ARCHIVES OF ACOUSTICS
21, 1, 99–107 (1996)

PIEZOELECTRIC INTERFACIAL WAVES IN LANGASITE AND DILITHIUM TETRABORATE

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Waves propagating along a perfectly conducting plane embedded in a piezoelectric medium are investigated. Numerical analysis of trigonal langasite (LGS) and cubic dilithium tetraborate (LBO) crystals shows that such waves exist for many orientations of the conducting plane with respect to the crystallographic axes, and for many directions of propagation. The wave velocity is close to that of the slowest shear wave for the same direction, and the piezoelectric coupling coefficient can be as high as 1.65% for LBO and 0.25% for LGS. The conditions of existence, and the properties of the most interesting waves are presented. Expected applications are discussed.

1. Introduction

SAW devices must be encapsulated to protect the free substrate surface (required for SAW to propagate) against the adverse environmental influence. Otherwise, contamination would cause ageing effects. The encapsulation, in turn, creates problems of its own: the case and the crystal have different thermal expansion coefficients; the difference causes stress in the crystal (all-quartz encapsulation has been devised to avoid this problem). Some interesting crystals cannot be used because of their high sensitivity to the environment.

The piezoelectric interfacial wave (PIW), which has been proposed in [1] as an alternative to SAW, is immune to these problems. PIW propagates inside the piezoelectric crystal, and is guided by the conducting plane embedded in it. The wave energy density is high in the immediate vicinity of the plane, and decreases exponentially with the distance from the plane.

From the practical point of view, one can cut the crystal into two pieces, put metal on the faces, and attach the pieces together with perfect mechanical contact (the metal diffusion technique can be used for that purpose). The metallization need not be full, there can be periodic metal strips instead, and still PIW will exist. The periodic metal strips can be further employed to form interdigital transducers, like these applied to

SAW. Another possibility is the application of a fragile sheet of high-temperature superconductor [2].

PIW can be easily excited and detected in both langasite and dilithium tetraborate due to high piezoelectric coupling. Although the highest coupling is accompanied by rather high beam steering (27° for LBO and 12° for LGS), there exist waves with high coupling (1% for LBO and 0.2% for LGS) and zero beam steering.

The wave polarization at the conducting plane depends on the orientation of the plane, and can be almost aribtrary. This opens certain interesting possibilities of application of PIW in acousto-optical devices like Bragg cells.

2. Eigenvalue problem

The system of coordinates (x, y, z) is chosen so that the perfectly conducting plane is described by the equation $z=0$, and is an interface between two half-spaces (for $z>0$ and $z<0$) of a homogeneous piezoelectric medium. It is assumed that the field is independent of y, and that the time dependence is given by the factor $\exp(j\omega t)$.

The electro-mechanical field satisfies the equations

$$T_{ij,j}=-\omega^2\rho u_i\,, \tag{2.1}$$

$$D_{k,k}=0\,, \tag{2.2}$$

complemented with the constitutive relations

$$T_{ij}=c_{ijkl}\,u_{k,l}+e_{kij}\phi_{,k}\,, \tag{2.3}$$

$$D_k=e_{kij}u_{i,j}-\varepsilon_{ki}\phi_{,i}\,, \tag{2.4}$$

where $i, j, k, l=1, 2, 3$ $(x_1=x, x_2=y, x_3=z)$. The medium is described by the following constants: elastic tensor c_{ijkl}, piezoelectric tensor e_{kij}, dielectric tensor ε_{ki}, and mass density ρ. The field variables are: displacement u_i, electric potential ϕ, stress tensor T_{ij}, electric induction D_k.

If we eliminated T_{ij} and D_k from Eqs. (2.1)–(2.4), we would get the conventional system of four second-order partial differential equations for the four functions u_i and ϕ. Instead, we introduce the four additional field variables T_i and D_3, where $T_i=T_{3i}$ (cf. [3]). Then we assume that the field depends on x and z through the factor $\exp(-j\omega rx-j\omega sz)$. We obtain the system of eight linear algebraic equations

$$H_{KL}(r)F_L=qF_K\,, \tag{2.5}$$

where $q=s/r$, $(F_K)=(j\omega ru_i, j\omega r\phi, T_i, D_3)$, and $K, L=1,...,8$. The matrix H_{KL}, which is real and non-symmetric for real r, depends on material constants.

After solving the eigenvalue problem defined by Eq. (2.5) we get eight eigenvectors $\tilde{F}_K^{(J)}(r)$ corresponding to eight eigenvalues $q^{(J)}(r)$ for $J=1,...,8$. Each eigenwave has the form $F_K^{(J)}=\tilde{F}_K^{(J)}\exp(j\omega t-j\omega r(x+q^{(J)}z))$, and the solution of Eqs. (2.1)–(2.4) is a linear combination of these waves.

We are interested in solutions for real r such that no eigenvalue is real (this is true for $r>r_s$ where $1/r_s$ is the phase velocity of the slowest shear wave). Therefore, the solution in the upper half-space, F_K^+, and the solution in the lower half-space, F_K^-, should consist of the eigenwaves that are decaying for $z\to\infty$ and $z\to-\infty$, respectively.

At the plane $z=0$, the complex amplitudes of the two solutions are

$$\tilde{F}_K^{\pm}=\pm\sum_J{}^{\pm}\, C_J\tilde{F}_K^{(J)}, \tag{2.6}$$

where the summation is performed over J such that $\operatorname{Im} q^{(J)}<0$ (for $\tilde{F}_K^+$) and $\operatorname{Im} q^{(J)}>0$ (for $\tilde{F}_K^-$); C_J are constant coefficients. The boundary conditions imply that all the field variables are continuous across the plane $z=0$ except D_3. The amplitude $\tilde{D}_3$ suffers a jump $\Delta\tilde{D}_3$ (assumed to be real). Thus $\Delta\tilde{F}_K=\tilde{F}_K^+-\tilde{F}_K^-=0$ for $K=1,...,7$, and $\Delta\tilde{F}_8=\Delta\tilde{D}_3$. Using these equalities and Eq. (2.6) we can find the coefficients C_J, and then the amplitudes $\tilde{F}_K^+$ and $\tilde{F}_K^-$ for $K=1,...,8$. In particular, for $K=4$,

$$j\omega r\tilde{\phi}=Z(r)\Delta\tilde{D}_3, \tag{2.7}$$

where $Z(r)$ is a complex-valued function determined by the set of eigenvectors for the given r.

It can be shown that the function $Z(r)$ is purely imaginary for $r>r_s$, and that $\operatorname{Im}Z(r)$ tends to a positive value as $r\to\infty$. PIW exists if the dispersive equation $Z(r)=0$ is satisfied for a particular value of r, say r_p. Then $\tilde{\phi}=0$ (i.e. ϕ satisfies the boundary condition for $z=0$), and $1/r_p$ is the phase velocity of PIW.

3. Calculations

The dispersive equation gives different solutions for different triplets of Euler angles. These triplets may be represented as points in a three-dimensional space. We scan the space at discrete points so that the angles change in steps of 2°. The symmetry of the piezoelectric crystal suggests that it is sufficient to scan the following ranges: 0°–45°, 0°–180°, 0°–180° for LBO, and 0°–30°, 0°–180°, 0°–180° for LGS.

At each chosen point in the angle space, the eigenvalue problem is solved numerically (using EisPack routines [4]) for several values of r in search of r_s, and then in search of r_p in a small neighbourhood of r_s, just above r_s (simple zero-finding routines).

If PIW exists then its parameters are calculated: $\delta=(r_p-r_s)/r_s$, $v_p=1/r_p$, beam steering angle ψ, effective permittivity ε_{eff}, piezoelectric coupling coefficient κ (see [1]), and normalized complex amplitudes (with respect to the time average of the total energy flux of the wave) for $z=0$, i.e., $\tilde{D}_3^+$, $\tilde{u}_k$ and $\tilde{T}_k$. The calculations are performed for $\omega=10^6\ \text{s}^{-1}$. The material constants for LBO and LGS are from [5].

4. Conclusion

PIW exists for a high percentage of points in the angle space (93% for LBO and 60% for LGS), although many of these points are not very interesting in view of applications (low piezoelectric coupling).

The most important features of PIW are illustrated in Figure 1 and Figure 2. Table 1 and Table 2 give PIW parameters for eight points in the angle space (representative of eight groups of points), selected from over 120 thousand points where PIW exists.

Table 1. PIW parameters for LBO

Euler angles	δ	v_p	ψ	$\varepsilon_{\mathrm{eff}}$	κ	$\tilde{D}_3^+$	$\tilde{u}_1$	$\tilde{u}_2$	$\tilde{u}_3$	$\tilde{T}_1$	$\tilde{T}_2$	$\tilde{T}_3$
deg	%	m/s	deg	—	%	$\frac{\mathrm{nC/m^2}}{\mathrm{P^{1/2}}}$	$\frac{\mathrm{pm}}{\mathrm{P^{1/2}}}$	$\frac{\mathrm{pm}}{\mathrm{P^{1/2}}}$	$\frac{\mathrm{pm}}{\mathrm{P^{1/2}}}$	$\frac{\mathrm{N/m^2}}{\mathrm{P^{1/2}}}$	$\frac{\mathrm{N/m^2}}{\mathrm{P^{1/2}}}$	$\frac{\mathrm{N/m^2}}{\mathrm{P^{1/2}}}$
42 90 70	1.18	3628	−27	18.8	1.65	13.4∠0°	−60.7	77.2	−0.15	−j3.33	−j2.40	j118.
0 90 90	0.50	3905	0	18.9	1.00	9.50∠0°	−63.2	0.00	−0.05	j0.28	j0.00	−j365.
30 90 90	0.45	3807	0	18.9	0.80	9.27∠0°	−62.8	0.00	0.01	−j0.06	−j33.2	−j348.
45 90 30	0.05	3818	37	18.6	0.10	2.84∠0°	0.02	32.2	0.00	j0.00	j0.00	j37.7

NOTES. Row 1: maximum κ. Row 2: $\kappa \geqslant 1\%$, $\psi = 0$, Row 3: $\kappa \geqslant 0.4\%$, $\psi = 0$, Row 4: maximum $|\psi|$. P = 1 mW/m.

Table 2. PIW parameters for LGS

Euler angles	δ	v_p	ψ	$\varepsilon_{\mathrm{eff}}$	κ	$\tilde{D}_3^+$	$\tilde{u}_1$	$\tilde{u}_2$	$\tilde{u}_3$	$\tilde{T}_1$	$\tilde{T}_2$	$\tilde{T}_3$
deg	%	m/s	deg	—	%	$\frac{\mathrm{nC/m^2}}{\mathrm{P^{1/2}}}$	$\frac{\mathrm{pm}}{\mathrm{P^{1/2}}}$	$\frac{\mathrm{pm}}{\mathrm{P^{1/2}}}$	$\frac{\mathrm{pm}}{\mathrm{P^{1/2}}}$	$\frac{\mathrm{N/m^2}}{\mathrm{P^{1/2}}}$	$\frac{\mathrm{N/m^2}}{\mathrm{P^{1/2}}}$	$\frac{\mathrm{N/m^2}}{\mathrm{P^{1/2}}}$
0 146 26	0.17	2851	−12	57.5	0.25	12.2∠-3°	7.34	49.5	24.9	−j379.	j36.6	j43.5
7 144 35	0.13	2936	0	57.9	0.20	10.9∠-2°	11.5	42.1	28.8	−j433.	j48.7	j105.
0 14 90	0.08	2816	0	62.7	0.15	10.1∠-6°	0.00	−40.4	0.00	j0.00	j1.14	j0.00
0 132 18	0.00	2681	−31	51.9	0.00	2.59∠-50°	−0.87	15.6	5.47	−j84.2	−j11.4	j18.8

NOTES. Row 1: maximum κ. Row 2: $\kappa \geqslant 0.2\%$, $\psi = 0$, Row 3: $\kappa \geqslant 0.1\%$, $\psi = 0$, Row 4: maximum $|\psi|$. P = 1 mW/m.

Some asymmetry seen in the figures, especially at the boundary of PIW existence, is an effect of the finite precision of calculation.

PIW exists in many other piezoelectric crystals, including well known bismuth germanium oxide (BGO) and quartz (SIO). Figure 3 and Figure 4 illustrate PIW existence and features for these two media.

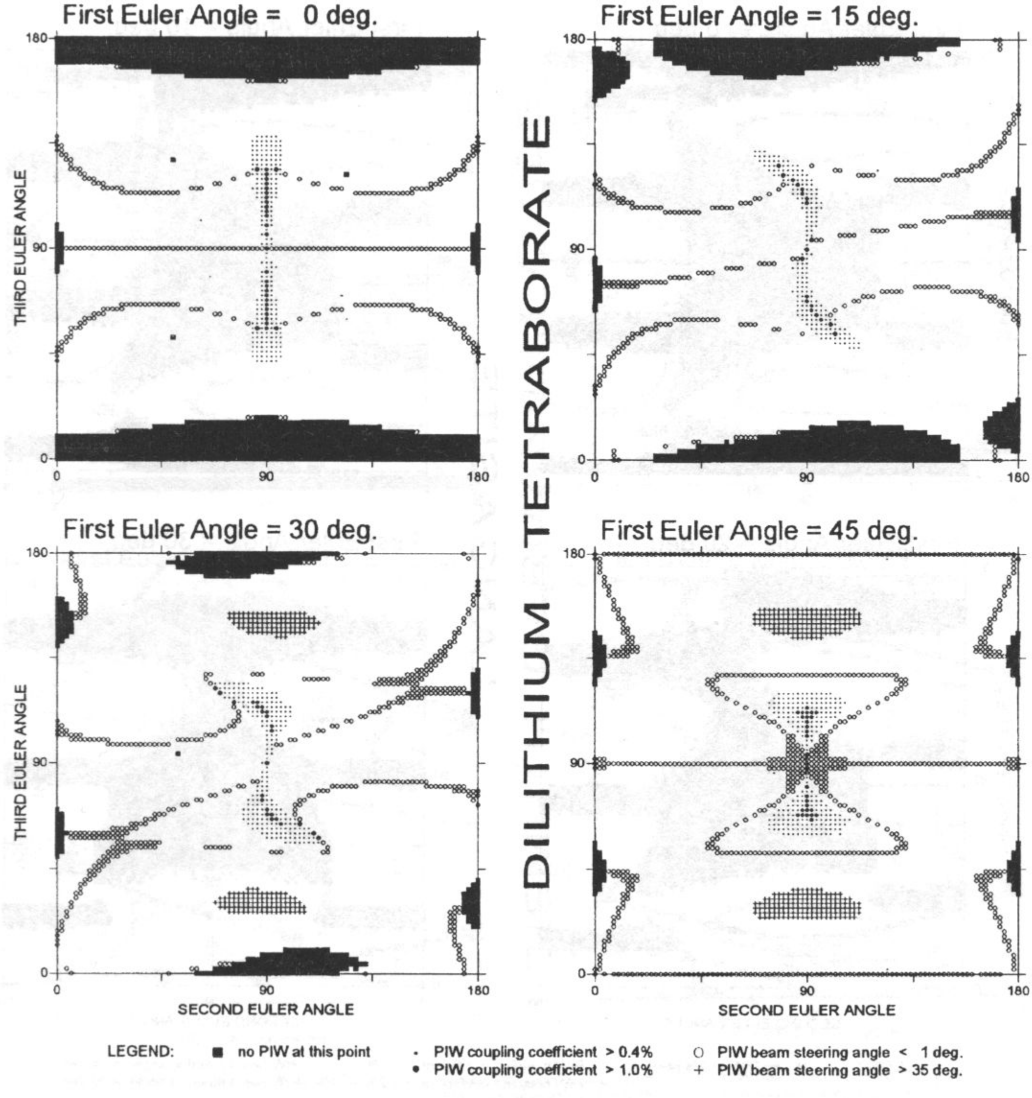

Fig. 1. Maps of PIW properties for LBO.

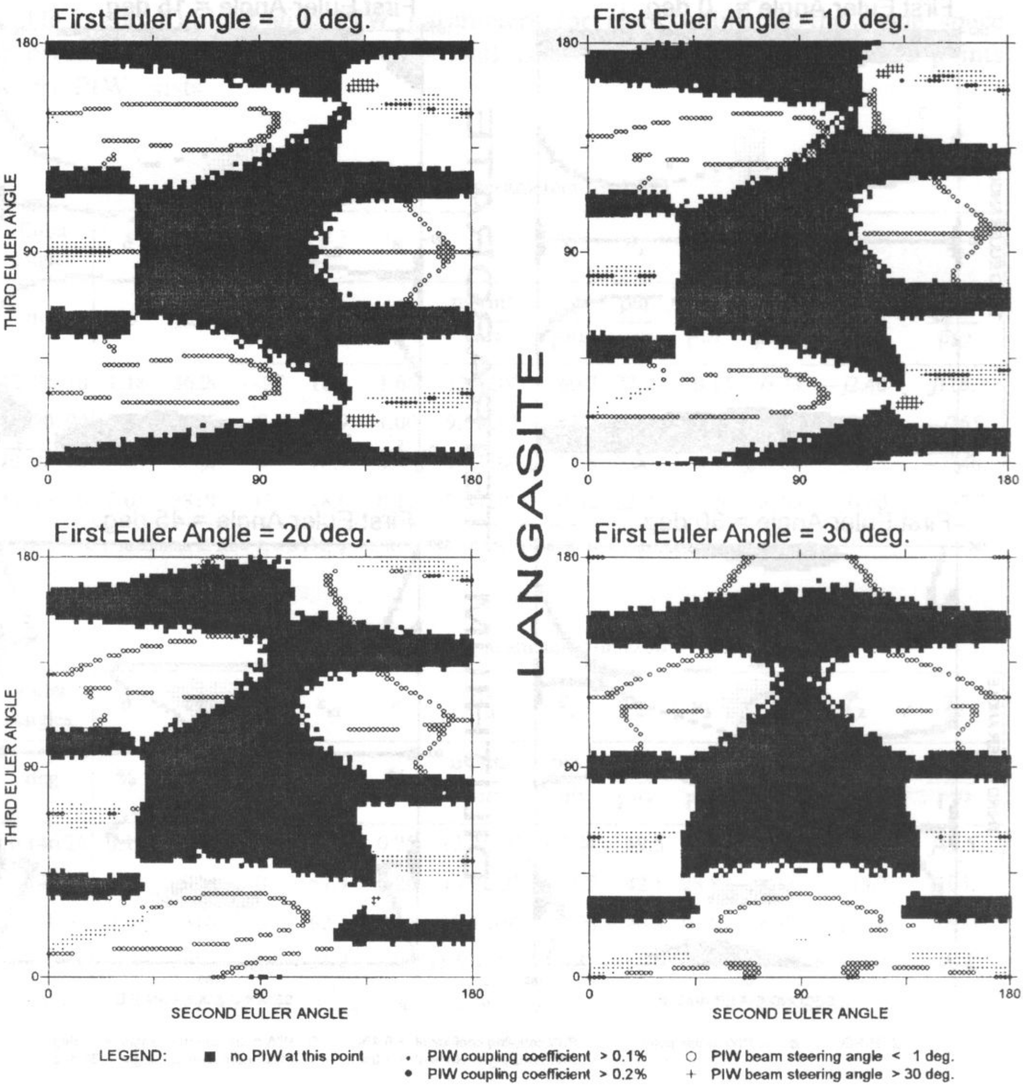

Fig. 2. Maps of PIW properties for LGS.

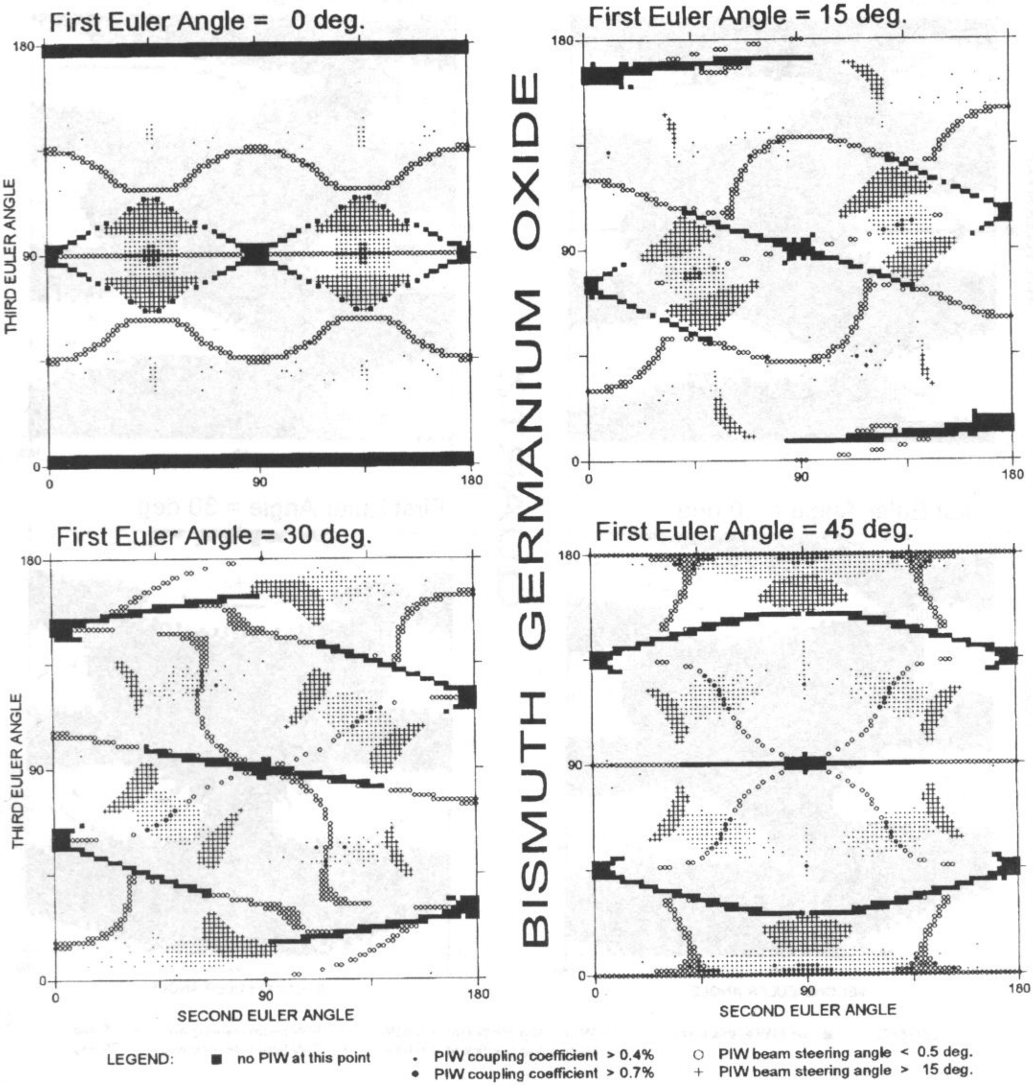

Fig. 3. Maps of PIW properties for BGO.

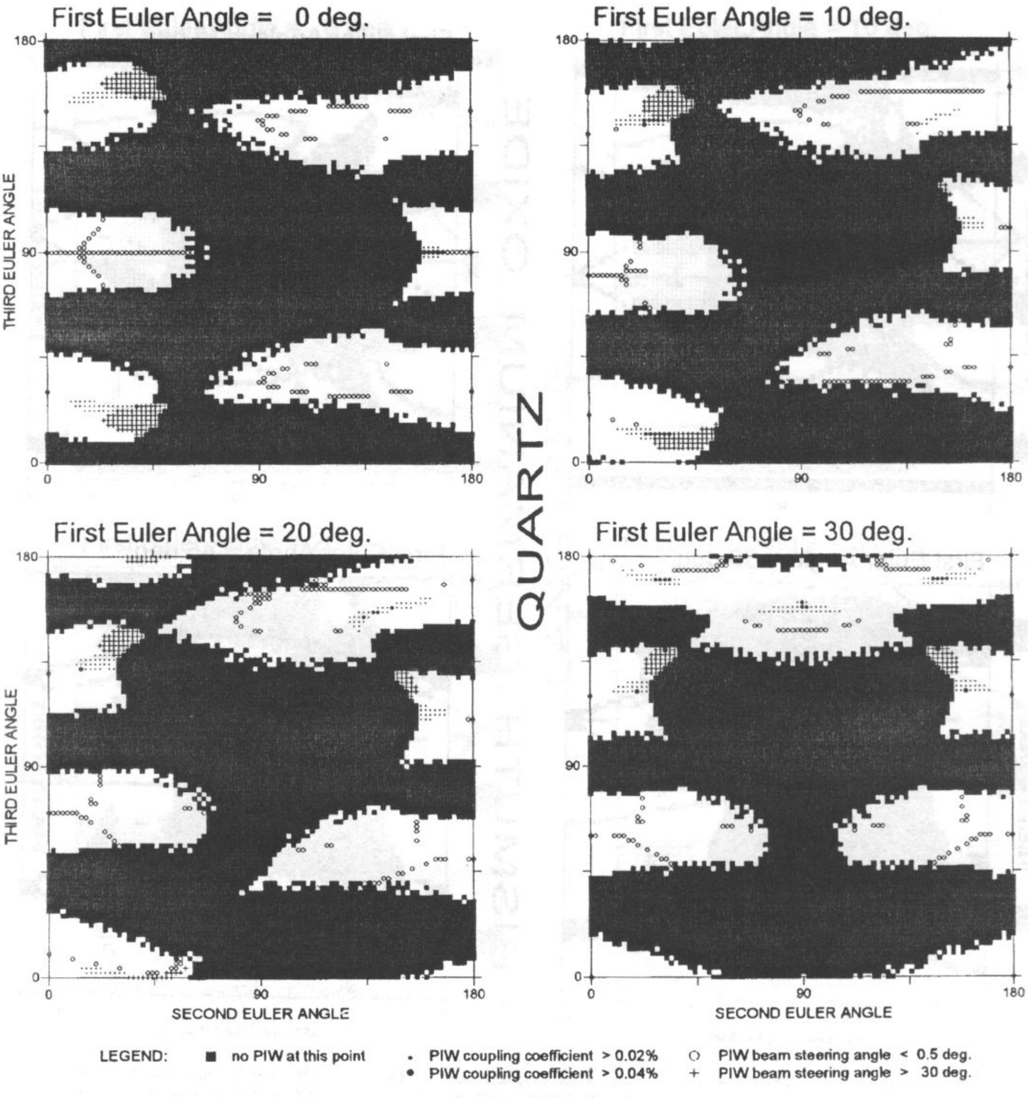

Fig. 4. Maps of PIW properties for SIO.

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