ARCHIVES OF ACOUSTICS
22, 4, 437–444 (1997)

# ON THE RELATION BETWEEN THE INERTIAL COAGULATION AND THE AMPLITUDAL EFFECT

H. CZYŻ

Institute of Physics, Pedagogical University of Rzeszów
(35-310 Rzeszów, Rejtana 16a, Poland)
e-mail: hczyz@atena.univ.rzeszow.pl

The phenomenon of drift of small particles suspended in a gaseous medium in which the acoustic wave propagates, is known since long. In the present work, which refers to [7], we present an outline of a theory allowing for full interrelation of all quantities characterizing the particle, the medium and the acoustic field.

## 1. Introduction

In the paper published in 1963 by Roman Wyrzykowski [7] there is at the beginning a serious printing fault, which spuriously could missinterprete the results. The theory of sound coagulations of aerosols, formulated by Roman Wyrzykowski allows to obtain formulae, which give mutual dependance between the acoustical data and the data of the aerosol. In this paper we present this theory in new shape, of course with the assistance of the author.

The coagulating action of the acoustic wave on aerosols is known since long [1], as well as theories explaining partially this phenomenon [2–5].

We will assume in the following that the coagulation occurs always in the polydispersion aerosols, as even monodispersion aerosols become polydispersional as a result of heat motions. Particles of greater dimensions vibrate in the acoustic field with smaller amplitudes, while smaller particles amplitudes are greater. As a result, relative velocities occur which in turn result in collision of particles (if only the amplitude of vibration is sufficiently great; a problem which will be discussed in the following) and in the so-called inertial sedimentation (coagulation) of small particles on bigger ones.

In practice, sedimentation of aerosols takes place in a settling tank, which is a tower long enough to assure that the dusted gas, turning round along helical lines, spends necessarily long time (3 to 5 seconds) in the acoustic field, produced by a generator located at the top of the purifier [6, 8].

## 2. The amplitude effect, or proper acoustic coagulation

In the present section we will deal with the problem of selection of acoustic field parameters such that for a given aerosol one would obtain amplitude of vibration great enough for occurrence of the acoustic coagulation.

The average distance between aerosol particles may be estimated temporarily as

$$l_0 = \frac{1}{\sqrt[3]{n_0}}, \tag{2.1}$$

where $n_0$ is the number of particles in 1 cubic centimeter of the gas.

In reality, we deal with some statistical distribution in both mutual distances and velocities of particles. We assume that in unit volume of the gas, the number of particles which are able to get in contact with each other is expressed by the integral

$$\int_0^A n(l)\,dl, \tag{2.2}$$

where $n(l)$ is the distribution function, and $A$ is the amplitude of particle vibrations:

$$\int_0^\infty n(l)\,dl = n_0. \tag{2.3}$$

The aerosols conform themselves in general to the Gauss type distribution, therefore we apply

$$n(l) = n_m e^{-\left(\frac{l-l_0}{L}\right)^2}. \tag{2.4}$$

The efficiency of the dust removal process as an result of what we call here the amplitude effect, may be described as

$$\eta_A = \frac{\displaystyle\int_0^A n(l)\,dl}{\displaystyle\int_0^\infty n(l)\,dl}. \tag{2.5}$$

By substitution of Eq. (2.4) into (2.5), the constant $n_m$ is being reduced, and the $L$ constant will be determined from experimental data.

Making use of definition of the error function:

$$\mathrm{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt \tag{2.6}$$

we may write

$$\eta_A = \frac{\mathrm{Erf}\left(\dfrac{l_0}{L}\right) + \mathrm{Erf}\left[\dfrac{l_0}{L}(\psi - 1)\right]}{\mathrm{Erf}\left(\dfrac{l_0}{L}\right) - 1}, \tag{2.7}$$

where $\psi$ denotes the relative amplitude of particle vibrations:

$$\psi = \frac{A}{l_0}. \tag{2.8}$$

We calculate the value of the $L$ constant by means of the following consideration: it is known from experiment [6, 7] that even at $\psi = 1.5$, the efficiency of acoustic purification is very high. Therefore, adopting arbitrarily the value $\eta_A = 0.99$ we obtain, by means of numerical solution of Eq. (2.7),

$$\frac{l_0}{L} = 3.3 \tag{2.9}$$

and

$$\eta_A = \frac{1}{2}\left\{1 + \mathrm{Erf}[3.3(\psi - 1)]\right\}. \tag{2.10}$$

Equation (2.10) is an estimation formula, but one thing is for sure: the efficiency of acoustic dedusting depends on relative amplitude (2.8), therefore it is worthwhile to calculate this quantity here.

The vibration maximum velocity amplitude of an aerosol particle in the acoustic field $v_0$ is expressed with the so-called drag coefficient $\mu$ and with the vibration velocity amplitude of the medium $U_0$ by means of a simple formula [5]:

$$v_0 = \mu U_0, \tag{2.11}$$

where

$$\mu = \frac{1}{\sqrt{1 + (\omega\tau)^2}} \tag{2.12}$$

$\omega$ is the angular frequency of vibrations, $\tau$ is the particle relaxation time given by (assumed applicability of the Stokes law):

$$\tau = \frac{2\varrho_p r^2}{9\eta}. \tag{2.13}$$

$\varrho_p$ is the density of the aerosol particle, being $r$ its radius and $\eta$ the medium viscosity.

In dust removing devices we use in practice the plane wave, the intensity $I$ of which is expressed by means of the formula [3]:

$$I = \frac{1}{2}\varrho_0 c_0 U_0^2, \tag{2.14}$$

where $\varrho_0$ is the rest density of the medium, $c_0$ is the acoustic wave velocity in this medium. Therefore, assuming that in practice the wave intensity is given, we have for the value of the medium vibration velocity amplitude:

$$U_0 = \sqrt{\frac{2I}{\varrho_0 c_0}}. \tag{2.15}$$

By Eqs. (2.11) and (2.15), the maximum amplitude of particle vibration is:

$$A = \frac{1}{\omega}\sqrt{\frac{2I}{\varrho_0 c_0}}\,\mu \tag{2.16}$$

and the dimensionless relative maximum amplitude $\psi$ (2.11), (2.8):

$$\psi = \frac{\mu}{\omega}\sqrt{\frac{2I}{\varrho_0 c_0}}\sqrt[3]{n_0}\,. \tag{2.17}$$

In practice, we define usually the mass concentration of an aerosol $s$ as the mass of dust particles contained in unit volume. We have obviously

$$n_0 = \frac{3}{4}\,\frac{s}{\pi r^3 \varrho_p}\,. \tag{2.18}$$

Substituting (2.18) to (2.17), we obtain the relative amplitude in its final form:

$$\psi = \frac{\mu}{r\omega}\sqrt{\frac{2I}{\varrho_0 c_0}}\sqrt[3]{\frac{3s}{4\pi \varrho_p}}\,. \tag{2.19}$$

During the process of dust removal, the above value should remain constant. For given acoustic wave, parameters $\mu$ and $\omega$ are defined, therefore we have a condition:

$$I \cdot s^{2/3} = \text{const}\,. \tag{2.20}$$

Thus, greater concentrations require smaller intensities and *vice versa*, which was confirmed by numerous experiments [7]. On the other hand, establishing all parameters except for the angular frequency $\omega$, or frequency of vibrations $\nu$, substituting Eq. (2.19) into (2.10) we obtain the dependence $\eta_A(\nu)$.

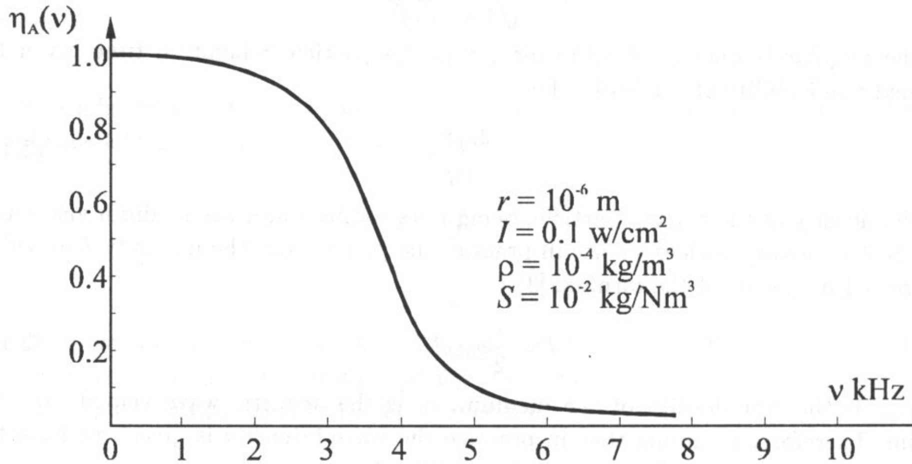


Fig. 1. An example of dependence $\eta_A(\nu)$ for conditions typical for acoustic dust removal process, described in the legend of the graph.

Figure 1 represents an example of this dependence for conditions typical for acoustic dust removal process, described in the legend of the graph. One can see that $\eta_A$ is practically equal to unity up to several kHz and then rapidly falls to zero. This phenomenon is also well known from experiment [7]. From Eqs. (2.19) and (2.10), we may calculate a maximum value of the wave frequency which gives the value of $\eta_A$ yet close to unity.

### 3. The inertial coagulation

Presently we proceed with consideration concerning the second factor influencing the overall efficiency of the acoustic coagulation the efficiency of the process of inertial sedimentation of smaller particles on bigger ones. Denoting this efficiency by $\eta_i$ we may write the overall efficiency of the process $\eta$ as

$$\eta = \eta_i \cdot \eta_A \,. \tag{3.1}$$

Numerous examinations show [1] that the quantity $\eta_i$ is a function of the so called Stokes number, which for bigger particle of radius $R$ is given by

$$n_{\mathrm{St}} = \frac{\tau v_w}{2R}, \tag{3.2}$$

where $\tau$ is the relaxation time of the settled (smaller) particle, $v_w$ being the maximum amplitude of relative velocity of particles.

The problem of the $\eta_i(n_{\mathrm{St}})$ dependence has been discussed in numerous experimental and theoretical papers, based on assumption of potential flow-around and viscous flow-around [5]. In any case, this is a function growing from 0 to 1, however it reaches the upper value for $n_{\mathrm{St}} \approx 1$ according to experimental data, and $2-3$ at theoretical curves. Therefore, in practice the problem is reduced to the value of the Stokes number, Eq. (3.2). We start form an analysis of relative velocity amplitude $v_w$ as a function of angular frequency $\omega$. From Eqs. (2.11) and (2.12) we see instantly that the function has to have a maximum – for $\omega = 0$ any particle, small or big, has the same velocity amplitude $U_0$, while the relative velocity is zero. At $\omega \to \infty$, the velocities of both particles tend to zero, and therefore the relative velocity is equal to zero also in this case.

For simplification of the following calculations, we assume that the relaxation time of the bigger particle is expressed by:

$$\tau_1 = \alpha\tau, \tag{3.3}$$

where obviously $\alpha > 1$.

Based on Eq. (3.4), we write formula for vibration velocity amplitude of the bigger particle:

$$v_{01} = \frac{U_0}{\sqrt{1 + \omega^2\alpha^2\tau^2}} \tag{3.4}$$

and of the smaller one:

$$v_{02} = \frac{U_0}{\sqrt{1 + \omega^2\tau^2}} \,. \tag{3.5}$$

The amplitude $v_{01}$ is shifted in phase with respect to $U_0$ by an angle $\varphi_1$ defined by equation:

$$\varphi_1 = \tan^{-1}(\omega\alpha\tau) \tag{3.6}$$

and amplitude $v_{02}$ is shifted by an angle

$$\varphi_2 = \tan^{-1}(\omega\tau). \tag{3.7}$$

Obviously, the relative velocity of both particles is shifted in phase by an angle $\varphi$:

$$\varphi = \varphi_1 - \varphi_2\,, \tag{3.8}$$

or by an angle, tangent of which is equal to:

$$\tan\varphi = \frac{\omega\tau(\alpha-1)}{1+\omega^2\tau^2\alpha^2}\,. \tag{3.9}$$

The velocities $v_{01}$ and $v_{02}$ should be subtracted geometrically (because of the phase shift), therefore, introducing the relative drag coefficient $\mu_w$

$$\mu_w = \frac{v_w}{U_0} \tag{3.10}$$

we may calculate it out from equation

$$\mu_w^2 = \frac{1}{1+\omega^2\alpha^2\tau^2} + \frac{1}{1+\omega^2\tau^2} - \frac{2\cos\varphi}{\sqrt{1+\omega^2\alpha^2\tau^2}\sqrt{1+\omega^2\tau^2}} \tag{3.11}$$

$\mu_w(\omega\tau)$ has, as one can easily prove, a maximum for the same value for which the function $\tan\varphi(\omega\tau)$ has its maximum, which makes the following calculations much easier. Namely, we have a condition:

$$\frac{d}{d(\omega\tau)}\tan\varphi = \frac{(\alpha-1)(1+\omega^2\tau^2\alpha) - 2\omega^2\tau^2\alpha(\alpha-1)}{(1+\omega^2\tau^2\alpha)^2} = 0, \tag{3.12}$$

or, after performing elementary calculations, we have from (2.13)

$$\omega\tau = \frac{1}{\sqrt{\alpha}} = \frac{r}{R}\,. \tag{3.13}$$

This value of $\omega\tau$ refers to

$$(\tan\varphi)_{\max} = \frac{1}{2}\frac{\alpha-1}{\sqrt{\alpha}}\,, \tag{3.14}$$

$$(\cos\varphi)_{\max} = \frac{2\sqrt{\alpha}}{\alpha+1}\,, \tag{3.15}$$

and

$$\mu_{w\ \max} = \frac{\alpha-1}{\alpha+1}\,. \tag{3.16}$$

For value $\omega\tau$ given by Eq. (3.13) we have the Stokes number equal to (3.13):

$$n_{\mathrm{St}} = \frac{\tau U_0}{2r} f(\alpha), \tag{3.17}$$

where

$$f(\alpha) = \frac{\alpha-1}{(\alpha+1)\sqrt{\alpha}}\,. \tag{3.18}$$

Figure 2 represents the dependence $\eta_A(\nu)$. The function has a maximum, which we presently calculate form the condition

$$\frac{d\,f(\alpha)}{d\alpha} = 0, \tag{3.19}$$

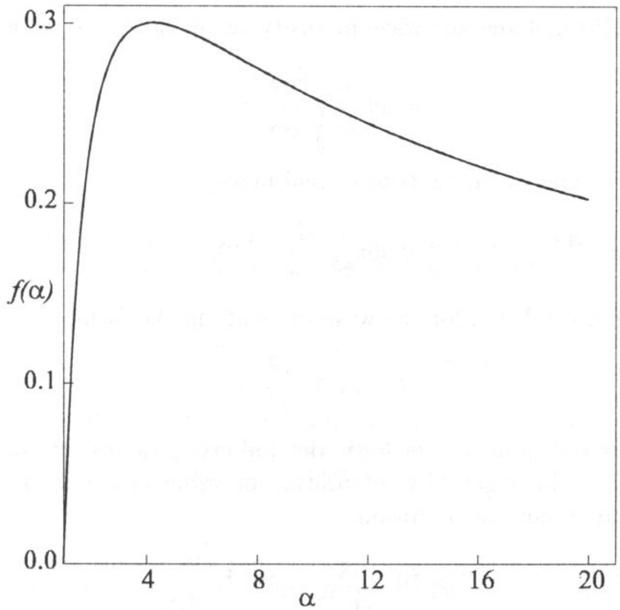


Fig. 2. The dependence $f(\alpha)$.

or

$$\frac{(\alpha+1)\sqrt{\alpha}-(\alpha-1)\sqrt{\alpha}-(\alpha-1)(\alpha+1)\dfrac{1}{2\sqrt{\alpha}}}{(\alpha+1)^2\alpha}=0. \tag{3.20}$$

We adopt here

$$\alpha_{\mathrm{extr}}=2+\sqrt{5}=4.236 \tag{3.21}$$

as the other solution would be negative, which makes no sense. Therefore, the maximum value of $f_{\max}(\alpha)$ is equal to (3.18):

$$f_{\max}(\alpha)=0.305 \tag{3.22}$$

and the respective value $\omega\tau$ (3.13), referring to the maximum of relative velocity, is

$$\omega\tau=\frac{1}{\sqrt{2+\sqrt{5}}}=0.493 \tag{3.23}$$

and

$$\mu_{w\ \max}=0.618\,. \tag{3.24}$$

Taking into account that the real processes are realized statistically, one should assume that the inertial coagulation $\alpha<\alpha_{\mathrm{extr}}$ (3.21) is very little probable. At the value of $\alpha_{\mathrm{extr}}$ we have an optimum course of the process. At $\alpha>\alpha_{\mathrm{extr}}$, as can be seen from Fig. 2, function $f(\alpha)$ decreases very slowly. The condition $n_{\mathrm{St}}>1$ now takes the form:

$$\frac{\tau U_0}{2r}\cdot 0.305>1. \tag{3.25}$$

The condition (3.25) includes the wave intensity, as by Eq. (2.15) we have

$$0.305\frac{\tau}{2r}\sqrt{\frac{2I}{\varrho_0 c}} > 1, \tag{3.26}$$

or, by raising both sides of Eq. (2.2) to second power,

$$0.465\frac{\tau^2}{r^2}\frac{I}{\varrho_0 c} > 10. \tag{3.27}$$

Finally, we obtain a condition for the wave intensity in the form:

$$I > 21.5 \cdot \frac{r^2}{\tau^2}\varrho_0 c\,. \tag{3.28}$$

For average industrial aerosols we have the following values: $r^2 \approx 10^{-8}$ [cm$^2$] and $\tau^2 \approx 10^{-8}$ [s$^2$] [7], which give, by adaptation of value of $\varrho_0 c = 42$ [g/cm$^2$s] (corresponding to the air in normal conditions):

$$I \geq 10^3\ \frac{\mathrm{g}}{\mathrm{cm^2 s}} = 10^{-4}\ \frac{\mathrm{W}}{\mathrm{cm^2}}\,. \tag{3.29}$$

The calculated intensity value is tens thousand times weaker than the intensity required for occurrence of proper acoustical coagulation, i.e. related to the amplitude effect.

## 4. Conclusions

The described phenomenon is fully "responsible" for coagulating action of the acoustic field, and provides, with great excess, conditions in which the efficiency of the inertial coagulation may be considered as equal to unity.

## References

[1] H. Czyż, *On the concentration of aerosol particles by means of drift forces in a standing wave field*, Acustica, **70**, 23–28 (1990).

[2] N.A. Fuchs, *The mechanics of aerosols*, Pergamon Press, London 1964.

[3] L.V. King, *On the acoustic radiation on spheres*, Proc. Roy. Soc., **A 147**, 212–241 (1934).

[4] I. Malecki, *Theory of acoustic waves and systems* [in Polish], PWN, Warszawa 1964.

[5] E.P. Mednikov, *Acoustic coagulation and precipitation of aerosols*, Consultants Bureau, New York 1966.

[6] R. Wyrzykowski, *Analysis of the possibility of increasing of sirens efficiency*, part II [in Polish], Technical University, **95**, 59–71, Wrocław, (1964).

[7] R. Wyrzykowski, *Comparative criteria and bases of calculatings of dust collectors* [in Polish], Technical University of Wrocław, **80**, 79–103, Wrocław (1963).

[8] R. Wyrzykowski, *A theory of acoustic coagulation* [in Polish], University of Rzeszów, **1**, 125–135, Rzeszów (1969).