THE MOTION OF GAS BUBBLES IN A LIQUID UNDER THE INFLUENCE OF ACOUSTICAL STANDING WAVE

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This paper comprise the results of theoretical analysis of the problem of the forces influencing the gas bubbles in a liquid in a stationary wave field. The problem of the motion of small particles suspended in the gaseous medium (aerosols and fumes) has been studied intensively starting from the forties of our century in connection with the technical application of the acoustic coagulation for the precipitation of gases. This paper considers the equation of the motion of a gas bubble in a standing wave field under the influence of the drift and the resistance forces in the Stokes and Oseen approximation. We take into account the drift related to the radiation pressure, periodic viscosity changes and the asymmetry of motion of gas bubble vibrating in a standing wave field. This study considers an estimation of the intensity of the drift forces of types R, L and A as a function of the bubble radius at $10-100$ kHz frequency of the wave. We give the general properties of the solutions of the motion equation of the gas bubble in the case of large attenuation constants, corresponding for typical values of the drift forces.

Notations

F_D – drift force,
A_D – drift intensity amplitude,
m_p – gas bubble mass,
r – gas bubble radius,
μ_g – gas bubble flow-around coefficient,
μ_p – gas bubble entrainment coefficient,
E – acoustic wave energy density,
k – wave number,
x – gas bubble position,
η – medium viscosity,
ω – angular frequency,
f – frequency,
t – time,
ϱ_g – medium density,
ϱ_p – gas bubble density.

1. Introduction

The drift forces, a consequence of interaction between the gas bubble and the vibrating medium, result from such phenomena as the radiation pressures, the asymmetric

vibration motion of the gas bubble or periodic changes in the viscosity of the liquid. The first mechanism of the drift has been presented by KING [6] and is called the radiation drift, the R – type drift in short. It is connected with the radiation pressure acting on the gas bubble as the result of momentum carried out by acoustic wave diffracted on the particle. This type of drift is important for large gas bubbles.

For small gas bubbles of radii of order of microns, authors introduced different mechanisms [7]. The most important models concern the effect of variations of viscosity in the wave field caused by local changes of the temperature during periodical compressions and decompressions of the medium [7]. This type of drift is called the viscosity drift, and the L – type drift in short.

Another type of drift is characteristics only of a standing wave. It results from the fact that in such a wave the vibration amplitude of the medium depends on position being greater in the area of loops. In view of their inertia, gas bubbles do not keep pace with the motion of the medium and are affected by variable forces over the time of their oscillation. Asymmetry drift is the most natural one and its mechanism is physically the most fundamental, it is called the A – type drift in short [2].

Different kinds of drift have a common property: the forces applied on the gas bubble depend in the same way on its position with respect to the loops and nodes of the standing wave and are proportional to the density of the wave energy. The drift forces depend strongly on the gas bubble size [4]. Below, consideration is given only to the problem of the motion of a single gas bubble. This means that the effect of interaction of gas bubbles is neglected, i.e., the process leading to augmentation itself, namely the fact that smaller gas bubbles link to form larger aggregates. Causing gas bubbles to gather near points of stable equilibrium (minima of the potential of the drift forces), the phenomena caused by the drift forces assist in a way the elementary acts of augmentation by increasing the concentration of gas bubbles near the nodes or loops of the standing wave.

2. Analysis of the equation of motion

It appears that, irrespective of the mechanism of the occurrence of drift forces, they can be described by the formula [1]

$$F_D(x) = F_0 \sin(2kx), \qquad (2.1)$$

where F_0, constant, denotes the value of the drift force amplitude. The forces of this type are called the drift forces. The position of the potential minima

$$U_D(x) = F_0(2k)^{-1} \cos(2kx) + \text{const} \qquad (2.2)$$

depends on the sign of the constant describing the maximum value of drift force. It is clear, that the sign of the constant has no effect on the kinetics of the process of gas bubble transport.

The equation of motion of the average position of the gas bubble is [1]

$$m_p \frac{d^2x}{dt^2} = -C_{\text{St}} \frac{dx}{dt} - C_{\text{Os}} \frac{dx}{dt} \left| \frac{dx}{dt} \right| + F_0 \sin(2kx) \qquad (2.3)$$

or

$$m_p \frac{d^2x}{dt^2} = -6\pi\eta r \frac{dx}{dt} - \frac{9}{4}\pi r^2 \varrho_p \frac{dx}{dt}\left|\frac{dx}{dt}\right| + F_0 \sin(2kx).$$ (2.4)

The first term on the right side of the equation represents the Stokes force, the second one represents the nonlinear Oseen correction which is significant for large Reynolds numbers. This equation is nonlinear in view of the last two terms. The above simple differential equation has no elementary solution. To estimate on the character of the solution, let us reduce to a minimum the number of constants in the equation (2.5) by replacing the position and time by the nondimensional variables

$$y = \pi - 2kx \qquad \theta = (2kA_D)^{1/2},$$ (2.5)

where the quantity $A_D = F_0/m_p$ which by analogy to other interactions, can be called the intensity of the drift force field.

On the basis of the formulae for the radiation drift [2]

$$F_R = \frac{8}{3}\pi kr^3 \mu_g^2 E \sin(2kx)$$ (2.6)

the asymmetry drift [2]

$$F_A = -\frac{1}{2}m_p \varrho_g^{-1} k\mu_p^2 \sin(2kx)$$ (2.7)

and the viscosity drift [2]

$$F_L = 3\pi(\kappa - 3)r\mu_g^2\eta(\varrho_g c)^{-1} E \sin(2kx)$$ (2.8)

we obtain the amplitudes of the drift intensity A_{DR}, A_{DA}, A_{DL} (the symbols R, L, A distinguish the considered types of the drift).

$$A_{DR} = 2\varrho_p^{-1} k\mu_g^2 E,$$ (2.9)
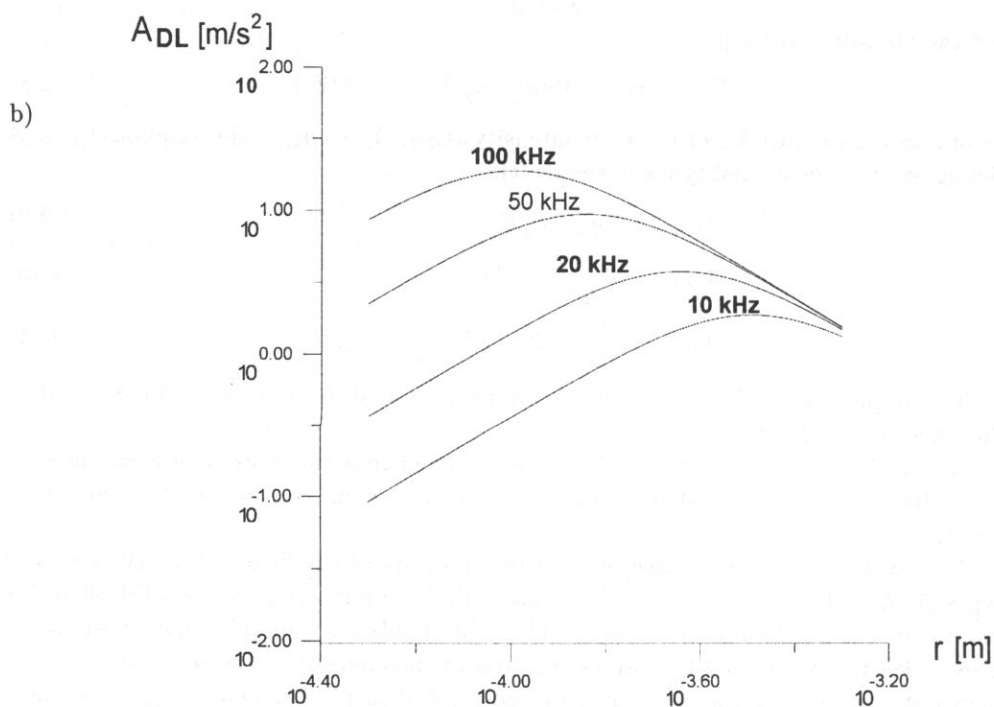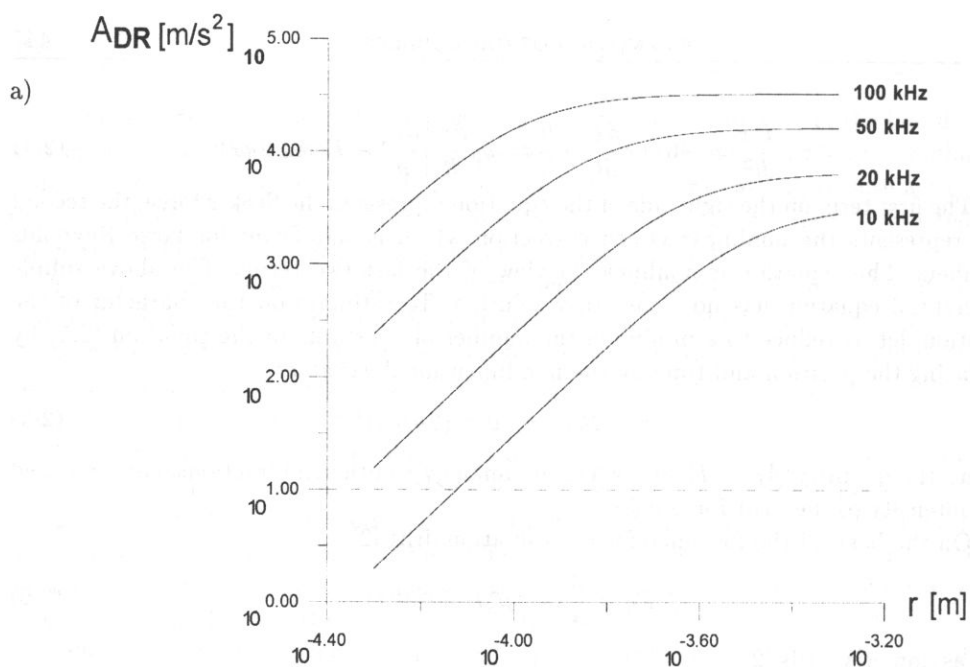
$$A_{DA} = -\frac{1}{2}\varrho_g^{-1} k\mu_p^2 E,$$ (2.10)

$$A_{DL} = \frac{9}{4}(\kappa - 3)\eta r^{-2}(\varrho_p \varrho_g c)^{-1}\mu_g^2 E.$$ (2.11)

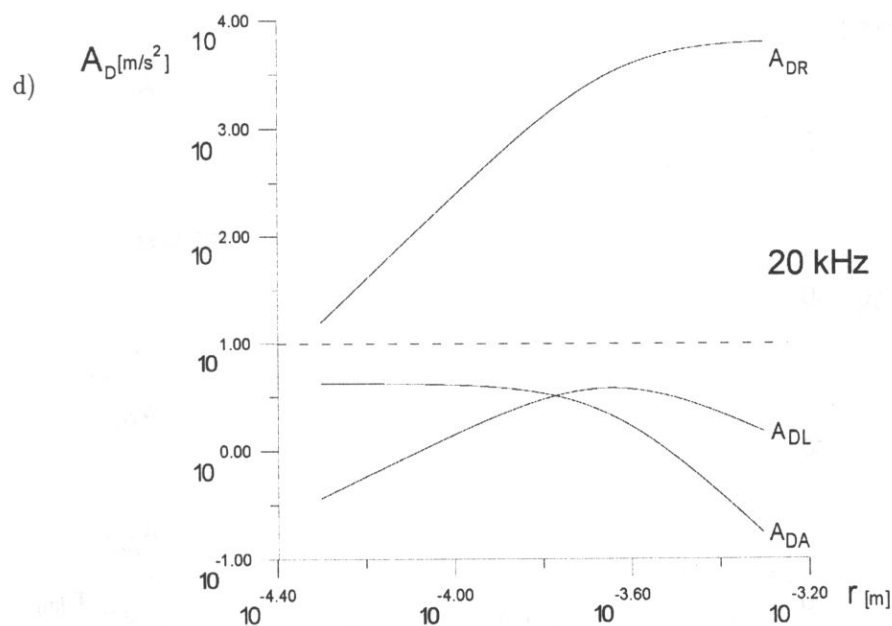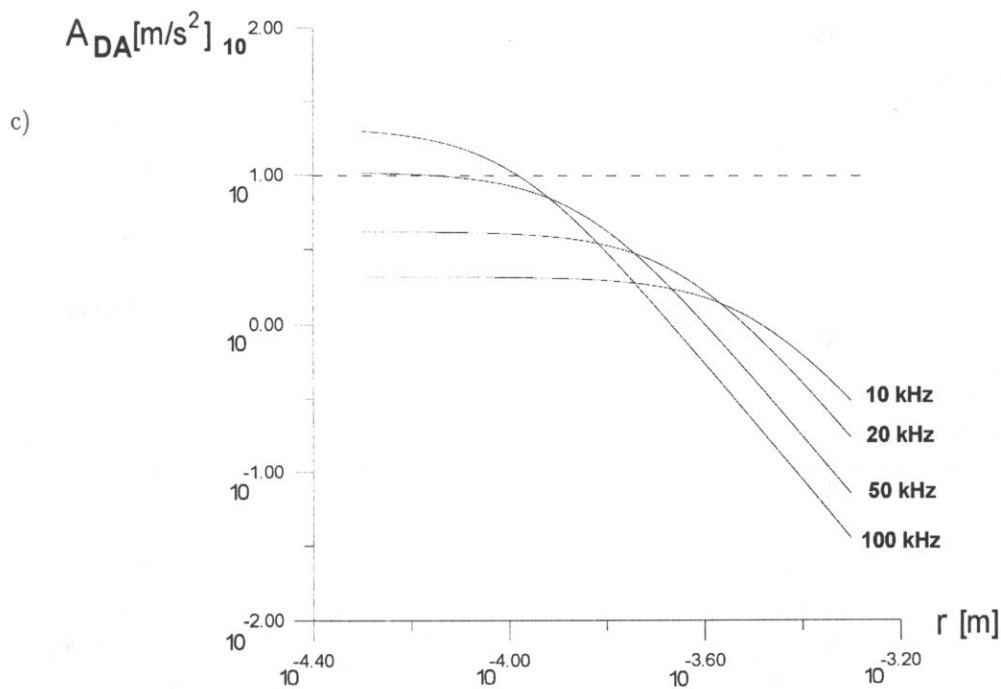The amplitudes of the drift intensity are proportional to the wave number k and to the wave energy density E.

In calculating the numerical values it is assumed that the density of acoustic wave $E = 100$ J/m^3, $c = 1500$ m/s, $\varrho_p = 1.249 \cdot 10^3$ kg/m^3, $\varrho_g = 1000$ kg/m^3, $\eta = 1.8 \cdot 10^{-3}$ N s/m^2.

Figures 1 a, b, c, d, e, f show plots of the intensity of the field of the drift forces of types R, L and A as a function of the gas bubble radius. The horizontal dashed line represents the acceleration of gravity. The plots made on a double logarithmic scale. This makes it easy to read the values of intensity for a density of the wave energy other than the given one, since the quantities represented in plots depend on E in a linear way and an increase or decrease in this value by one order of magnitude causes the same change in the value of the intensity drift fields.

Analysis of the plots indicates that particular kinds of drift dominate various intervals of variation of the particle radius.

a)

A_{DR} [m/s²]

100 kHz
50 kHz
20 kHz
10 kHz

r [m]

b)

A_{DL} [m/s²]

100 kHz
50 kHz
20 kHz
10 kHz

r [m]

[Fig. 1 a, b]

[448]

c)

A_DA[m/s²] 10^2.00

10^1.00

10^0.00

10^-1.00

10^-2.00

10^-4.40   10^-4.00   10^-3.60   10^-3.20

r [m]

10 kHz
20 kHz
50 kHz
100 kHz

d)

A_D[m/s²] 10^4.00

10^3.00

10^2.00

10^1.00

10^0.00

10^-1.00

10^-4.40   10^-4.00   10^-3.60   10^-3.20

r [m]

A_DR

20 kHz

A_DL

A_DA

[FIG. 1 c, d]

[449]

e) $A_D[m/s^2]$

A_{DR}

50 kHz

A_{DL}

A_{DA}

f) $A_D[m/s^2]$

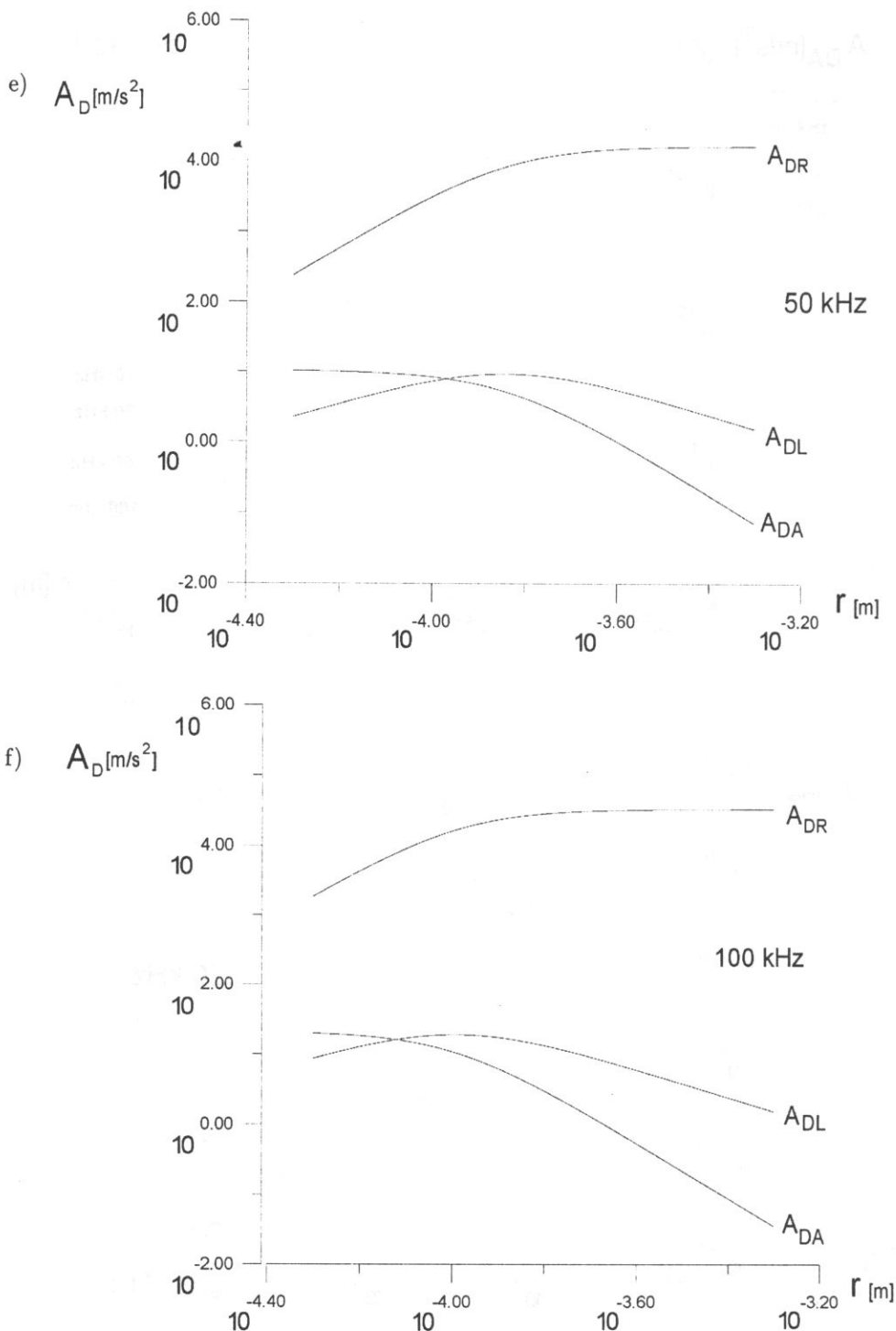A_{DR}

100 kHz

A_{DL}

A_{DA}

Fig. 1. Plots of the intensity of the field of the drift forces a) of type – R, b) of type – L, c) of type – A, as a function of the gas bubble radius, at 20 kHz, 50 kHz, 100 kHz, and of types R, L and A as a function of the gas bubble radius d) at 20 kHz, e) at 50 kHz, f) at 100 kHz. The horizontal dashed line represents the acceleration of gravity.

[450]

The equation of motion becomes

$$\frac{d^2y}{d\theta^2} + \alpha\frac{dy}{d\theta} + \beta\frac{dy}{d\theta}\left|\frac{dy}{d\theta}\right| + \sin(y) = 0, \tag{2.12}$$

where the following was introduced

$$\alpha = \tau^{-1}(2kA_D)^{-1/2}, \tag{2.13}$$

$$\beta = \frac{27\varrho_g}{32kr\varrho_p}. \tag{2.14}$$

The equation of motion in this form (2.12) contains only two constant, whereas the initial equation (2.3) contained four of them (including the wavenumber). The gas bubble mass, the drift force and the wavelength, are normalized in a way in this equation making it easier to analyze the effect of the two dissipation terms on the solution. The constants α and β depend on the parameters characterizing the gas bubble and the wave. The constant α depends on the intensity of the drift force field, and on the kind of drift.

Assuming that same numerical values which were used in calculating the quantity A_D, one can estimate the constants α and β, and, thus, evaluate the two terms representing friction in the equation of motion.

Figures 2 a, b, c show plots of the constants α and β of equation (2.12) as a functions of the gas bubble and frequency. The symbols R, L and A distinguish the considered types of drift.
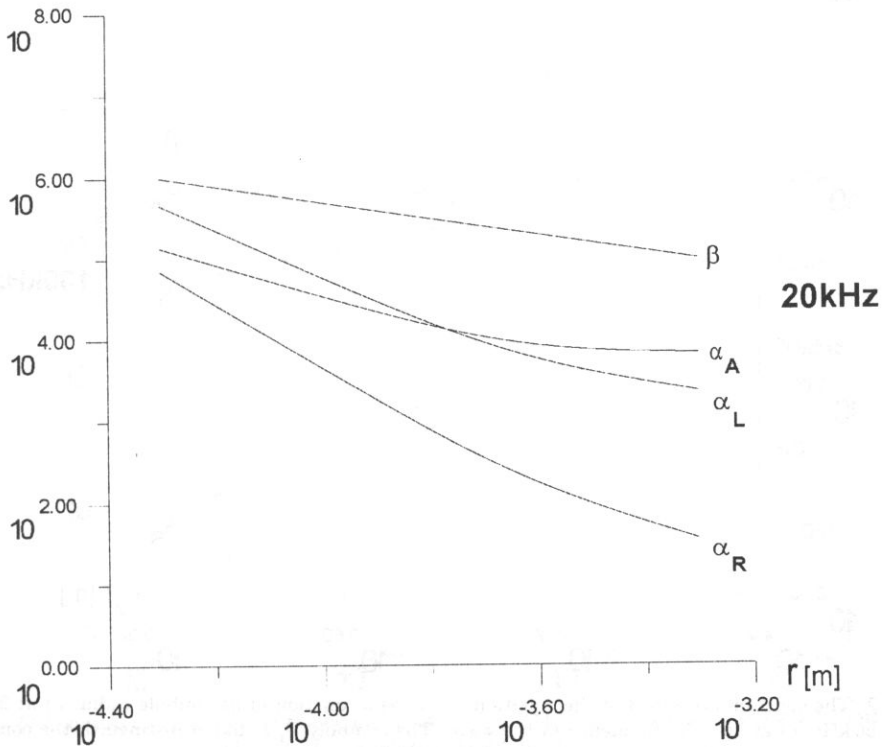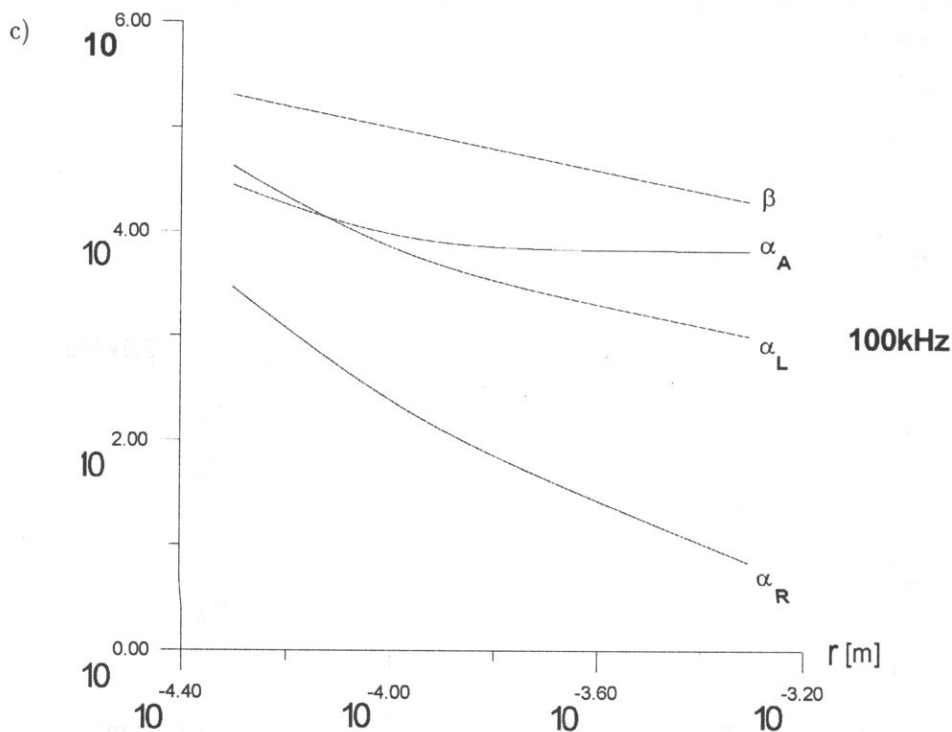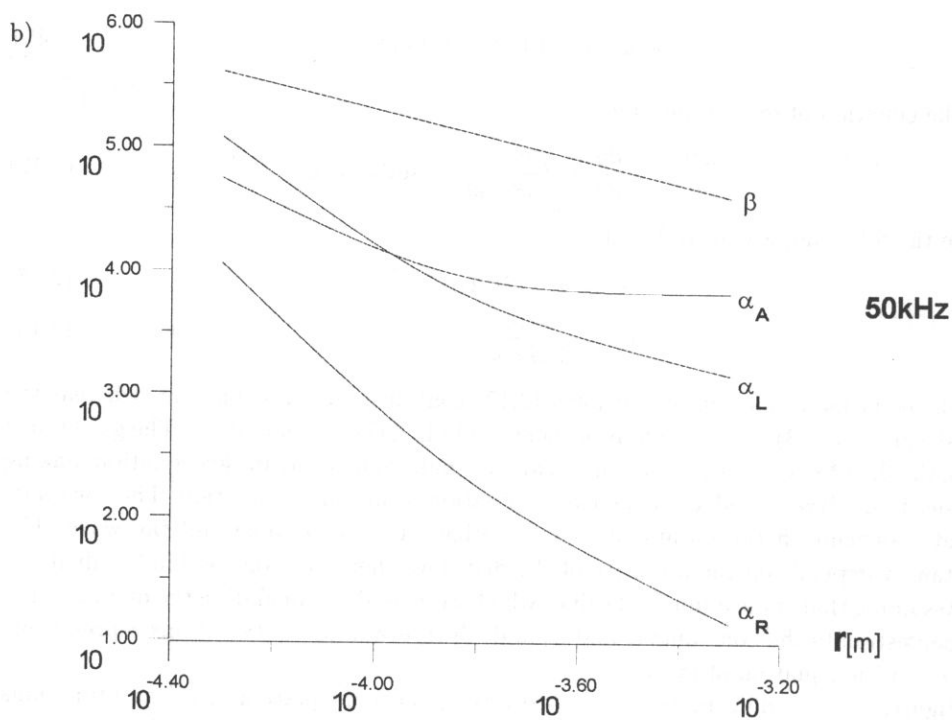

[FIG. 2 a]

Fig. 2. The constants α and β of the equation (2.12) as a function of gas bubble radius a) at 20 kHz, b) at 50 kHz, c) at 100 kHz frequency of the wave. The symbols R, L and A distinguish the considered types of the drift.
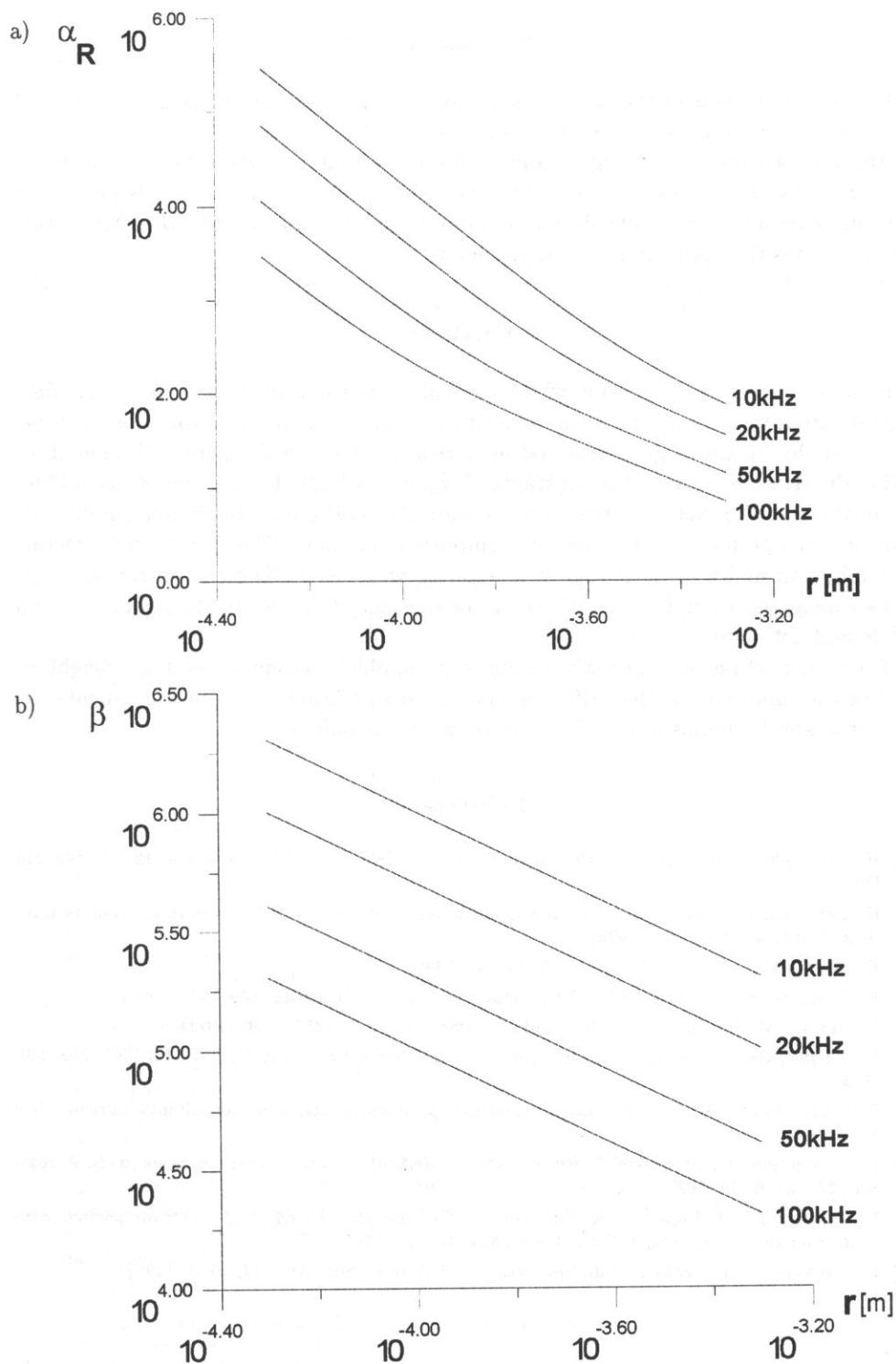
[452]

Fig. 3. The constants a) α, b) β of the equation of motion (2.12) as a function of the gas bubble radius at 10 kHz, 20 kHz, 50 kHz, 100 kHz, for R – type of drift.

Figures 3 a, b show of the constants α and β of the equation (2.12) as a function of gas bubble radius and frequency of the wave for the R – type drift.

Analysis of these plots (Figs. 2 and 3) indicates that the nonlinear friction term, represented by the constant β (the Ossen correction) can play a role in this motion. It gives the general properties of solutions of motion equations for considerable attenuation and formulates the applicability of approximation.

3. Conclusion

Equation (2.12) of the motion of the gas bubble in the acoustic standing wave field of great intensity does not have an elementary solution. Analysis of the solution type, carried out by means of graphical and numerical methods in the paper [1] allowed for finding the relation between the constants of equation (2.12). The motion of gas bubbles in acoustic standing field consists in monotonically approaching the stable equilibrium point or quasi-periodical vibration with amplitude damping. The drift forces directing gas bubbles to nodes or antinodes of a standing wave [8, 9]. The concentration of gas bubbles increases around the equilibrium position and decreases in the region between a node and antinode.

The transport phenomenon which causes gas bubbles concentration in the neighborhood of the minimum of the drift force potential significantly supports augmentation microprocesses by reducing the distances between gas bubbles.

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