{4.11}$$

$$L_3 = -\frac{1}{\omega \varrho h} \iint_\sigma \sum_n f_n v_n(\xi_0, \eta) v_m^*(\xi_0, \eta) \, d\sigma.$$

In order to find the solutions of the above integrals we must take into account that the element of spheroid surface $d\sigma$ is equal to

$$d\sigma = h_\eta h_\varphi \, d\eta \, d\varphi, \tag{4.12}$$

where h_η, h_φ denotes the scaling factors (components of measurement tensor) [14]:

$$h_\eta = b\sqrt{\frac{\xi_0^2 + \eta^2}{1 - \eta^2}},$$

$$h_\varphi = b\sqrt{(1 - \eta^2)(1 + \xi_0^2)}. \tag{4.13}$$

As a result of this calculation, the previous equation (4.7) turns into a system of linear algebraic equations [7, 9]

$$c_m \left(\frac{k_m^4}{k_p^4} - 1 \right) - \varepsilon_1 \sum_n^\infty i \zeta_{mn} c_n = f_m, \qquad (4.14)$$

where

$$f_m = -\frac{2i}{\omega m \gamma_m b^2} v_{0n} F_0 a^2 J_1(\gamma_m) \qquad (4.15)$$

is the expansion coefficient of the external excitation into the Fourier series which has been obtained according to expression (4.3) and quantity

$$\zeta_{mn} = \sum_l \frac{W_{nl} W_{ml}}{N_l} ih \frac{R_{0l}^{(3)}(-ih, i0)}{\partial R_{0l}^{(3)}(-ih, i0)/\partial \xi} \qquad (4.16)$$

means normalised impedance of the plate [7].

The solution of the system (4.14) is possible with the application of numerical methods. In order to determine expansion coefficients c_n the system (4.14) has been solved using Crout algorithm, which enables us to find the velocity distribution on the surface of the plate, in accordance with the analysed case.

5. Calculation of the acoustic power

The total acoustic power of the vibrating plate is calculated according to the definition [15]

$$N = \frac{1}{2} \iint_\sigma p v^* \, d\sigma, \qquad (5.1)$$

where p is the acoustic pressure and v^* denotes the amplitude of vibration velocity distribution, which is coupled with the velocity of the considered source.

In the case of flat circular sources vibrating in a finite baffle, the total acoustic power can be calculated by the application of Eq. (5.1) in oblate spheroidal coordinate. In this case the surrounded spheroidal surface $\sigma = \sigma_0$ becomes the surface of a considered vibrator system, including the baffle. In the oblate spheroidal coordinate system the definition (5.1) can be expressed as

$$N = \pi b^2 \int_{-1}^{1} p(0, \eta) v^*(\eta) \eta \, d\eta. \qquad (5.2)$$

The quantities that appear in the integrand function can be described as follows:

$$v^*(\eta) = \sum_n^\infty c_n^* v_n(\eta). \qquad (5.3)$$

where c_n^* denotes complex coefficients coupled with c_n obtained above.

Basing our calculations on the relation (2.3) and expression (2.6) together with (2.7) and (2.8), the formula of the acoustic pressure of the circular plate with the finite baffle takes the form [7]

$$p(0,\eta) = -2i\varrho_0 hc \sum_{n=1}^{\infty} c_n \sum_{l=1}^{\infty} / \frac{W_{nl}}{N_l \partial R_{0l}^{(3)}(-ih,i0)/d\xi} S_{0l}(-ih,\eta) R_{0l}^{(3)}(-ih,i0). \qquad (5.4)$$

Symbol "/" in the second sum is associated with the manner in which the waves are emitted by a system with the plate as a vibration source. In our analysis the acoustic field is radiated by both upper and lower surfaces of the plate, so in the expression (5.4) only odd index of l can be taken into account.

Introducing the transfer impedance [11]

$$\chi_l(-ih) = -ih \frac{R_{0l}^{(3)}(-ih,i0)}{R_{0l}^{(3)/}(-ih,i0)}, \qquad (5.5)$$

where $R_{0l}^{(3)/}(-ih,i0)$ denotes $\partial R_{0l}^{(3)}(-ih,i0)/\partial\xi$, the acoustic pressure takes the simpler form

$$p(0,\eta) = 2\varrho_0 c \sum_{n=1}^{\infty} c_n \sum_{l=1}^{\infty} / \frac{W_{nl}}{N_l} S_{0l}(-ih,\eta)\chi_l(-ih). \qquad (5.6)$$

In order to obtain the pattern for the acoustic power radiated by a plate let us replace the above quantities into definition (5.2)

$$N = 2\pi b^2 \varrho_0 c \sum_{n=1}^{\infty} c_n \sum_{l=1}^{\infty} / \frac{W_{nl}}{N_l} \chi_l(-ih) \int_{-1}^{1} v^*(\eta) S_{0l}(-ih,\eta)\eta \, d\eta. \qquad (5.7)$$

Regarding expression (5.3), we obtain

$$N = 2\pi b^2 \varrho_0 c \sum_{n=1}^{\infty} c_n \sum_{l=1}^{\infty} / \frac{W_{nl}}{N_l} \chi_l(-ih) \sum_{m} c_m^* \int_{-1}^{1} v_m(\eta) S_{0l}(-ih,\eta)\eta \, d\eta. \qquad (5.8)$$

The integral that appears in the above relation can be separated into two parts, according to the boundary condition (3.2)

$$N = 2\pi b^2 \varrho_0 c \sum_{n=1}^{\infty} c_n \sum_{l=1}^{\infty} / \frac{W_{nl}}{N_l} \chi_l(-ih) \sum_{m} c_m^*$$
$$\times \left[\int_{-1}^{-\eta_0} v_m(\eta) S_{0l}(-ih,\eta)\eta \, d\eta + \int_{\eta_0}^{1} v_m(\eta) S_{0l}(-ih,\eta)\eta \, d\eta \right] \qquad (5.9)$$

which leads to the change of integration limits after applying the following property of angle oblate spheroidal functions [14]

$$S_{0l}(-ih,\eta) = (-1)^l S_{0l}(-ih,-\eta). \qquad (5.10)$$

The calculations explained above result in:

$$N = 2\pi b^2 \varrho_0 c \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} c_m^* c_n \sum_{l=1}^{\infty} \!{}'\, \frac{W_{nl} W_{ml}}{N_l} \chi_l(-ih). \tag{5.11}$$

In this way the final formula for the total acoustic power radiated by the plate located in a finite baffle has been derived. It can easily be separated into real and imaginary part because there is only one complex quantity $\chi_l(-ih)$ inside:

$$
\begin{aligned}
\mathrm{Re}(N) &= 2\pi b^2 \varrho_0 c \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} c_m^* c_n \sum_{l=1}^{\infty} \!{}'\, \frac{W_{nl} W_{ml}}{N_l} \mathrm{Re}[\chi_l(-ih)], \\
\mathrm{Im}(N) &= 2\pi b^2 \varrho_0 c \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} c_m^* c_n \sum_{l=1}^{\infty} \!{}'\, \frac{W_{nl} W_{ml}}{N_l} \mathrm{Im}[\chi_l(-ih)],
\end{aligned}
\tag{5.12}
$$

where

$$
\begin{aligned}
\mathrm{Re}[\chi_l(-ih)] &= \frac{1}{\left[R_{0l}^{(1)\prime}(-ih, i0)\right]^2 + \left[R_{0l}^{(2)\prime}(-ih, i0)\right]^2}, \\
\mathrm{Im}[\chi_l(-ih)] &= \frac{R_{0l}^{(2)}(-ih, i0) R_{0l}^{(2)\prime}(-ih, i0)}{\left[R_{0l}^{(1)\prime}(-ih, i0)\right]^2 + \left[R_{0l}^{(2)\prime}(-ih, i0)\right]^2}.
\end{aligned}
\tag{5.13}
$$

6. Figures and conclusions

It is convenient for calculations to introduce the normalised factor N^{∞}, described as the acoustic resistance when wavenumber $k_0 \to \infty$ [13]

$$N^{\infty} = \lim_{k_0 \to \infty} N = 2\pi b^2 \varrho_0 c \sum_{n=1}^{\infty} c_n c_n^*. \tag{6.1}$$

Then the normalised acoustic power can be calculated as follows:

$$N' = \frac{N}{N^{\infty}} = \frac{\displaystyle\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} c_m^* c_n \sum_{l=1}^{\infty} \!{}'\, \frac{W_{nl} W_{ml}}{N_l} \chi_l(-ih)}{\displaystyle\sum_{n=1}^{\infty} c_n c_n^*}. \tag{6.2}$$

On the basis of the above formula the real and imaginary part of the total acoustic power radiated by the plate in question has been calculated. Since the series in the formula (6.2) are infinite, the number of terms ensuring adequate accuracy of results have been numerically determined.

The largest considered value of h was 15, for which it was found that the series with index l converged in approximately 30 terms. The number of terms required for convergence of this series was always greater for the real part than for the imaginary
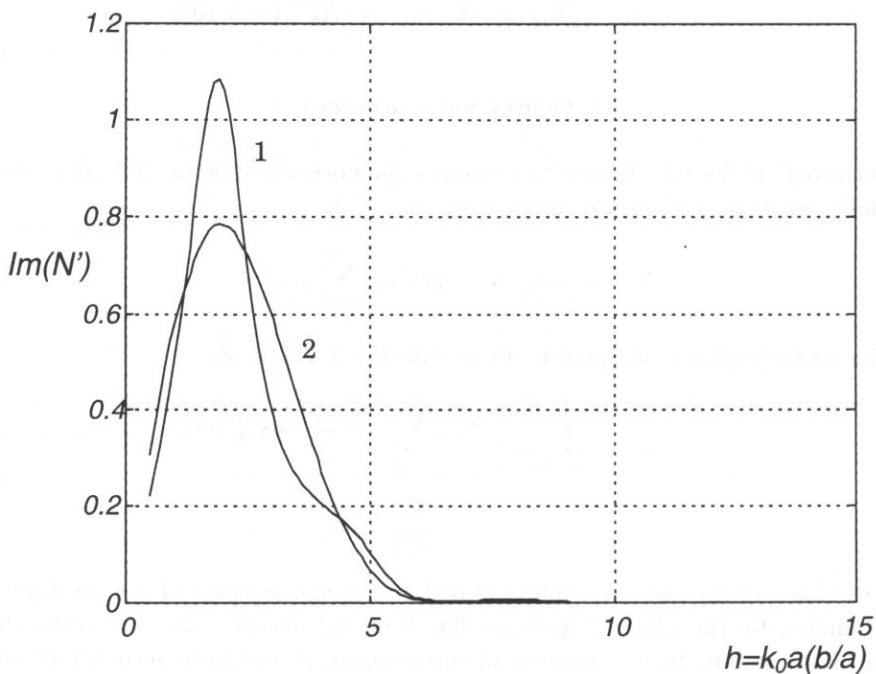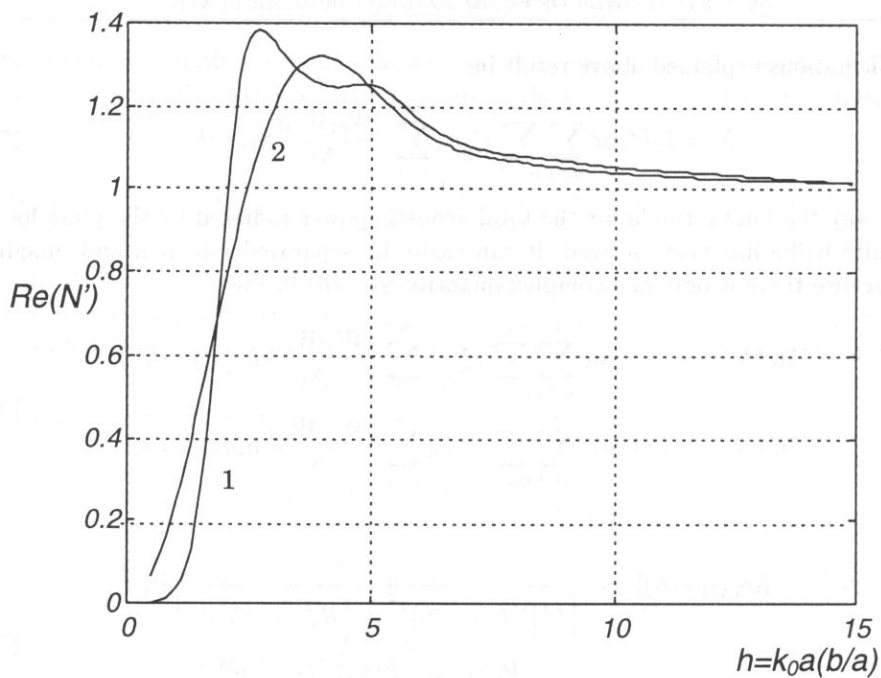
Fig. 2. Acoustic power radiated by the circular plate in terms of the acoustic parameter $h = k_0 b$.
1 – plate without the baffle, $a/b = 1$, $H/2a = 0.08$; 2 – plate with the unlimited baffle – the curve
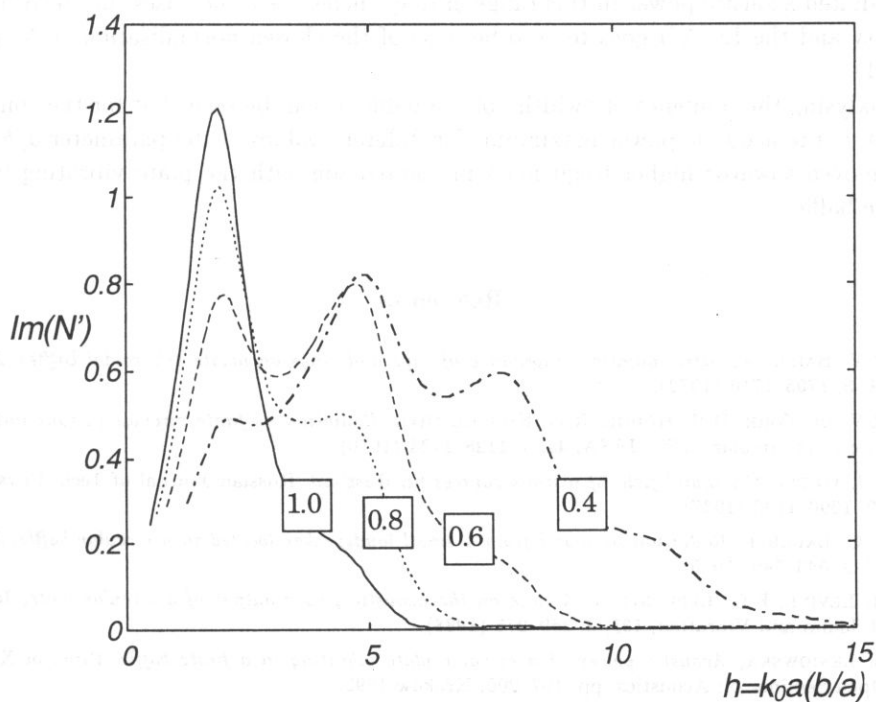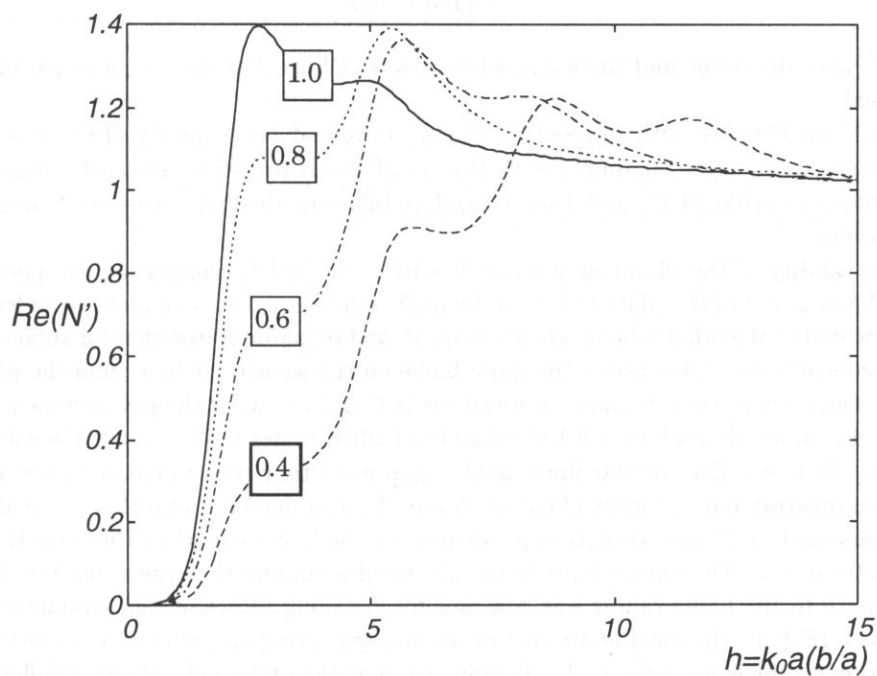was calculated from formula $(11')$ in [9], $H/2a = 0.08$.

Fig. 3. Acoustic power radiated by the circular plate in terms of acoustic the parameter $h = k_0 b$, $H/2a = 0.08$. The numbers on the curves denote the values of the ratio a/b of the plate radius to the baffle size.

part of acoustic power and increased when $h = k_0 a(b/a)$, the acoustic size parameter increased.

The double series with indexes m and n converged very quickly. The expansion coefficients c_n, c_m were computed with the aid of Crout procedure suited for algebraic equations system like (4.14) and it was enough to take only the first few terms in practical calculations.

The validity of the obtained solution has been checked by comparing the no-baffle case (the ratio a/b of the plate radius to the baffle size is equal 1) with plots given for the plate located in the infinite baffle [9, 10], (Fig. 2), and it can be stated that for sufficiently high frequencies the influence of the finite baffle on the acoustic field around the planar sources can be neglected. It can be noticed easily (Fig. 2) that for the parameters $h > 10$ obtained characteristics for both baffled and unbaffled plates $(a/b = 1)$ are the same.

The effect of a flat circular finite baffle upon acoustic power radiated by the plate has been illustrated by a family of curves in the Fig. 3. It demonstrates that the real and imaginary part of N' are strongly dependent upon the baffle size when the acoustic size parameter $h < 7$. The curves have been calculated assuming that the ratio λ/b of the wavelength to the baffle radius was constant for changing values of the parameter a/b. For $h \approx 6$ (Fig. 3), the local additional maximum appearing on each curve is caused by diffraction on the edge of the baffle. It indicates that the finite baffle strongly influences the radiated acoustic power in this range of frequencies. As h increases the $\mathrm{Re}(N')$ goes to unity and the $\mathrm{Im}(N')$ goes to zero because of the chosen normalisation of N' given by (6.1).

Analysing the influence of "width" of the baffle, it can be seen that for the constant ratio λ/b, the acoustic power maximums for different values of the parameter a/b have been moved towards higher frequencies in comparison with the plate vibrating in the infinite baffle.

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