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ANALYSIS OF THE PROPAGATION OF A SPHERICAL WAVE WITH FINITE AMPLITUDE IN AN IDEAL GAS BY THE RENORMALISATION METHOD

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This paper gives an approximate solution of the hydrodynamic equations in the case of a spherical wave with finite amplitude. Using the perturbation renormalisation method it gives the desired description of the acoustic field of a spherical wave generated by a spherical source which pulsates monochromatically with finite amplitude in an infinite, lossless gaseous medium. The solutions obtained for the acoustic velocity and the acoustic pressure have the form of asymptotic expansion of the first order relative to a small perturbation parameter and are valid both for the near and the far field. The analysis of the acoustic field has for the first time been performed directly using the perturbation renormalisation method for a spherical wave.

Notation

$\hat{u}$ — acoustic velocity
u — normalised acoustic velocity
$\hat{p}$ — pressure
p — normalised acoustic pressure
p_0 — pressure in the unperturbed medium
$\hat{\varrho}$ — density of the medium
ϱ — relative density of the medium
ϱ_0 — density in the unperturbed medium
c_0 — weak-signal sound velocity
$\hat{t}$ — time
t — normalised time
t^* — formation time of the shock wave
$\hat{r}$ — length of the tracing radius of a given point
r — normalised length of the tracing radius of a given point in spherical coordinates
r^* — formation distance of the shock wave

R — static radius of the source
τ — normalised time
η — deformed coordinate
ϕ — potential of the acoustic velocity
Φ — parameter
γ — ratio of specific heats
O — large Landau symbol
δ — deformation parameter
ω — angular frequency
Ω — normalised angular frequency
ε — perturbation parameter
A — amplitude of the pulsating sphere
φ, φ_0, ν — constants

1. Introduction

An analytical solution of the hydrodynamic equations, which are the basis for consideration of such problems as the generation and propagation of acoustic waves with small but finite amplitude, is known only in the case of a plane wave propagating in an acoustically ideal medium. This solution has been given independently by EARNSHOW and RIEMANN (e.g. [11]). The hydrodynamic equations for spherical and cylindrical waves of finite amplitude have been considered in a relatively large number of papers, mainly concerned with the description of the propagation in the far field. Experimental work has also been performed, e.g. on spherical waves propagated in water [17] and air [4]. In general, two theoretical approaches to these problems can be distinguished. Some authors use a method which consists in approximating exact equations and seeking exact solutions (e.g. [1, 3]), others employ approximation methods (e.g. [5, 8, 9, 12, 16]. The solution of exact hydrodynamic equations for spherical and cylindrical waves of finite amplitude in an ideal medium has been given by AUGUSTYNIAK [2], with the assumption, however, that the velocity is of one sign. BLACKSTOCK [3] approximated a nonlinear wave equation which is valid for onedimensional travelling waves: plane, spherical and cylindrical, in a lossless medium to the form of the lossless Burgers equation and subsequently for the far field he reduced this equation to one analogous to the equation for plane waves. LOCKWOOD [12] carried out approximation of the second order of the hydrodynamic equations and subsequently solved these equations using the method of multiple scales [14] for a spherical source in a lossless medium, achieving a parametric description of the profile of the pressure wave valid for the far field. GINSBERG [8, 9] gave a description of the profile of the pressure wave and the acoustic wave generated by a monochromatic cylindrical source, taking into account a moving boundary condition, in the case of two and three-dimensional motion of the source. Using the renormalisation method he obtained asymptotic expansions of the first order of the expressions defining the pressure and the acoustic velocity in the far field and gave a matching procedure with which he achieved a description of the near field based on the description of the far field achieved previously. An extension of this analysis to the case when the motion of the source is a superposition of harmonic excitations was given by NAYFEH and KELLY [16].

2. Formulation of the problem

The equations of motion and the equation of state of the lossless gaseous medium will be given with dimensionless variables defined by the relations

$$r=\frac{\hat{r}}{R},\quad t=\frac{c_0}{R}\hat{t},\quad p=\frac{\hat{p}-p_0}{p_0},\quad u=\frac{\hat{u}}{c_0},\quad \varrho=\frac{\hat{\varrho}}{\varrho_0},\quad \Omega=\frac{\omega R}{c_0},\tag{1}$$

where $\hat{u}$ is the radial component of the acoustic velocity, $\hat{p}$ is the pressure, $\hat{\varrho}$ is the density of the medium, $\hat{r}$ is the distance from the centre of the sphere, p_0 and ϱ_0 are respectively the pressure and density in the unperturbed medium, c_0 is the weak — signal sound velocity, γ is the exponent of the adiabate, ω is the angular frequency and R is the static radius of the spherical source.

The hydrodynamic equations for the spherical wave in dimensionless variables are respectively

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\varrho\gamma}\frac{\partial p}{\partial r}=0;\tag{2}$$

$$\frac{\partial \varrho}{\partial t}+u\frac{\partial \varrho}{\partial r}+\varrho\frac{\partial u}{\partial r}+\frac{2}{r}\varrho u=0;\tag{3}$$

$$1+p=\varrho^{\gamma}.\tag{4}$$

The motion of the lossless gaseous medium, under the assumption of irrotationality of the field, can be described with the dimensionless velocity potential function $\phi(r,t)$ such that $u=\partial\phi/\partial r$. The equations describing the motion of the gas, the equation of the potential and the equation of the pressure p, as derived from equations (2)-(4) [19] are given in the following form

$$\frac{\partial^2\phi}{\partial r^2}-\frac{\partial^2\phi}{\partial t^2}+\frac{2}{r}\frac{\partial\phi}{\partial r}=2\frac{\partial\phi}{\partial r}\frac{\partial^2\phi}{\partial r\partial t}+\left[\frac{1}{2}(\gamma-1)+1\right]\left(\frac{\partial\phi}{\partial r}\right)^2\frac{\partial^2\phi}{\partial r^2}+$$
$$+(\gamma-1)\frac{\partial\phi}{\partial t}\frac{\partial^2\phi}{\partial r^2}+\frac{2}{r}(\gamma-1)\frac{\partial\phi}{\partial r}\frac{\partial\phi}{\partial t}+\frac{1}{2}(\gamma-1)\frac{2}{r}\left(\frac{\partial\phi}{\partial r}\right)^3;\tag{5}$$

$$(1+p)^{(\gamma-1)/\gamma}=1-(\gamma-1)\left[\frac{\partial\phi}{\partial t}+\frac{1}{2}\left(\frac{\partial\phi}{\partial r}\right)^2\right].\tag{6}$$

In turn, the motion of an oscillating spherical source with finite amplitude which generates a wave in an infinite medium, written in dimensionless variables, is defined by the relation

$$r_k(t) = 1 + \varepsilon \cos(\Omega t + \varphi); \tag{7}$$

where $\varepsilon = |A/R| \ll 1$ is a small perturbation parameter and A is the amplitude of the pulsating sphere.

The desired moving boundary condition is such that at each moment the normal component of the velocity of the medium (in the present case only this component of velocity occurs) is equal to the normal component of the velocity of the surface of the source, for all points of the surface of the source [13].

$$\left.\frac{\partial \phi}{\partial r}\right|_{r=r_k=1+\varepsilon\cos(\Omega t+\varphi)} = \frac{dr_k(t)}{dt}. \tag{8}$$

3. Description of the potential of the acoustic velocity

In the problems which involve parametric perturbations the quantities to be expanded can depend on one or more independent variables, apart from the perturbation parameter. Construction of the asymptotic representation of the function $f(x;\varepsilon)$, where x is a scalar or vector variable, independent from the parameter ε in the terms of the asymptotic sequence $\delta_m(\varepsilon)$, gives [14]

$$f(x;\varepsilon) \sim \sum_{m=0}^{\infty} a_m(x)\,\delta_m(\varepsilon), \quad \varepsilon \to 0, \tag{9}$$

where $a_m(x)$ are terms which depend only on x.

This expansion will be called asymptotic if

$$f(x;\varepsilon) = \sum_{m=0}^{N-1} a_m(x)\,\delta_m(\varepsilon) + R_N(x;\varepsilon); \tag{10}$$

$$R_N(x;\varepsilon) = O[\delta_N(\varepsilon)], \quad \lim_{\delta_N(\varepsilon)\to 0} \{O[\delta_N(\varepsilon)]/\delta_N(\varepsilon)\} = 0 \tag{11}$$

for all the considered values of x.

In the contrary case it is said that the expansion is singular.

For small but finite pulsation amplitude of the source the potential of the acoustic field generated (and also such quantities as u, p, ϱ) is a quantity of low value and can therefore be expanded in a power series with respect to the small parameter ε,

$$\phi(r,t;\varepsilon) = \varepsilon\phi_1(r,t) + \varepsilon^2\phi_2(r,t) + \dots \tag{12}$$

and similarly

$$u(r,t;\varepsilon) = \varepsilon u_1(r,t) + \varepsilon^2 u_2(r,t) + \ldots; \tag{13}$$

$$p(r,t;\varepsilon) = \varepsilon p_1(r,t) + \varepsilon^2 p_2(r,t) + \ldots. \tag{14}$$

Substitution of the relation of ϕ in the form of expansion (12) into equation (8) with the right side expanded into a Taylor series and comparison of the terms with the same powers of ε give linear equations for $\phi_1(r,t)$ and $\phi_2(r,t)$ and the boundary conditions

order ε:

$$\frac{\partial^2\phi_1}{\partial r^2} - \frac{\partial^2\phi_1}{\partial t^2} + \frac{2}{r}\frac{\partial\phi_1}{\partial r} = 0; \tag{15}$$

$$\left.\frac{\partial\phi_1}{\partial r}\right|_{r=1} = -\Omega\sin(\Omega t+\varphi); \tag{16}$$

order ε^2:

$$\frac{\partial^2\phi_2}{\partial r^2} - \frac{\partial^2\phi_2}{\partial t^2} + \frac{2}{r}\frac{\partial\phi_2}{\partial r} = 2\,\frac{\partial\phi_1}{\partial r}\frac{\partial^2\phi_1}{\partial r\,\partial t} + (\gamma-1)\frac{\partial\phi_1}{\partial t}\frac{\partial^2\phi_1}{\partial r^2} + $$
$$+\frac{2}{r}(\gamma-1)\frac{\partial\phi_1}{\partial r}\frac{\partial\phi_1}{\partial t}; \tag{17}$$

$$\left.\frac{\partial\phi_2}{\partial r}\right|_{r=1} = -\cos(\Omega t+\varphi)\left.\frac{\partial^2\phi_1}{\partial r^2}\right|_{r=1}. \tag{18}$$

Equations (15) and (17) are linear equations. The first is a linearized equation of the velocity potential for the spherical wave, whereas the second is a linear, heterogeneous equation which describes a nonlinear correction for the potential function.

Solution of these equations, with relevant boundary conditions, gave the sought expansion of the velocity potential according to the powers of the perturbation parameter.

$$\phi(r,t;\varepsilon) = -\varepsilon\Omega(\Omega^2+1)^{-1/2}r^{-1}\cos\{\Omega[t-(r-1)]+\varphi+\varphi_0\}+$$
$$+\varepsilon^2[\Omega(\Omega^2+1)^{-1/2}[(\Omega^2-2)\cos\varphi_0+2\Omega\sin\varphi_0]r^{-1}\cos^2\{\Omega[t-(r-1)]+\varphi\}+$$
$$+\Omega[(\Omega^2+1)(4\Omega^2+1)^{-1/2}\left[\Omega(\Omega^2-1)\cos\varphi_0+\left(\frac{3}{2}\,\Omega^2+1\right)\sin\varphi_0\right]r^{-1}\sin 2\{\Omega[t-$$
$$-(r-1)]+\varphi+\nu\}+$$
$$+\frac{1}{2}\,\Omega^3(\Omega^2+1)^{-1}(1+4\Omega^2)^{-1/2}r^{-1}\sin 2\{\Omega[t-(r-1)]+\varphi+\varphi_0+\nu\}+$$
$$-\frac{1}{2}\,\Omega^3(\Omega^2+1)^{-1}r^{-2}\sin 2\{\Omega[t-(r-1)+\varphi+\varphi_0\}+$$

$$+\frac{1}{2}\,\Omega^3(\Omega^2+1)^{-1}r^{-1}\sin 2\{\Omega[t-(r-1)]+\varphi+\varphi_0\}+$$

$$+\frac{1}{8}\,\Omega^4(\Omega^2+1)^{-1}(\gamma+1)r^{-1}\ln r\cos 2\{\Omega[t-(r-1)]+\varphi+\varphi_0\}+$$

$$-\frac{1}{8}\,\Omega^4(\Omega^2+1)^{-1}(\gamma+1)r^{-1}\sin 2\{\Omega[t-(r-1)]+\varphi+\varphi_0\}\cos 4\Omega r[Si(4\Omega r)-Si(4\Omega)]+$$

$$+\frac{1}{8}\Omega^4(\Omega^2+1)^{-1}(\gamma+1)r^{-1}\sin 2\{\Omega[t-(r-1)+\varphi+\varphi_0\}\sin 4\Omega r[Ci(4\Omega r)-Ci(4\Omega)]-$$

$$-\frac{1}{8}\,\Omega^4(\Omega^2+1)^{-1}(\gamma+1)r^{-1}\cos 2\{\Omega[t-(r-1)]+\varphi+\varphi_0\}\sin 4\Omega r[Si(4\Omega r)-Si(4\Omega)]-$$

$$-\frac{1}{8}\,\Omega^4(\Omega^2+1)^{-1}(\gamma+1)r^{-1}\cos 2\{\Omega[t-(r-1)]+\varphi+\varphi_0\}\cos 4\Omega r[Ci(4\Omega r)-$$

$$-Ci(4\Omega)]+\ldots, \tag{19}$$

where

$$\varphi_0=\tan^{-1}\Omega^{-1},\qquad \nu=-\frac{1}{2}\tan^{-1}2\Omega.$$

The expansion of the dimensionless function of the potential of the acoustic velocity as defined by relation (19) is not asymptotic. The singularity of this expansion results from the presence in the second-order terms of the secular term (the sixth term in the second-order terms of the expansion) which causes the second-order terms of the expansion to take values of the same order or greater than those of the first-order terms with large distances from the source. This term occurs in the solution of equation (17). In turn, the nonlinear effects which result from the moving boundary condition are of the second order of magnitude. Therefore, in order to obtain the asymptotic expansion of the first order in the case of a linearized boundary condition, it is enough to consider the problem in the form

$$\left.\frac{\partial\phi}{\partial r}\right|_{r=1}=-\varepsilon\Omega\sin(\Omega t+\varphi). \tag{20}$$

4. Description of the field of the acoustic pressure

Nayfeh and Kluwick [15] and Ginsberg [7] have proved that in seeking the correct expressions of such physical quantities as the acoustic velocity or pressure it is necessary to eliminate the secular term from the expressionsy of these quantities and not from the expression of the acoustic velocity potential.

The acoustic velocity, defined as $u = \partial\phi/\partial r$ and derived from the expansion of the velocity potential, is given by the relation

$$u(r,t;\varepsilon) = \varepsilon[\Omega(\Omega^2+1)^{-1/2}r^{-2}\cos\{\Omega[t-(r-1)]+\varphi+\varphi_0\} - \Omega^2(\Omega^2+1)^{-1/2}\times$$
$$\times r^{-1}\sin\{\Omega[t-(r-1)]+\varphi+\varphi_0\}]+$$
$$+\varepsilon^2\frac{1}{4}\Omega^5(\Omega^2+1)^{-1}(\gamma+1)r^{-1}lnr\sin 2\{\Omega[t-(r-1)]+\varphi+\varphi_0\}+NST+\dots, \quad (21)$$

where $\varphi_0 = \tan^{-1}\Omega^{-1}$ and NST are the nonsecular terms of the expansion.

In order to eliminate the secular term from expansion (21), according to the renormalisation method chosen, new independent variables of time and distance, τ and η, were introduced. GINSBERG [8] and NAYFEH [15] have shown that in the problems which involve travelling waves it is sufficient to transform only one variable. It is convenient to assume the following form of the transformation of r and t,

$$r = \eta+\varepsilon r_1(\eta,\tau)+\dots, \quad (22)$$

$$t = \tau. \quad (23)$$

After insertion of the expressions of r and t given by relations (22) and (23) into equation (21) and expansion of the right side for small ε, with a definite value of the new coordinate η the velocity $u(\eta,\tau;\varepsilon)$ is given by expression (24):

$$u(\eta,\tau;\varepsilon) = \varepsilon\big[\Omega(\Omega^2+1)^{-1/2}\eta^{-2}\cos\{\Omega[\tau-(\eta-1)]+\varphi+\varphi_0\} - \Omega^2(\Omega^2+1)^{-1/2}\times$$
$$\times\eta^{-1}\sin\{\Omega[\tau-(\eta-1)]+\varphi+\varphi_0\}\big]+$$
$$+\varepsilon^2\big[2\Omega^2(\Omega^2+1)^{-1/2}\eta^{-2}\sin\{\Omega[\tau-(\eta-1)]+\varphi+\varphi_0\}-2\Omega(\Omega^2+1)^{-1/2}\times$$
$$\times\eta^{-3}\cos\{\Omega[\tau-(\eta-1)]+\varphi+\varphi_0\}+$$
$$+\Omega^3(\Omega^2+1)^{-1/2}\eta^{-1}\cos\{\Omega[\tau-(\eta-1)]+\varphi+\varphi_0\}\big]r_1(\eta,\tau)+$$
$$+\varepsilon^2\frac{1}{4}\Omega^5(\Omega^2+1)^{-1}(\gamma+1)\eta^{-1}\ln\eta\sin 2\{\Omega[\tau-(\eta-1)]+\varphi+\varphi_0\}+NST+\dots \quad (24)$$

It would seem apparently that in order to eliminate the secular term from expansion (24) the function $r_1(\eta,\tau)$ should be chosen (according to the renormalisation method) so that the secular term and the terms containing the function $r_1(\eta,\tau)$ would zero one another. It can readily be shown that in such a case $r_1(\eta,\tau)\to\infty$ for given values of the variable η, which would lead to infinitely great deformation of the profile of the wave. It follows from the analyses carried out in the case of a plane travelling wave [6, 15] and a cylindrical wave [8, 9] that the function of the deformation of the profile is the product of the funciton of distance from the source and of the acoustic velocity $u_1(\eta,\tau)$

$$r_1(\eta,\tau) = h_1(\eta)u_1(\eta,\tau), \quad (25)$$

where $h_1(\eta)$ is a function which depends only on η.

It was assumed that in order to obtain $r_1(\eta, \tau)$ in the same form as occurs in relation (25), it is necessary to take also into consideration, apart from the secular term, other terms in the second-order terms of expansion (24), although they do not cause the singularity

$$r_1(\eta,\tau)[\Omega^3(\Omega^2+1)^{-1/2}\eta^{-1}\cos\{\Omega[\tau-(\eta-1)]+\varphi+\varphi_0\}+2\Omega^2(\Omega^2+1)^{-1/2}\times$$
$$\times\eta^{-2}\sin\{\Omega[\tau-(\eta-1)]+\varphi+\varphi_0\}-2\Omega(\Omega^2+1)^{-1/2}\eta^{-3}\cos\{\Omega[\tau-(\eta-1)+\varphi+\varphi_0)\}]=$$
$$=-\frac{1}{4}\Omega^5(\Omega^2+1)^{-1}(\gamma+1)\eta^{-1}\ln\eta\sin 2\{\Omega[\tau-(\eta-1)]+\varphi+\varphi_0\}+$$
$$+\sum_n f_n(\eta)g_n\{\Omega[\tau-(\eta-1)]+\varphi+\varphi_0\}, \qquad (26)$$

where $f_n(\eta)g_n\{\Omega[\tau-(\eta-1)]+\varphi_0+\varphi\}$ are asymptotic terms in the second-order terms of the expansion.

On the basis of relations (24)-(26), the function $h_1(\eta)$,

$$h_1(\eta)=\frac{1}{2}(\gamma+1)\eta\ln\eta, \qquad (27)$$

and the following elements of the second term of expansion (24), which should be taken into account together with the secular term,

$$f_1g_1=\frac{1}{2}\Omega^4(\Omega^2+1)^{-1}(\gamma+1)\eta^{-2}\ln\eta\cos^2\{\Omega[\tau-(\eta-1)]+\varphi+\varphi_0\}; \qquad (28)$$

$$f_2g_2=-\Omega^4(\Omega^2+1)^{-1}(\gamma+1)\eta^{-2}\ln\eta\sin^2\{\Omega[\tau-(\eta-1)]+\varphi+\varphi_0\}; \qquad (29)$$

$$f_3g_3=-\Omega^2(\Omega^2+1)^{-1}(\gamma+1)\eta^{-4}\ln\eta\cos^2\{\Omega[\tau-(\eta-1)]+\varphi+\varphi_0\}; \qquad (30)$$

$$f_4g_4=\Omega^3(\Omega^2+1)^{-1}(\gamma+1)\eta^{-3}\ln\eta\sin 2\{\Omega[\tau-(\eta-1)]+\varphi+\varphi_0\}. \qquad (31)$$

were obtained.

From equations (22)-(27), the following asymptotic expansion of the first order of the acoustic velocity $u(r, t)$ was obtained in parametric form, valid for both the near and far field,

$$u(\eta,\tau)=\varepsilon\Omega(\Omega^2+1)^{-1/2}\eta^{-2}\cos\{\Omega[\tau-(\eta-1)]+\varphi+\varphi_0\}-$$
$$-\varepsilon\Omega^2(\Omega^2+1)^{-1/2}\eta^{-1}\sin\{\Omega[\tau-(\eta-1)]+\varphi+\varphi_0\}+O[\varepsilon^2\Omega^4(\Omega^2+1)^{-1}]; \qquad (32)$$

$$r=\eta+\frac{1}{2}(\gamma+1)\eta\ln\eta u(\eta,\tau)+O[\varepsilon^2\Omega^4(\Omega^2+1)^{-1}]; \qquad (33)$$

$$t=\tau. \qquad (34)$$

5. Description of the field of the acoustic pressure

It is convenient to represent the dimensionless equation of the acoustic pressure (6) in the following form,

$$1+p = \left\{1-(\gamma-1)\left[\frac{\partial\phi}{\partial t}+\frac{1}{2}\left(\frac{\partial\phi}{\partial r}\right)^2\right]\right\}^{\gamma/\gamma-1}. \quad (35)$$

Expansion of the binomial on the right side of this equation and insertion of the expression of $\phi(r, t; \varepsilon)$, given by relation (12), gave the following form of the equation of the acoustic pressure

$$p(r, t; \varepsilon) = -\gamma\left[\varepsilon\frac{\partial\phi_1}{\partial t}+\varepsilon^2\frac{\partial\phi_2}{\partial t}+\frac{1}{2}\varepsilon^2\left(\frac{\partial\phi_1}{\partial r}\right)^2-\frac{1}{2}\varepsilon^2\left(\frac{\partial\phi_1}{\partial t}\right)^2+\ldots\right]. \quad (36)$$

In a way analogous to the expression defining the acoustic pressure, an asymptotic expansion of the first order of the acoustic pressure was derived in parametric form from equations (36) and (19),

$$p(\eta, \tau) = -\varepsilon\Omega^2(\Omega^2+1)^{-1/2}\gamma\eta^{-1}\sin\{\Omega[\tau-(\eta-1)]+\varphi+\varphi_0\}+O[\varepsilon^2\Omega^4(\Omega^2+1)^{-1}]; \quad (37)$$

$$r = \eta+\frac{1}{2}\frac{\gamma+1}{\gamma}\eta\ln\eta\, p(\eta, \tau)+O[\varepsilon^2\Omega^4(\Omega^2+1)^{-1}]; \quad (38)$$

$$t = \tau. \quad (39)$$

This description is valid for the near and far field up to to the place where the discontinuity (r^*, t^*) occurs in the profile of the wave. From equations (32)-(34) and (37)-(39), it is possible to determine the relation between the original coordinate and the deformed one. As an example, this dependence is shown graphically in Fig. 1.

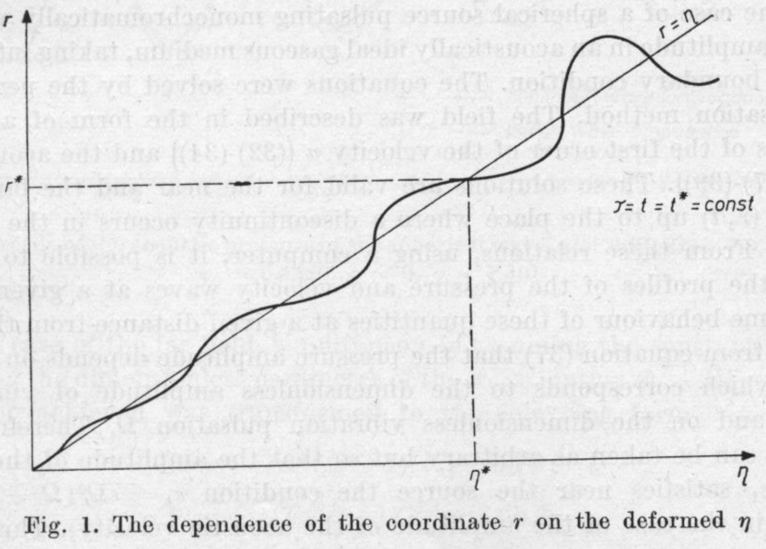

Fig. 1. The dependence of the coordinate r on the deformed η

When one value of the variable r corresponds to one value of the variable η, this then corresponds to one value of the profile of the wave (for $t = \text{const.}$). It follows from Fig. 1 that, from some value of the variable η, the transformation becomes singular, since the same value of the variable r corresponds to several values of the variable η. The profile of the wave becomes a multivalent function of the variable r, which corresponds to the formation of a shock wave. Determination of the shortest distance at which the discontinuity of the profile of the wave occurs can be reduced to the determination of the lowest value of r^* and of the time t^*, according to relation (40).

$$\left.\frac{\partial r}{\partial \eta}\right|_{r^*,t^*} = 0. \tag{40}$$

The use of relations (40) and (38), under the assumption that $\Omega r \gg 1$ (far field) gave

$$r^* = \eta^* \sim \exp \frac{2(\Omega^2+1)^{-1/2}}{\varepsilon(\gamma+1)\Omega^3}; \tag{41}$$

$$\cos\{\Omega[t^* - (r^*-1)] + \varphi + \varphi_0\} = -1. \tag{42}$$

For another definite observation time $t \neq t^*$ the discontinuity forms at a farther distance from the source. Expressions (41) and (42), as defined for the pressure wave for the far field, are also valid for the velocity wave.

6. Conclusions

This paper presented an approximate solution of the hydrodynamic equations in the case of a spherical source pulsating monochromatically with finite vibration amplitude in an acoustically ideal gaseous medium, taking into account a moving boundary condition. The equations were solved by the perturbation renormalisation method. The field was described in the form of asymptotic expansions of the first order of the velocity u ((32)-(34)) and the acoustic pressure p ((37)-(39)). These solutions are valid for the near and the far field for all points (r, t) up to the place where a discontinuity occurs in the profile of the wave. From these relations, using a computer, it is possible to calculate and plot the profiles of the pressure and velocity waves at a given moment and the time behaviour of these quantities at a given distance from the source. It follows from equation (37) that the pressure amplitude depends on the parameter ε which corresponds to the dimensionless amplitude of vibration of a sphere and on the dimensionless vibration pulsation Ω. Therefore, these quantities can be taken as arbitrary but so that the amplitude of the acoustic pressure ε_1 satisfies near the source the condition $\varepsilon_1 = \varepsilon\Omega^2(\Omega^2+1)^{-1/2} \ll 1$ (similarly in the case of the amplitude of the acoustic velocity). This solution is valid for the amplitudes $\gamma\varepsilon_1 \leqslant 0.1$, which corresponds to a level of the acoustic pressure $\leqslant 174$ dB in air in normal conditions. As an example, Figs. 2 and 3 show profiles of the acoustic velocity and acoustic pressure in air for $\varepsilon = 0.01$ and $\Omega = 7$ and the observation time $t = t^*$.

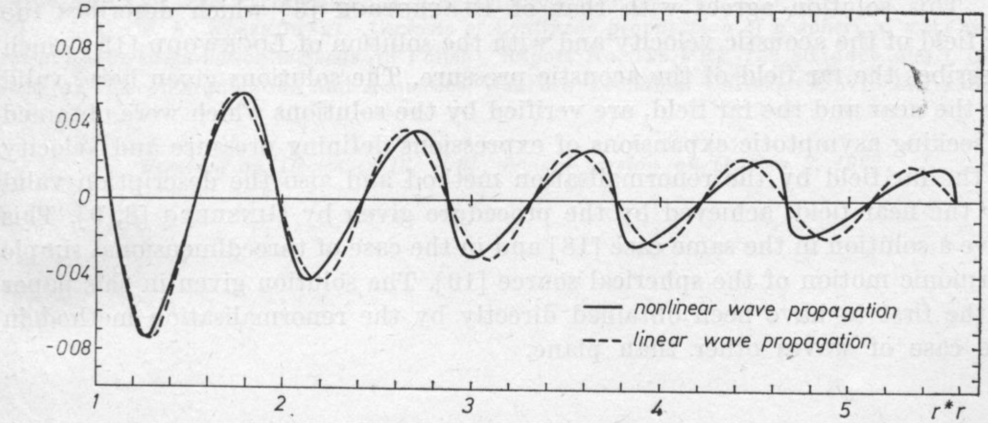

Fig. 2. The profile of the acoustic velocity of the spherical wave, $\varepsilon = 0.01$, $\Omega = 7.0$, $t = 4.996$ $\varphi_0 = 0.1420$, $\varphi = 0$, $\gamma = 1.401$

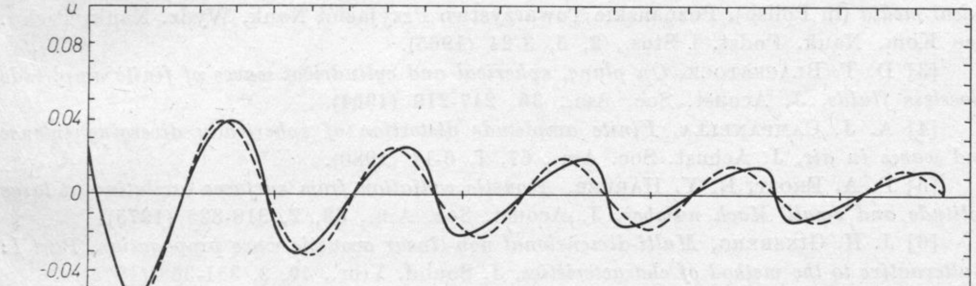

Fig. 3. The profile of the acoustic pressure of the spherical wave, $\varepsilon = 0.01$, $\Omega = 7.0$, $t = 4.996$, $\varphi_0 = 0.1420$, $\varphi = 0$, $\gamma = 1.401$

In the case of the far field, simultaneously assuming the constant $\varphi = -\varphi_0$ and defining the dimensionless parameter of the wave motion $\Phi = \Omega[\tau-(\eta-1)]$, the solution achieved was transformed to the following form

$$u = -\varepsilon\Omega^2(\Omega^2+1)^{-1/2}r^{-1}\sin\Phi; \tag{43}$$

$$p = -\varepsilon\Omega^2(\Omega^2+1)^{-1/2}\gamma r^{-1}\sin\Phi; \tag{44}$$

$$\Phi = \Omega[t-(r-1)] - \sigma \sin\Phi; \tag{45}$$

$$\sigma = \varepsilon \frac{1}{2} \Omega^3 (\Omega^2+1)^{-1/2} (\gamma+1) \ln r. \tag{46}$$

This solution agrees with that of BLACKSTOCK [3] which describes the far field of the acoustic velocity and with the solution of LOCKWOOD [12] which describes the far field of the acoustic pressure. The solutions given here, valid for the near and the far field, are verified by the solutions which were obtained in seeking asymptotic expansions of expressions defining pressure and velocity in the far field by the renormalisation method and also the description valid for the near field, achieved by the procedure given by GINSBERG [8, 9]. This gave a solution in the same case [18] and in the case of threedimensional simple harmonic motion of the spherical source [10]. The solution given in this paper is the first to have been obtained directly by the renormalisation method in the case of waves other than plane.

References

[1] V. A. AKULICHEV, Y. Y. BOGUSLAVSKI, A. Y. YOFFE, K. A. NAGOLNYKH, *Izlucheniye sfericheskikh voln konechnoy amplitudy*, Ak. Zhurn., **13**, 3, 321-328 (1967).

[2] S. AUGUSTYNIAK, *Propagation of cylindrical and spherical waves of finite amplitude in ideal media* (in Polish), Poznańskie Towarzystwo Przyjaciół Nauk, Wydz. Nauk. Techn. Prace Kom. Nauk. Podst. i Stos., **2**, 5, 3-24 (1965).

[3] D. T. BLACKSTOCK, *On plane, spherical and cylindrical waves of finite amplitude in lossless fluids*, J. Acoust. Soc. Am., **36**, 217-219 (1964).

[4] A. J. CAMPANELLA, *Finite amplitude distortion of spherically diverging intense sound waves in air*, J. Acoust. Soc. Am., **67**, 1, 6-14 (1980).

[5] P. A. FROST, E. Y. HARPER, *Acoustic radiation from surfaces oscilating at large amplitude and small Mach number*, J. Acoust. Soc. Am., **58**, 2, 318-325 (1975).

[6] J. H. GINSBERG, *Multi-dimensional non-linear acoustic wave propagation. Part I: An alternative to the method of characteristics*, J. Sound. Vibr., **40**, 3, 351-358 (1975).

[7] J. H. GINSBERG, *A re-examination of the non-linear interaction between an acoustic fluid and flat plate undergoing harmonic excitation*, J. Sound. Vibr., **60**, 3, 449-458 (1978).

[8] J. H. GINSBERG, *Propagation of nonlinear acoustic waves induced by a vibrating cylinder. I. The two-dimensional case*, J. Acoust. Soc. Am., **64**, 6, 1671-1678 (1978).

[9] J. H. GINSBERG, *Propagation of nonlinear acoustic waves induced by a vibrating cylinder. II. The three-dimensional case*, J. Acoust. Soc. Am., **64**, 6, 1679-1687 (1978).

[10] S. G. KELLY, A. H. NAYFEH, *Non-linear propagation of directional spherical waves*, J. Sound. Vibr., **71**, 1, 25-37 (1980).

[11] H. LAMB, *Hydrodynamics*, 6-th ed., Camb. Univ. Press, New York 1945.

[12] J. C. LOCKWOOD, *Two problems in high intensity sound*, Univ. Texas Austin, Appl. Res. Lab., ARL-TR-71-26 (1971).

[13] R. E. MEYER, *Introduction to mathematical fluid dynamics*, J. Wiley and Sons, New York 1971.

[14] A. H. NAYFEH, *Perturbation methods*, J. Wiley and Sons, New York 1973.

[15] A. H. NAYFEH, A. KLUWICK, *A comparison of three perturbation methods for nonlinear hyperbolic waves*, J. Sound. Vibr., **48**, 2, 293-299 (1976).

[16] A. H. NAYFEH, S. G. KELLY, *Nonlinear propagation of waves induced by an infinite vibrating cylinder*, The 8th International Symposium on Nonlinear Acoustics, Paris 1978.

[17] E. V. ROMANENKO, *Eksperimentalnoe issledovaniye raspostraneniya sfericheskikh voln konechnoy amplitudy*, Ak. Zhurn., **5**, 1, 101-105 (1959).

[18] Cz. A. ROSZKOWSKI, *Nonlinear propagation of spherical acoustic waves* (in Polish), Proc. XXVII OSA, Warszawa – Puławy 1980.

[19] Cz. A. ROSZKOWSKI, *Analysis of nonlinear propagation of a spherical wave in an ideal gas by perturbation methods* (in Polish), Report No. 128/PRE-128/80 (doct. diss.), Institute of Telecommunication and Acoustics, Wrocław Technical University, Wrocław 1980.

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ULTRASONIC INVESTIGATIONS OF THE RELAXATION PROCESSES RELATED TO THE PRESENCE OF THE GROUP $-C\langle{}^{O}_{NH}$ IN MOLECULES

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Measurements of the velocity and the absorption coefficient of ultrasound were taken in monosubstituted amides as a function of temperature and frequency. It was found that a single relaxation process related to rotation of molecules around the C—N bond occurs over the temperature and frequency range investigated. Termodynamic parameters which characterize the process observed were determined.

1. Introduction

Measurements of the velocity and absorption of ultrasound as a function of temperature and frequency are a valuable tool in the investigations of the structure of the substance and permit the determination of the parameters characteristic of the processes which occur in the given medium.

The aim of the present investigation is to determine the relaxation parameters characteristic of the process of hindered rotation in monosubstituted amides. The investigations of dipole moments [1], dielectric constants [2], infrared and Raman spectra [3] and NMR investigations [4-6] showed the existence of two conformation states which occur as a result of hindered rotation around the C—N bond.

The present paper is an attempt to explain the processes of ultrasonic relaxation in this group of compounds on the basis of acoustic investigations of amides with different length of hydrocarbon chains which occur within a molecule.

2. Experimental part

The absorption coefficient α/f^2 was determined on the basis of measurements taken with a US-4 set and a high-frequency CSU-250 system. This equipment permitted measurements of the absorption coefficient α/f^2 over the frequency range 10-74 MHz. The velocity was measured using a pulse-phase UI-14 interferometer.

The measurement method was described in previous papers [7, 8]. N-propylformamide, N-propylacetamide, N-propylpropionamide and N-butylpropionamide were used in the investigations. All these reagents were synthesized by the authors.

3. Results and conclusions

As an example, Fig. 1 shows the dependence of α/f^2 on frequency (at 50°C) for N-propylacetamide, N-propylpropionamide and N-butylpropionamide.

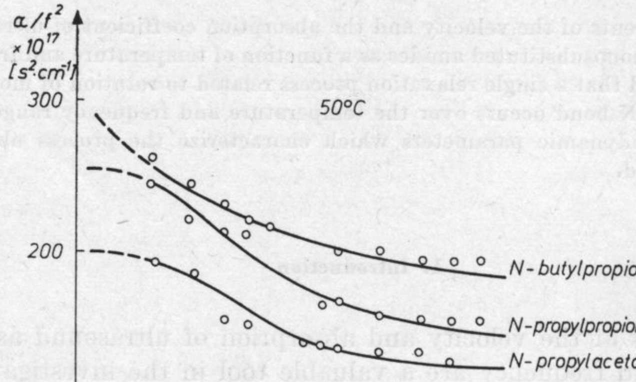


Fig. 1. The dependence of α/f^2 on $\log f$ (temp. 50°C) for N-butylpropionamide, N-propylpropionamide and N-propylacetamide

In the compounds under investigation, except N-propylformamide, there occurs a relaxation process which is characterized by a single relaxation time. In the case of N-propylformamide no dispersion of the absorption coefficient was observed over the whole frequency and temperature range. For the other substances the dependence of the absorption coefficient α/f^2 on frequency is given by the relaxation equation

$$a/f^2 = \frac{A}{1+(f/f_c)^2} + B, \tag{1}$$

where A is the relaxation parameter depending on the equilibrium characteristic and B represents the contribution of the classical absorption and other relaxation processes with characteristic frequency much greater than f_c.

On the basis of the expression

$$\mu = (a/f^2 - B) f c_0, \tag{2}$$

the values of the coefficients a/f^2 served to determine the excess absorption coefficients μ per wavelength. c_0 is the sound velocity for low frequencies at which no dispersion is observed. Fig. 2 shows the dependence of μ on frequency at a temperature of 50°C. Tables 1-3 give the values of the velocity c_0, the coefficients A, B and μ and the relaxation frequency f_c.

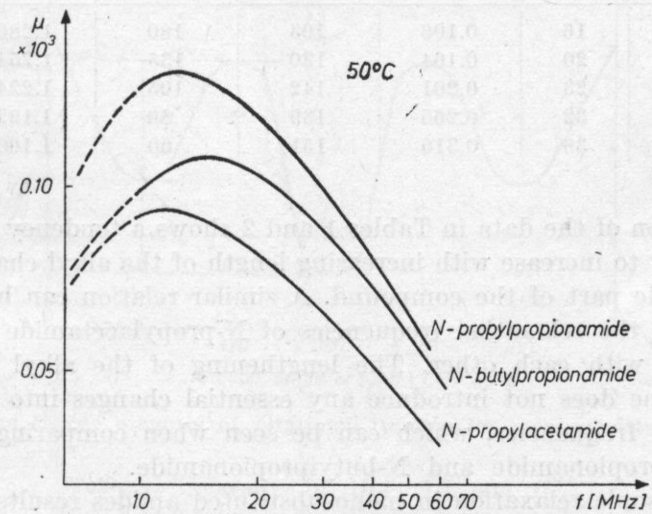


Fig. 2. The dependence of the excess absorption coefficient μ on log f (temp. 50°C) for N-butylpropionamide, N-propylpropionamide and N-propylacetamide

Table 1. The values of the relaxation parameters for N-propylacetamide

T [°C]	f_c [MHz]	$\mu_{max} \cdot 10^3$	$A \cdot 10^{17}$ [cm$^2 \cdot$s^{-1}]	$B \cdot 10^{17}$ [cm$^2 \cdot$s^{-1}]	$c_0 \cdot 10^5$ [cm$\cdot$s^{-1}]
40	11	0.66	87	173	1.388
50	12	0.92	117	122	1.305
60	18	0.80	69	110	1.275
70	22	0.84	62	84	1.240
80	30	0.89	49	72	1.205

Table 2. The values of the relaxation parameters for N-propylpropionamide

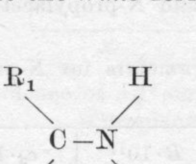
T [°C]	f_c [MHz]	$\mu_{max}\cdot 10^3$	$A\cdot 10^{17}$ [$cm^2\cdot s^{-1}$]	$B\cdot 10^{17}$ [$cm^2\cdot s^{-1}$]	$c_0\cdot 10^5$ [$cm\cdot s^{-1}$]
40	10	1.129	170	184	1.330
50	13	1.283	153	148	1.290
60	21	1.402	106	126	1.260
70	25	1.345	88	95	1.220
80	39	1.301	56	86	1.185
90	42	1.601	67	73	1.150

Table 3. The values of the relaxation parameters for N-butylpropionamide

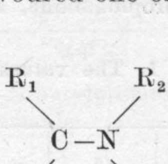
T [°C]	f_c [MHz]	$\mu_{max}\cdot 10^3$	$A\cdot 10^{17}$ [$cm^2\cdot s^{-1}$]	$B\cdot 10^{17}$ [$cm^2\cdot s^{-1}$]	$c_0\cdot 10^5$ [$cm\cdot s^{-1}$]
50	16	0.106	103	180	1.286
60	20	0.164	130	135	1.254
70	23	0.201	142	105	1.224
80	32	0.265	139	80	1.192
90	36	0.316	151	60	1.160

Comparison of the data in Tables 1 and 2 shows a tendency of the relaxation frequency to increase with increasing length of the alkyl chain originating from the amide part of the compound. A similar relation can be observed in comparison of the relaxation frequencies of N-propylacetamide and N-butylpropionamide with each other. The lengthening of the alkyl chain coming from the amine does not introduce any essential changes into the values of the relaxation frequencies, which can be seen when comparing these values for N-propylpropionamide and N-butylpropionamide.

The ultrasonic relaxation in monosubstituted amides results from limited rotation around the C$-$N bond. The results of the hitherto investigations have shown that the *trans* form (I) is the more favoured one of the two possibilities.



trans (I)



cis (II)

Changes in the enthalpy ΔH for this type of conformation transformation can be illustrated with a diagram.

The value of the energy barrier ΔH_2^+ which a molecule has to overcome in its transition from a state of higher energy to the primary state and the values of the enthalpy difference ΔH_0 were determined for all the compounds investigated. Table 4 gives the values of ΔH_2^+, ΔH_0 and $\Delta H_1^+ = \Delta H_2^+ + \Delta H_0$.

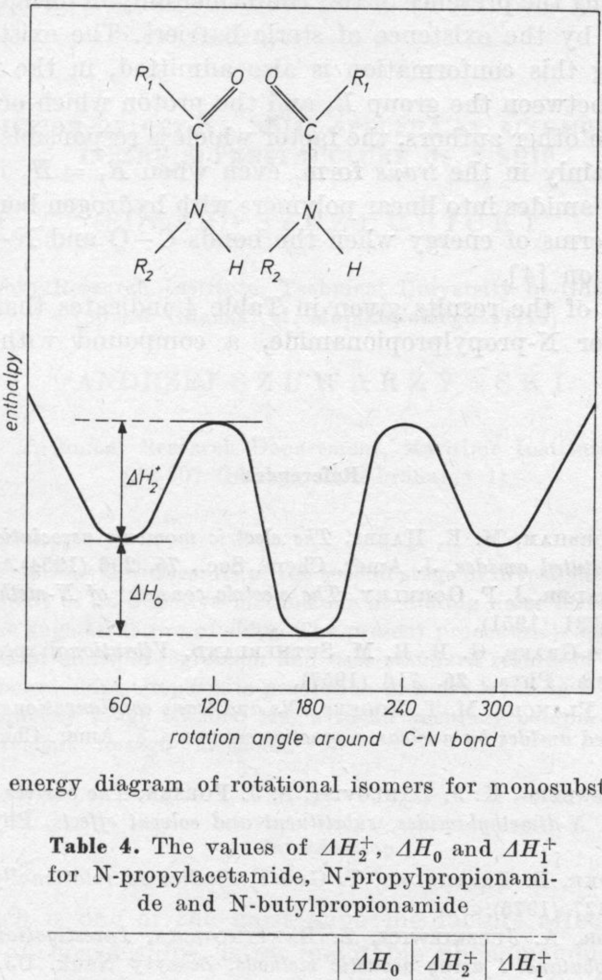


Fig. 3. The energy diagram of rotational isomers for monosubstitud amides

Table 4. The values of ΔH_2^+, ΔH_0 and ΔH_1^+ for N-propylacetamide, N-propylpropionamide and N-butylpropionamide

	ΔH_0	ΔH_2^+	ΔH_1^+
N-propylacetamide	3.3	5.1	8.4
N-propylpropionamide	3.8	6.2	10.0
N-butylpropionamide	8.2	4.2	12.4

ΔH_2^+ takes its lowest value for N-butylpropionamide. This is caused by the relatively large volumes of alkyl groups, which causes an increase in steric interaction between the alkyl substituent at nitrogen and the alkyl substituent which comes from the amide. The durability of the *cis* configuration decreases, and accordingly there is an decrease in the value of the energy ΔH_2^+. The result of this is the parallel increase in the value of the enthalpy difference ΔH_0 between forms I and II. The enthalpy difference ΔH_0 increases as the sums R_1 and R_2 increase, which indicates that the *trans* form stabilizes as the alkyl groups enlarge.

The NMR investigations [4] exclude the existence of the *cis* configuration in amides, allowing the presence of this conformer only in formamides ($R_1 = H$). This is justified by the existence of steric barriers. The existence of another factor stabilizing this conformation is also admitted, in the form of a weak hydrogen bond between the group R_1 and the proton which occurs at nitrogen [9]. According to other authors, the factor which is responsible for maintaining the molecule mainly in the *trans* form, even when $R_1 = H$, is the possibility of association of amides into linear polymers with hydrogen bonds. This process is favoured in terms of energy when the bonds C–O and N–H occur in the *trans* configuration [4].

Comparison of the results given in Table 4 indicates that ΔH_2^{+} takes its highest value for N-propylpropionamide, a compound with a symmetrical structure.

References

[1] J. E. Worsham, M. E. Habbs, *The electric moments association and structure of some N-mono-substituted amides*, J. Amer. Chem. Soc., **76**, 206 (1954).

[2] G. R. Leader, J. F. Gormley, *The electric constant of N-methylamides*, J. Amer. Chem. Soc., **73**, 5731 (1951).

[3] D. E. De Graff, G. B. B. M. Sutherland, *Vibrational spectrum of N-methylformamide*, J. Chem. Phys., **26**, 716 (1957).

[4] L. A. La Planche, M. T. Rogers, *Cis and trans configurations of the peptide bond in N-monosubstituted amides by nuclear magnetic resonance*, J. Amer. Chem. Soc., **86**, 1, 337 (1964).

[5] T. Drakenberg, K. J. Dahlgvist, S. J. Forsen, *The barrier to internal rotation in amides. IV. N. N-dimethylamides, substituent and solvent effects*, Phys. Chem., **76**, 15, 2178 (1972).

[6] W. Doepke, B. Bartholme, T. Gross, *Studies on rotationally hindered systems*, Z. Chem., **16**, 8, 327 (1976).

[7] B. Zapiór, A. Juszkiewicz, Z. Bartynowska, *Investigation of the relaxation processes in 2-nitrobutanol-1 using acoustic methods*, Zeszyty Nauk. UJ, Prac. Chem., **21**, 191 (1976).

[8] A. Juszkiewicz, *System for measurements of the ultrasonic absorption coefficient at 10-60 MHz over the temperature range −60°C − +200°C* (in Polish), Zeszyty Nauk. UJ, Prac. Chem., **21**, 257 (1976).

[9] R. A. Nyquist, *The structural configuration of some a-substituted secondary acetamides in dilute CCl_4 solution*, Spectrochim. Acta, **19**, 509 (1963).

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PREDICTION OF OCTAVE NOISE SPECTRA IN ACCOMODATIONS IN THE SUPERSTRUCTURE OF A SHIP

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This paper presents the results of the second stage of investigations aimed at the development of an effective method for predicting noise levels in accomodations in the superstructure of ships. The present procedure is based on a statistical model of multiple regression and uses standard results of noise measurements on board ships. It permits prediction of noise levels in 6 octave bands over the frequency range 63-2000 Hz, with an accuracy comparable to those of the equivalent foreign methods.

1. Introduction

Noise, which is one of the most annoying factors affecting the life and work of crews on board ships, is an inevitable phenomenon. The activity of man concentrates therefore on the development of methods for acceptable its harmful effect. Because of the necessity of providing the crew with correct conditions of rest after work, this applies particularly to accomodations in the superstructure. The elimination cf excess noise levels is secured by the consideration of the acoustical aspect at the early stage of the ship design.

In view of its simplicity, the method presented in [6] for predicting A-weighted noise levels can be used already at the preliminary stage of the ship design. Its efficient application can provide a valuable tool for the designer in arranging accomodations in the superstructure of a ship. The case may arise when for some reasons (functional requirements, rules of classification institutions etc.) some accomodations are located in the regions where the predicted A-weighted noise level exceeds the permissible value. In such cases it is necessary to undertake some technical means of noise control, which consist in

– reduction of the noise transmitted by the source to the structure (e.g. by the use of elastic mountings);

– decrease of the noise level in accomodations by insulation (e.g. floating floors).

The design of noise reduction means requires the knowledge of its frequency characteristic in a given accomodation. The results of octave bands noise prediction over the frequency range 31.5-8 kHz are commonly used for this purpose [5, 8]. Thus, the present paper gives an extension and modification of the method presented in paper [6], in order to permit such prediction in accomodations in the superstructures of ships.

2. Modification of the calculation algorithm

The model of multiple regression according to which the calculations were made was presented in [6, 7]. The assumptions of this model and the manner of estimation of its parameters remained unachanged, with one exception. It is now assumed that the values of the dependent variable observed are implementations of the $(n \times m)$ – dimensional random matrix Y (whereas previously Y was a n-element random vector), where n is the number of measurements and m is the number of octave bands at which noise levels were measured. On the basis of the results obtained, it was found that

1. The procedure of matrix inversion used in [6], which was based on the traditional Gauss algorithm, proved to be numerically unstable with a considerable increase in the column dimension of the matrix X. A numerically unstable algorithm does not assure that a solution can be achieved with error at the level of the inevitable error which results from the approximate representation of data in a computer. It most often involves large relative errors which prevent the use of the calculated results in practice.

2. For all the octave bands different systems of parameters have an essential effect on the noise level. In the course of calculations it is therefore necessary to change continuously the dimension of the matrix X in relation to the elimination of variables not essential for a given octave. The traditional Gauss algorithm prevents efficient performance of this operation.

3. The expansion of the set of explanatory variables brought the large probability of there being a linear or close to linear relationship among them. With a limited accuracy of computer calculations, this can prevent the satisfaction of the basic condition required by the standard procedures for the solution

of systems of linear equations (such being the Gauss method), i.e. a full column order of the matrix of the coefficients of a system of equations.

In view of the problems given above, the method for noise level prediction in octave bands was based on a standard model of multiple linear regression, with, however, some essential modifications to the algorithm given in [6]. The most important of these modifications is the use of the Gauss-Jordan algorithm to solve a system of normal equations.

The modified Gauss-Jordan algorithm was developed by EFFROYMSON and BEAL in the 1960s. In the present paper the authors have used the procedure given by BARTKOWIAK in [1, 2]. This algorithm offers the following possibilities, which are particularly convenient in regression analysis,

1. Direct estimation of the coefficient of multiple correlation, on the basis of which it is possible to conclude about the quality of the approximation of the variable Y by the calculated regression equation.

2. Successive introduction of dependent variables into the regression set.

3. When the diagonal element of the matrix of the coefficients of a system of normal equations which corresponds to the variable to be introduced into the regression set is close to zero, this signifies that this variable is an almost linear function of the variables which are already in the regression set. Such a variable is automatically neglected by the algorithm, since it does not bring any new information about the dependent variable Y, and its introduction into the set would prevent the matrix of the coefficients of the system of normal equations from being positively defined.

The programme for the calculations of octave spectrum prediction was written in Fortran 1900 and introduced into an Odra 1325 computer. In view of the required capacity of the operational memory, of the order of 30 k words, the calculations were made on an Odra 1305 computer. The range of the calculations included

– estimation of the coefficients of regression equations,
– estimation of the coefficient of multiple correlation,
– evaluation of the coefficients of the regression equation,
– verification of the results obtained.

3. Data for calculations — selection of explanatory variables

Table 1 shows chosen results of measurements taken in 400 accomodations in 25 ships of different types and size. The differences $L_{\max}$-$L_{\min}$ in particular octave bands take values of the order of 40 dB. It can be stated that the permissible values defined by the curve $N55$ are most often exceeded for octave bands with centre frequencies from 63 Hz to 2 kHz, sometimes even reaching values of more than 20 dB. The considerations below are concerned with the frequency range given above.

Table 1. Selected results of noise measurements in 400 accomodations

Centre frequency of octave band [Hz]	Noise level [dB]			Number of accomodations with permissible level L_{dop} exceeded
	L_{min}	L_{max}	L_{dop}	
31.5	66	99	93	8
63	62	93	79	80
125	51	94	70	95
250	45	80	63	103
500	40	76	58	116
1000	32	67	55	80
2000	26	65	52	44
4000	22	58	50	20
8000	18	54	49	4
A-weighted noise level	46	77	60	128

L_{dop} – permissible level

The calculations aimed at the generation of regression equations and their verification were made with the results of noise level measurements in 6 octave bands for 422 accomodations in 20 ships. Table 2 gives the basic data for these ships. The measurements were taken according to the standard ISO 2923 – *Acoustics Measurement of Noise on Board Vessels*. Sound level meters complying with the requirements of the International Electronical Commission IEC 179, equipped with octave or 1/3 octave filters satisfying the requirements of IEC 225, were used in the measurements. All the available observations were divided into two sets: one used for the estimation of the parameters of the model and the other destined for prediction on the basis of the estimated regression equations with the view to their verification. Table 2 shows this division.

One line of the matrix X of observations made on the independent variables corresponds to each line of the matrix of observations made on the dependent variable Y (values of noise levels measured in 6 octave bands). Selection of the explanatory variables took into account the significance of their effect on the noise level and their availability at the early stage of the ship design. On the basis of the prediction methods known from the literature [3-5, 8] and analysis of the results of noise measurements on ships, five groups of parameters affecting the noise level in the superstructure were distinguished. These

Table 2. Characteristics of the ships whose data were used in the calculations

No	Ship type	Deadweight TDW	Main engine power [kW]	Main engine revolutions [min^{-1}]	Revolutions of generating set [min^{-1}]	Number of measurement points	Purpose
1	bulk carrier	37840	8824	115	750	46	estimation of parameters of regression model
2	bulk carrier	9810	3434	800	1800	23	
3	bulk carrier	32000	8832	122	750	20	
4	bulk carrier	5735	2502	242	1500	22	
5	bulk carrier	14180	5446	155	750	29	
6	bulk carrier	25500	7066	199	750	38	
7	bulk carrier	3610	1656	225	500	12	
8	bulk carrier	14036	5888	150	750	17	
9	general cargo ship	11760	5299	139	500	13	
10	general cargo ship	7490	3270	430	1000	14	
11	general cargo ship	7350	5299	135	500	22	
12	semi container ship	12000	12806	122	720	13	
13	semi container ship	16000	17075	122	720	5	
14	con-ro	22000	21344	122	750	28	
15	bulk carrier	23785	7066	119	500	13	verification of regression equations derived
16	bulk carrier	52020	10739	134	750	10	
17	bulk carrier	39900	8832	122	720	28	
18	general cargo ship	14000	7618	120	720	15	
19	general cargo ship	11630	5888	135	500	5	
20	semi container ship	17000	17060	122	750	49	

groups are related to the characteristics of:

– the main engine,

– the generating set,

– the screw propeller,

– the sound propagation paths in the superstructure,

– other technical and operational parameters of the ship.

As a result of the optimization (in a statistical sense) of the sought regression relation, the following set of explanatory variables was established,

x_{i1} – the rated power of the main engine [kW],

x_{i2} – the rated revolutions of the main engine [min^{-1}],

x_{i3} – the ratio of the operational revolutions to the rated ones,

x_{i4} – the producer of the ship (each shipyard uses some characteristic designs which are based on experience and result to a large extent from the availability of technical means and a general technological level),

x_{i5} – the rated revolutions of the generating set [min^{-1}],

x_{i6} – the shape of the stern part of the hull (the distances between the top of the propeller blade and the shell plating over the propeller are significant acoustically),

x_{i7} – the distance between the centre of the accomodation and the edge of the generating set, defined by the number of frames,

x_{i8} – the position of the generating set in terms of height, defined by the successive number of the deck or platform from the inner bottom up,

x_{i9} – the kind of mounting of the generating set (rigid or elastic),

x_{i10} – the blade frequency of the propeller (the product of the rotation frequency of the shaft and the number of propeller blades),

x_{i11} – the Froud number, $F = V/(gL)^{1/2}$, which is the relative speed of the ship, where V is the speed of the ship [m/s], g is the acceleration of gravity [m/s^2] and L is the length of the ship [m],

x_{i12} – the distance between the centre of the accomodation and the edge of the main engine, defined by the number of frames,

x_{i13} – the position of the accomodation along the axis of the ship, defined by the number of the frame on which the centre of the accomodation is,

x_{i14} – the position of the accomodation in terms of height, defined by the successive number of the deck or platform from the inner bottom up,

x_{i15} – the position of the accomodation with respect to the casing (four positions were distinguished: adjacent to the casing, separated from the casing by a corridor, separated from the casing by another accomodation, outside the area of the casing),

x_{i16} – the position of the accomodation with respect to other accomodations (three positions were distinguished: direct vicinity to an accomodation with a noise source, vicinity to a workshop or store, position over another accomodation),

x_{i17} – the correction coefficient accounting for the nonlinearity of the damping characteristic of structure-borne sound in terms of height, where $i = 1, 2, \ldots, n$ is the number of observations.

Some of the above variables define the properties which cannot be measured. For these a digital coding system was assumed, with the principle that the values of the independent variables encoded must be in direct proportion to the corresponding values of the dependent variable.

4. Discussion of the calculated results

The parameters of the model were estimated on the basis of the results of the noise measurements on ships whose data are given in Table 2. The following linear-logarithmic form of the regression relation was assumed,

$$y_i = \sum_{j=1}^{17} b_{ij} d_{ij} + b_{i0} \quad (i = 1, 2, \ldots, 6), \tag{1}$$

where y_i is the noise level in the ith octave band [dB], b_{ij} are the regression coefficients ($i = 1, 2, \ldots, 6$; $j = 0, 1, \ldots, 17$), d_{ij} are the parameters affecting the noise in the accomodation ($i = 1, 2, \ldots, 6$; $j = 1, 2, \ldots, 17$), where

$$d_{ij} = \begin{cases} x_{ij} & \text{for } j = 3, 4, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, \\ \log x_{ij} & \text{for } j = 1, 2, 5, 13; \ i = 1, 2, \ldots, 6. \end{cases}$$

In order to check the validity of the present model, the coefficients of multipl e correlation were estimated for particular octave bands. The estimated results, given in Table 3, confirm the correctness of the set of explanatory

Table 3. The values of the coefficients of multiple correlation

Centre frequency of octave band [Hz]	63	125	250	500	1000	2000
Value of coefficient of multiple correlation	0.783	0.843	0.828	0.840	0.850	0.837

variables and forms of the regression relation assumed here. In turn, Table 4 shows an evaluation of the significance of the estimated coefficients of regression equations. Statistical conclusions were drawn at the significance level $\alpha = 0.1$. When it was found that the regression coefficient under investigation is not different from zero at the significance level α assumed, the corresponding variable was eliminated from the regression set. Such a procedure optimizes the final solution. In most cases the significance of particular explanatory variables depend s on the octave band. This is a result of the physical nature of the generation and propagation of noise on board a ship.

Table 4. The results of the significance test of the coefficients of the regression equations

Number of regression coefficient	Centre frequency of octave band [Hz]					
	63	125	250	500	1000	2000
1	+	−	−	−	−	−
2	+	+	+	+	+	+
3	−	−	−	+	+	+
4	+	+	+	+	+	+
5	+	+	+	+	+	+
6	+	+	+	+	+	+
7	−	+	+	+	+	+
8	−	+	+	+	+	+
9	+	+	+	+	+	+
10	−	+	+	+	+	+
11	−	−	−	−	−	−
12	−	+	−	+	+	+
13	+	+	+	+	+	+
14	+	+	+	+	+	+
15	−	−	−	+	−	−
16	−	+	+	+	+	−
17	−	−	−	+	+	+

+ significant coefficient, − insignificant coefficient

The most significant test of the usefulness of the regression equations derived is the verification of the predictions calculated from these equations, by comparing them with the measured results. Such a comparison was performed for 120 accomodations of 6 ships whose data are given in Table 2. Table 5 shows, for particular octave bands, the parameters of the distribution of the differences obtained between the calculated and measured levels. The low mean

Table 5. The parameters of the distribution of the differences between the calculated and measured noise levels for 120 accomodations

Parameter of difference distribution	Centre frequency of octave band [Hz]					
	63	125	250	500	1000	2000
mean value	−1.6	0.1	1.3	1.2	1.7	3.4
standard deviation	4.0	4.0	2.8	3.9	4.1	5.4

values for the bands over the range 63-1000 Hz indicate the lack of an essential trend in these octaves. Only in the octave band of 2000 Hz it is possible to notice a distinct trend for the method to exaggerate the prediction results. However, it can readily be seen in Table 1 that in this band the permissible noise level is

Table 6. Comparison of the accuracy of three methods for prediction of octave noise spectra

Authors of method	Characteristic of samples used for verification of method		Parameters of distribution of set of differences	Centre frequency of octave band [Hz]					
	number of ships	number of accomodations		63	125	250	500	1000	2000
KIHLMAN ŁLUNT	2	12	mean value	−1.33	−2.00	−0.08	−0.50	−1.57	−1.17
			standard deviation	2.96	4.35	4.60	3.37	3.17	1.75
BUITEN	11	160	mean value	0.49	−1.31	0.55	0.80	0.56	no data
			standard deviation	4.49	3.89	3.86	4.12	4.35	no data
present method	6	120	mean value	−1.60	0.10	1.30	1.20	1.70	3.40
			standard deviation	4.00	4.00	2.80	3.90	4.10	5.40

rarely exceeded. The standard deviations given in Table 5 can be taken as some additional measure of the accuracy of the method, which indicates the scatter of the magnitude of the differences about their mean value. In the next section this accuracy is examined in greater detail.

5. Evaluation of the accuracy of the present prediction method

Among the many papers on the prediction of noise levels in octave bands (e.g. [3-5, 8]), none contains a full algorithm permitting a direct use of the methods described in them. In all cases, apart from the basic principles of the construction of the model used and a general description of its solution, these publications contain information on the accuracy of the prediction results obtained. Table 6 gives the parameters of the distributions of the differences between the calculated and measured levels, for three methods: of KIHLMAN

Table 7. The distribution of the magnitude of error in predicted noise levels for 120 accomodations

Range of error [dB]	Probability of error — centre frequency of octave band [Hz]					
	63	125	250	500	1000	2000
±2	0.39	0.46	0.40	0.32	0.28	0.27
±4	0.61	0.73	0.80	0.63	0.67	0.47
±6	0.85	0.82	0.97	0.85	0.82	0.62
±8	0.94	0.94	1.00	0.99	0.92	0.75
±10	1.00	0.99	1.00	1.00	0.99	0.88
±12	1.00	1.00	1.00	1.00	1.00	0.93

and PLUNT [4], BUITEN [3] and the one used here. In view of the small number of samples used to verify the first method, its results have only an illustrative character. Of BUITEN's method it can be said that it shows hardly any trend (the mean values being close to zero) but rather large standard deviations, which cause prediction error of the order of ±4-10 dB in the 95 per cent confidence interval [3]. It can be seen from the data in Table 6 that the present method does not differ greatly in terms of accuracy from the other equivalent methods used at present for prediction of noise levels in octave bands.

The present prediction method permits an evaluation of its error ex ante, i.e. when the prediction is performed. The standard prediction errors estimated in the course of calculations vary between 3 and 4.5 dB for all octave bands. This assures a prediction accuracy of the order of ±6-8 dB in the 95 per cent confidence interval. This is confirmed by Table 7 which shows the probability of error occurring in particular intervals, calculated from the results of the verification of the present method which were discussed previously (Table 5). The fact that there may be some parameters which can essentially affect the noise level and which are not accounted for in the present model, should be given as the most probable cause of error in the present method. This is also the main explanation given by other authors for inaccuracies in their methods [3, 4, 8].

6. Conclusions

1. The method for prediction of octave spectra presented here and the method for A-weighted noise level prediction published previously [6] are based on a methodological approach which is different from those followed previously in the development of methods for noise level prediction. The present approach consists in omitting the stage of complicated and expensive laboratory research carried out on physical models and in concentrating on a deeper statistical analysis of standard measurement results. The results obtained to date confirm the validity of this direction of research.

2. The present method for octave spectrum prediction is efficient, which was shown in the course of verification. The results obtained using the regression prediction of noise levels are comparable to those of equivalent foreign methods.

3. The regression prediction is quite simple in application, since it involves one calculation step, whereas the foreign methods mentioned above require at least several steps.

4. None of the published foreign methods can be used directly. The relevant algorithms are unknown outside the research centres where they have been developed and their use by Polish shipyard has always the character of an expensive service.

5. The present method, as all empirical ones, requires continuous updating and supplementing of the set of measurement data from which the relevant regression equations are generated.

6. The accuracy of the present method will be increased and its small permanent trend eliminated in the next stage of the authors' investigations. It will be possible to attain this end when the set of independent variables has been extended and there are more measurement data.

References

[1] A. Bartkowiak, *Basic algorithms of mathematical statistics* (in Polish), PWN, Warsaw 1979.

[2] A. Bartkowiak, *Yearbooks of the Polish Mathematical Society*, Series III: Applied Mathematics VIII (in Polish), 1976.

[3] J. Buiten, *Experiences with structure borne sound transmission in sea going ships*, Proceedings Int. Symp. on Shipboard Acoustics, Amsterdam 1976.

[4] T. Kihlman, J. Plunt, *Prediction of noise levels in ships*, Proceedings Int. Symp. on Shipboard Acoustics, Amsterdam 1976.

[5] A. C. Nilsson. *Noise prediction and prevention*, Technical Report No 78-030, Det Norske Veritas Research Division, 1978.

[6] E. Szczerbicki, A. Szuwarzyński, *Noise prediction on ships*, Archives of Acoustics, **6**, 2, 111-122 (1981).

[7] H. Theil, *Principles of econometry* (in Polish), PWN Warsaw 1979.

[8] G. Ward, A. Hoyland, *Ship design and noise levels*, North East Coast Institution of Engineers Shipbuilders, **95**, 4, 177-196 (1979).

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DETECTION OF LOW INTENSITY AUDITORY EVOKED RESPONSES

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A signal processing technique based on the use of crosscorrelation functions is proposed for the analysis of the auditory evoked brainstem response (*ABR*). Detection of a response as well as estimation of its position along the time axis (group latency) are shown to be significantly improved after correlation analysis. Results indicate that the method may be applied to systematical investigations of the responses measured — with surface electrodes on the scalp — at extremely low intensity levels.

1. Introduction

Analysis of short latency auditory evoked responses (whole-nerve Action Potential and auditory evoked brainstem response, or *ABR*) generated at medium to low intensity levels is of particular interest for several reasons. First, at the lowest intensities, the frequency specific mechanisms active at cochlear levels, as those observed in the frequency threshold curves [11, 8] are such that the short latency responses originate mainly from the cochlear regions tuned to the peak(s) of the stimulus spectrum [2, 3, 13, 14, 10, 1]. At medium to low intensities, the region excited by the stimulus widens and consequently the frequency — or "place" — specificity is progressively lost. The "place" specificity of low intensity responses has been proposed for use in extracting information on the peripheral hearing at specific frequencies [4, 17]. Secondly, despite the fact that the literature accumulated on the early evoked responses during the last decade is very rich, many aspects concerned with the analysis of responses evoked at very low intensity levels are still uncovered.

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Thirdly, at medium to high intensities, in a variety of hearing losses, the response waveshapes and latencies tend to approximate the normative standards [6]. On the other hand, interpretation of the recordings at medium to low intensities is difficult because the signal-to-noise ratio is poor; at intensities of 30-20 dB *SL* the detection of a response and the identification of its components may be impossible, in particular for the *ABR*.

The aim of the present paper is twofold: 1) we want to show how and to what extent the use of a signal processing technique (crosscorrelation analysis) may improve the response detection in a given set of recordings, and 2) we present evidence for the constant presence of an *ABR* at intensity levels as low as 0-10 dB re the subject's threshold sensation level for the same stimulus. We shall confine our study to the *ABR* measured with electrodes on the scalp, in humans.

Some preliminar results have been presented elsewhere [9]. The use of crosscorrelation techniques for the analysis of evoked potentials from the cortex was introduced by WOODY [16] and MCGILLEM and AUNON [12] and by ROSENHAMER [15] and ELBERLING [7] for the *ABR*.

2. Methods

The proposed procedure

The regular and monotonic changes of amplitudes and latencies of the *ABR*s over the intensity range suggest that the response evoked at a given stimulus intensity I_1, $s_1(t)$, may be used to predict the presence of a signal and its time location (group latency) in a recording obtained at an intensty I_2, $s_2(t)$. It can be assumed that $I_2 < I_1$. A first approximation model of the response $s_2(t)$ is therefore given by

$$s_2(t)\, a s_1(t-\tau). \tag{1}$$

This expression means that $s_2(t)$ may be considered as a shifted version of $s_1(t)$, the relative time delay being τ, apart from a scale factor. Expression (1) is a reasonable approximation of the real situation at least as far as I_1 and I_2 are close together.

Then, evaluation of the crosscorrelation function between $s_1(t)$ and $s_2(t)$ will provide information on the presence of a response at the lower intensity I_2 and on its relative time delay. As it will be shown in the next sections, the results of correlation analysis do confirm *a posteriori* the assumptions above.

Recording and processing procedures

The responses were recorded with Ag–AgCl disk electrodes pasted on the ipsilateral (+) and contralateral (−) mastoid, with the forehead as ground. Rarefaction rectangular pulses (100 μs duration) were delivered through TDH-49 earphones, with MX-41/AR cushions, at a rate of 11/s, at levels ranging from 120 to 20 dB (peak equivalent at 3kHz) *SPL*, in 5 or, more frequently, 10 dB steps. The subjects were lying comfortable on a bed in a quiet room. Responses were sampled over a time window of 10 ms (512 sampled data points); filters were set at 200-2000 Hz (24 dB/oct). Recordings were averaged over 2048 sweeps and stored on floppy disks for offline processing. Stimulus generation, data acquisition and processing were under the control of a (Fortran programmable) Amplaid MK 6 system. Before the beginning of the recording session, the subjects (all trained normal hearing listernes involved in the Biomedical Engineering Program at the Polytechnic of Milan) were asked to make by themselves the determination of the threshold sensation level, by entering the stimulus intensity from the computer keyboard. Typical values were in the range of 35-40 dB *p.e. SPL*.

Offline processing of the signals included:

I. demean of the record;

II. tapering with a cosine window over 75 points, at the beginning and at the end of each record;

III. zero-phase shift bandpass digital filtering; bandpass of 100-2000 Hz or 150-2000 Hz (24 dB/oct) has been most commonly used;

IV. normalization in amplitude according to the *rms* value.

Crosscorrelation functions are then computed for the entire sequence of responses, in 10 dB steps, from the highest to the lowest intensity levels, respectively.

3. Results

A complete set of *ABR*s measured over a range of 100 dB is illustrated in Fig. 1a. It can be seen that the responses evoked at stimulus levels below approximately 70-60 dB *p.e. SPL* are hardly discernible. Fig. 1b and c illustrate the results obtained after application of the procedure above to the original recordings.

Interpretation of the crosscorrelation functions (*c.c.f.*) requires some comments. First of all it is noted that strong periodical components are present in the *c.c.f.*s over the whole intensity range – a reflection of the quasi-periodical time course of the *ABR*s as a result of our recording technique. It is noted also that the waveshapes of the *c.c.f.*s are remarkably constant, apart from a delay reflecting the response (group) latencies. Removal of the low frequency components ($< 150 : 200$ Hz, see *Methods*) from the original recordings emphasizes the periodicities in the *ABR*s and, consequently, in the *c.c.f.*s. We have deliberatedly chosen the mastoid-mastoid electrode derivation to the same purpose; in fact, with a vertex-mastoid or earlobe-vertex derivation, the *c.c.f.*s have a rather broad single-peaked waveshape, as a consequence of the relative

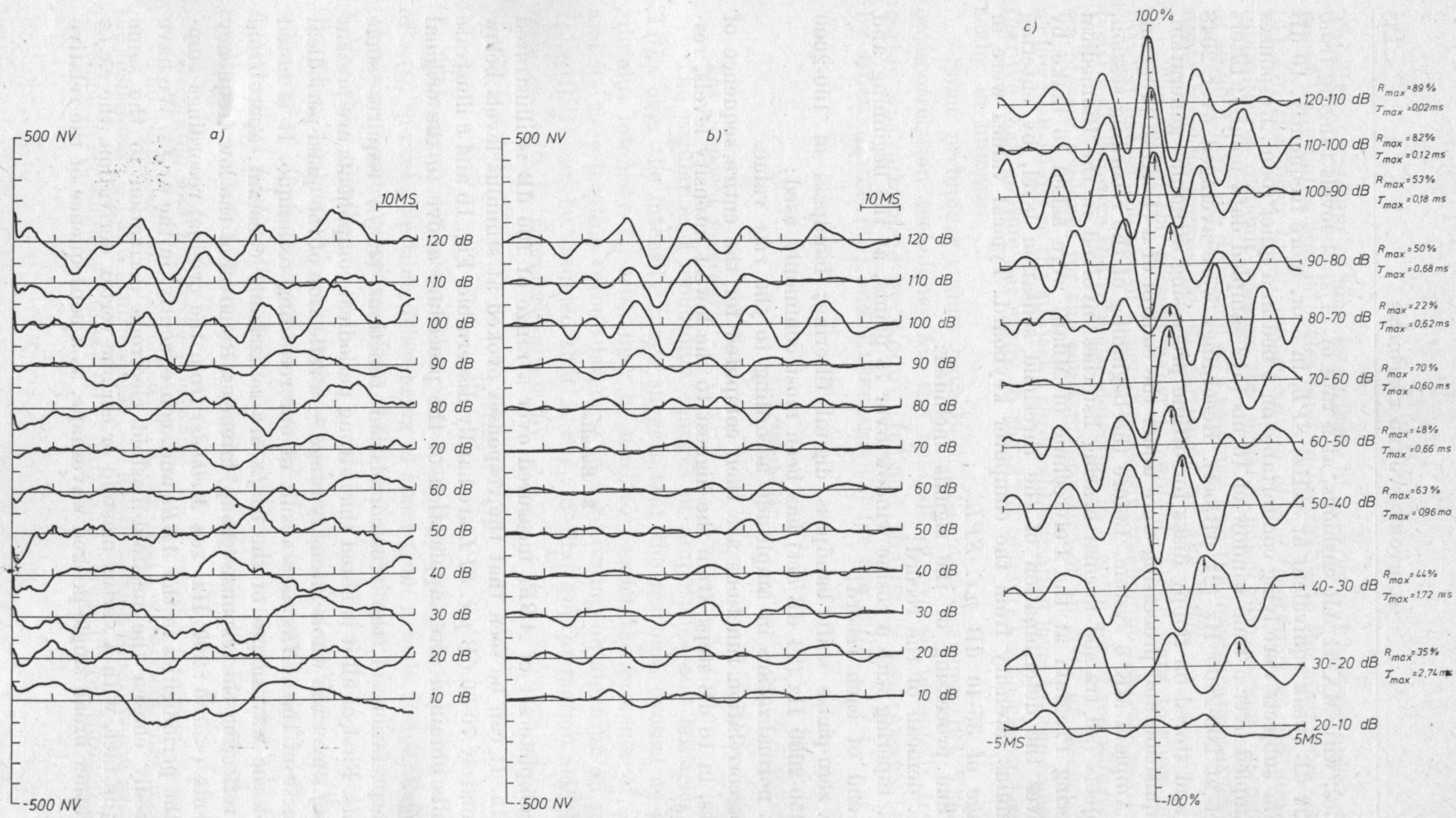

Fig. 1. a) *ABR*s from a normal hearing subject. Amplitudes are the same for all the recordings. b) Responses after bandpass filtering. c) Crosscorrelation functions computed from the responses of b), according to the present procedure. Arrows indicate the peak corresponding to the maximum correlation. For each function, the values of the maximum correlation (R_{max} %) and its abscissa (τ_{max}) are reported

dominance of wave *V*. By use of the present recording and processing procedures the similarities observed in the *c.c.f.*s are more easily observed over the whole intensity range.

It is seen from Fig. 1c that the values of the maximum correlation (R_{max}) are approximately a slowly decreasing function of the intensity level (see also Fig. 2 to be discussed later). This is due to the decrease of the signal-to-noise

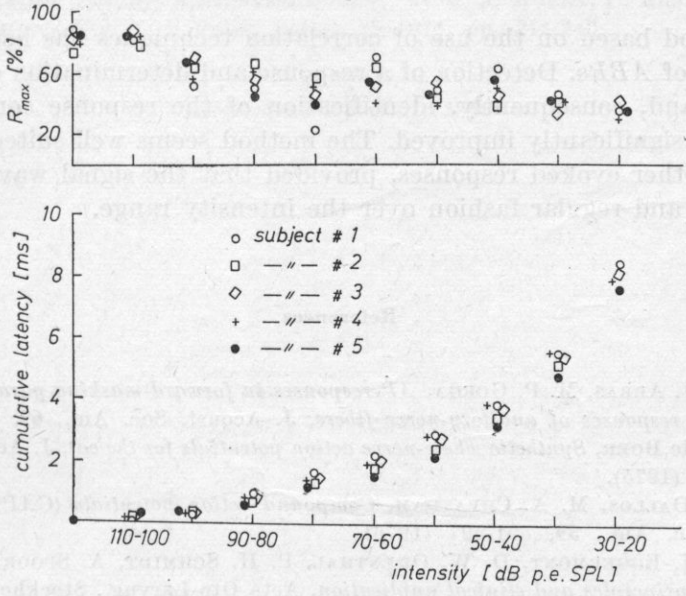

Fig. 2. Intensity dependence of R_{max} (top graph) and of the cumulative latency (bottom graph) from five subjects

the results of the correlation analysis of subject 1 are those reported in Fig. 1; the data points of subject 2 at 30-20 dB *p.e. SPL* have been disregarded since R_{max} was below 20 %. Threshold sensation levels were 37,35, 40,37 and 35 dB, *p.e. SPL* for subjects 1, 2, 3, 4 and 5, respectively

ratio with the decrease of the intensity as well as to some subtle changes of the *ABR* waveforms. However, the maximum correlation is still as high as 63 and 44 % at 50-40 and 40-30 dB *p.e. SPL*, whereas the threshold sensation level of the subject was 37 dB *p.e. SPL.*

Results as those illustrated in Figure 1 are typical for all the recordings analysed with the present procedure. Fig. 2 resumes the results of the correlation analysis for five subjects, in the form of R_{max} and of time delays. The top graph of Fig. 2 shows that, for intensity levels of 40-30 and 30-20 dB *p.e. SPL*, R_{max} is still high. With 10 dB steps between the input levels and with a time window of 10 ms, R_{max} falls abruptly at the lowest intensities. Data concerning the relative time delays have been processed as follows: a "cumulative latency" has been calculated, for each subject, by summing the values of τ_{max} from the *c.c.f.*s, step by step, orderly from the higest to the lowest intensities.

The results, from the same five subjects are shown in the bottom graph of Fig. 2; as expected, the cumulative latency increases regularly and monotonically as the intensity is decreased.

4. Conclusions

A method based on the use of correlation techniques has been applied to the analysis of *ABR*s. Detection of a response and determination of its relative time delay and, consequently, identification of the response components are shown to be significantly improved. The method seems well suited also for the analysis of other evoked responses, provided that the signal waveshapes vary in a smooth and regular fashion over the intensity range.

References

[1] P. J. Abbas, M. P. Gorga, *AP responses in forward-masking paradigms and their relationship to responses of auditory-nerve fibers*, J. Acoust. Soc. Am., **69**, 492-499 (1981).

[2] E. de Boer, *Synthetic whole-nerve action potentials for the cat*, J. Acoust. Soc. Am., **58**, 1030-1050 (1975).

[3] P. Dallos, M. A. Cheatham, *Compound action potentials (CAP) tuning curves*, J. Acoust. Soc. Am., **59**, 591-597 (1976).

[4] J. J. Eggermont, D. W. Odenthal, P. H. Schmidt, A. Spoor, *Electrocochleography. Basic principles and clinical application*, Acta Oto-Laryng., Stockholm, Suppl. 316.

[5] J. J. Eggermont, A. Spoor, D. W. Odenthal, *Frequency specificity of toneburst electro-cochleography* in: *Electrochleography*, R. J. Ruben, C. Elberling, G. Salomon (eds.), University Park Press, Baltimore 1976, pp. 215-246.

[6] C. Elberling, G. Salomon, *Action potentials from pathological ears compared to potentials generated by a computer model*, in: *Electrochleography*, R. J. Ruben, C. Elberling, G. Salomon (eds.), University Park Press, Baltimore 1976, pp. 439-455.

[7] C. Elberling, *Auditory electrophysiology. The use of templates and crosscorrelation functions in the analysis of brain stem potentials*, Scand. Audiol., **8**, 187-190 (1979).

[8] E. F. Evans, *The frequency response and other properties of single fibers in the guinea pig cochlea*, J. Physiol., 226, 263-287 (1972).

[9] F. Grandori, J. Szabs, *Detecting low intensity auditory evoked responses with signal processing techniques*, VII Symp. of the International Electric Response Audiometry Study Group, VI 1981, Bergamo, Italy.

[10] D. M. Harris, *Action potential suppression, tuning curves and thresholds: Comparison with single fibers data*, Hear. Res., **1**, 133-154 (1979).

[11] N. Y.-S. Kiang, *Discharge patterns of single fibers, in the cat's auditory nerve*, Monogr. No. 35, MIT Press, Cambridge 1965.

[12] C. D. McGillem, J. I. Aunon, *Measurements of signal components in single visually evoked brain potentials*, I. E. E. E. Trans. on B. M. E., **24**, 232-241 (1977).

[13] Ö. Özdamar, P. Dallos, *Input-output functions of cochlear whole-nerve action potentials: interpretation in terms of one population of neurons*, J. Acoust. Soc. Am., **59**, 143-147 (1976).

[14] Ö. ÖZDAMAR, P. DALLOS, *Synchronous responses of the primary auditory fibers to the onset of tone burst and their relation to compound action potential*, Brain Res., **155**, 169-175 (1978).

[15] H. J. ROSENHAMER, *Observation on electric brainstem responses in retrocochlear hearing loss,* Scand. Audiol, **6**, 179-183 (1977).

[16] C. J. WOODY, *Characterization of an adaptive filter for the analysis of variable latency neuroelectric signals,* Med. Biol. Engng., **5**, 539-553 (1967).

[17] S. ZERLIN, R. F. NAUNTON, *Whole-nerve response to third-octave audiometric clicks at moderate sensation level,* in: *Electrocochleography,* by R. J. RUBEN, C. ELBERLING, G. SALOMON (eds.), University Park Press, Baltimore 1976, pp. 215-246.

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EVALUATION OF ELECTROACOUSTIC DEVICES BY THE EQUIVALENT SCALE METHODS

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This paper presents the preliminary results of experiments on the possibility of using methods based on the mutual masking of signals compared, in auditory evaluation. An attempt was made to objectivize the measure of the similarity of the signals, or the transmission channel quality, and to evaluate the degree of differentiation of objects investigated on an equivalent physical scale. In the preliminary experiments a number of experimental procedures were examined, two of which, characterized by the relatively highest differentiation sensitivity, underwent more detailed evaluation. The results indicated the possibility of objectivizing the measure of the transmission circuit quality in the range of frequency response variation over a third-octave.

1. Introduction

The ultimate criterion of quality (usefulness) of devices for broadly understood sound generation and transmission is auditory evaluation of sound signals generated or transmitted by these devices. This evaluation can also extend to the sound signals themselves, considered apart from the forming devices used. This can occur, for example, in the evaluation of the naturalness of sound effects or the efficience of the performance of different warning signal types.

The purpose of both the evaluation of sound signals themselves (direct auditory evaluation) and the evaluation of sound generation and transmission devices (indirect auditory evaluation) which is based on the former, may be to determine the degree of mutual similarity between objects evaluated and the degree to which one of the objects dominates over the other. In such cases the basic kinds of auditory evaluation are differentiation and classification of evaluated objects, respectively. A discussion of the applications of the two kinds of evaluation and a review of the different forms of auditory evaluation in practice is given by ŁĘTOWSKI in paper [3].

The basic disadvantage of the above two kinds of auditory evaluation is the lack of a stable and unambiguously defined general measure of the quality (similarity) of objects which would permit the reliable reference of the results of different auditory experiments to one another and unambiguous determination of the degree of differentiation between given evaluated objects. Although the categorization of evaluated objects on interval scales characterized by the patterns (anchors) of some selected categories permits the comparison of the results of different (independent) auditory experiments, it is, however, a very imprecise tool and covers a rather limited range of practical application. A greater opportunity for the evaluation of objects is provided by the anchored absolute (ratio) scales, but in the case of evaluating most sensations and emotions the use of scales of this type raises justifield objections [6, 7].

In view of this situation, for some time attempts have been made to use in the field of auditory evaluation the procedures of differentiation and classification of objects, with a simultaneous representation of the results on some different types of equivalent substitute continua. They can be abstract (the auxiliary scale of evaluation reliability degrees), graphic (a linear distance scale) or expressed in physical units. Attempts have also been made to use for this purpose the procedures of cross modality scaling introduced into experimental psychology by STEVENS [9].

MUNSON and KARLIN [4] were the first to develop, in 1962, an equivalent quality scale of physical character in the field of auditory impressions. In their investigations of the evaluation of devices for speech transmission they introduced as the "objective" measure of the quality difference between two speech signals the difference in the sound intensity level (dB) between them which is necessary for the achievement of the same subjective quality of the two sounds. The paired comparison method was used as the scaling procedure. The signal at the output of the transmission channel investigated is compared with the original (input) signal masked by some additional random noise level. The difference between the levels of the output and input signals which were recognized as qualitatively equal with a specific random noise level interfering with the input signal is recognized as the measure of the quality of a given transmission channel. This method was later developed, among others, by ROTHAUSER *et al.* [8] and NAKATANI and DUKES [5].

Another method of physical equivalent scales which has been proposed for description of (electroacoustic) transmission channels consists in a controlled introduction of a given type of deformation into the input signal. The measure of the quality of the transmission channel is the degree of deformation which can be introduced into the input signal without causing a noticeable change in the output signal. Such qualitative scales were used by BORDONE-SACERDOTE and MODENA [1] and KULESZA [2], for example.

The aim of this paper is to investigate the possibility of using in the field of auditory evaluation methods based on the mutual interference (masking) between signals compared with each other and to evaluate the degree of differentiation between objects investigated on a corresponding physical equivalent scale. The methods proposed are based on the assumption that the sound intensity level of the reference signal X, or some additional stimulus M, which is necessary for a given perception threshold of the signal Y evaluated to be achieved, can be the "objectivized" measure of the similarity of the signal Y to the signal X, or the measure of the quality of the transmission channel under study.

2. General characteristic of the investigations

The previous experience of the authors in the field of auditory evaluation and investigations in the fundamentals of hearing led to the hypothesis that the hearing threshold for a given signal under given conditions of simultaneous or succesive masking can be the measure of the deviation of this signal from some "norm" (reference signal) assumed. In order to verify this hypothesis five experimental procedures based on the use of the masking effect were investigated. These procedures were named in the following way:

a. the method of alternative sequences I,
b. the method of alternative sequences II,
c. the method of alternative sequences III,
d. the pulsation method
e. the method of succesive masking.

The time paradigms of the particular procedures are shown schematically in Fig. 1.

The method of alternative sequences I consists in the presentation, against a continuous reference signal, of a sequence of pulses of a signal to be compared. The listeners' task is to find such a low level of the compared signal, or such a high level of the reference signal, at which a characteristic pulsation (modulation) of the resultant signal occurs. The measure of similarity between X and Y is the difference in level which occurs between these signals for the controlled signal level defined by the listener.

The method of alternative sequences II consists in the presentation, against a given continuous masking signal, of alternating sequences of equal-level pulses of the reference and comparative signals. The listeners' task is to select such a low level of the masking signal at which the difference between the sequences of the pulses becomes inaudible. The measure of similarity between the signals X and Y is the minimum level of the signal M defined by the listener according to the present instruction.

The method of alternative sequences III consists in the alternating presentation, against pulses of the masking signals, of pulses of the reference or comparative signals. The listeners' task is to find such a high level of the masking signal at which differences between the pulses begin to be audible. The measure of similarity between the signals X and Y is the level of the masking signal, defined by the listener, with a constant level of the reference and comparative signals.

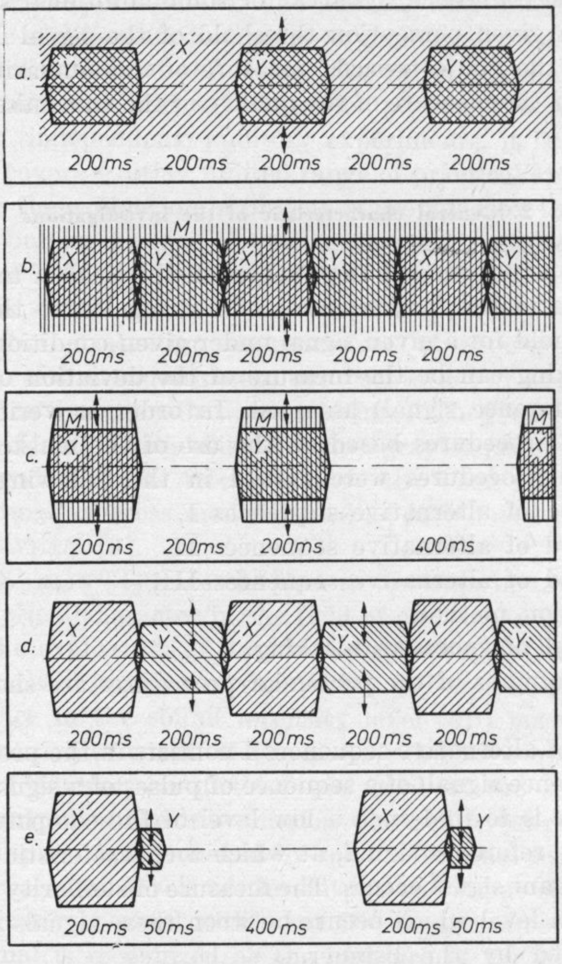

Fig. 1. Time paradigms of the measurement procedures: a. method of alternative sequences I, b. method of alternative sequences II, c. method of alternative sequences III, d. pulsation method, e. method of successive masking

The pulsation method consists in the alternative presentation of pulses of the reference signal X and the comparative signal Y. The listener's task is to adjust the level of the signal Y so as to obtain the sensation of continuity of this signal against the background of pulses of the signal X. The measure of difference between the signals X and Y is the difference between their levels when the pulsation threshold occurs.

The method of succesive masking consists in the presentation of sequences of pulses of the reference signal X, followed by short pulses of the comparative signal Y. The listener increases the level of the signal Y until pulses of this signal begin to be audible when a pulse of the signal X ends. The subjective measure of difference between the two signals is the difference between the levels of the signals X and Y selected for the above purpose by the listener.

Table 1. The physical signals in particular investigation methods

Investigation method	Variant	Reference signal	Comparative signal	Masking signal
method of alternative sequences I	*A*	*SL*	*FSL*	–
	B	*SR*	*FSR*	–
	C	*SRM*	*FSRM*	–
method of alternative sequences II	*D*	*SL*	*FSL*	*SL*
	E	*SL*	*FSL*	*SR*
	F	*SL*	*FSL*	*SRM*
method of alternative sequences III	*G*	*SL*	*FSL*	*SL*
	H	*SL*	*FSL*	*SR*
	I	*SL*	*FSL*	*SRM*
pulsation method	*J*	*SL*	*FSL*	–
method of succesive masking	*K*	*SL*	*FSL*	

SL – random noise (white noise)
SR – pink noise (with power density spectrum decreasing as a function of frequency at a rate of 3 dB/oct.)
SRM – uniformly masking noise (with power density spectrum stable up to the frequency $f = 500$ Hz and decreasing at a rate of 3 dB/oct. above this frequency)
FSL – filtered random noise
FSR – filtered pink noise
FSRM – filtered uniformly masking noise

Table 1 shows physical signals used by the authors in the implementation of particular investigation procedures. In all cases the transmission channel system investigated was simulated with a controlled Brüel and Kjaer 5587 spectrum shaper. The (output) comparative signals corresponded to four high-pass and four low-pass responses of the equaliser with the following cut-off frequencies (−3 dB):

1. the low-pass system: a. 7 kHz, b. 8.9 kHz, c. 11.2 kHz, d. 14.1 kHz;
2. the high-pass system: e. 179 Hz, f. 224 Hz, g. 282 Hz, h. 355 Hz.

The selection of such a "laboratory" transmission system was caused by the necessity of unambiguous description at this stage of investigations of physical transmission properties of the systems compared.

The investigations presented in this paper were carried out in two stages. In the first stage, the authors examined all the 8 "laboratory" transmission systems for all the 11 stimuli configurations ($A-K$) shown in Table 1. The investigations also included some "laboratory" systems with narrow-band responses (single 1/3 octave and octave bands). The loudness level of the monitoring of stable signals, i.e. those not adjusted by the listener, was in all cases 50-60 phons. In individual investigations both loudspeaker monitoring (loudspeaker GK 124) and earphone monitoring (earphones Peerless PMB-6) were used.

It was found in these preliminary investigations that among all the psychoacoustic procedures examined, only the version J of the method of alternative sequences III, called the method of alternative sequences below, and the pulsation method seem to provide the purpose-desired sensitivity in differentiating evaluated objects and the repeatability of the investigation results. Therefore, these methods underwent more formalized experiments whose results are the object of the present communication.

3. Pulsation method

The measurement system used in the pulsation method is shown schematically in Fig. 2. The reference signal X (channel I) was white noise recorded on magnetic tape (tape recorder Revox A77). The comparative signal Y (channel II) was a white noise signal (from Brüel and Kjaer 1024 generator) filtered by a set of 36 parallel 1/3 octave filters (Spectrum Shaper 5587). Both signals

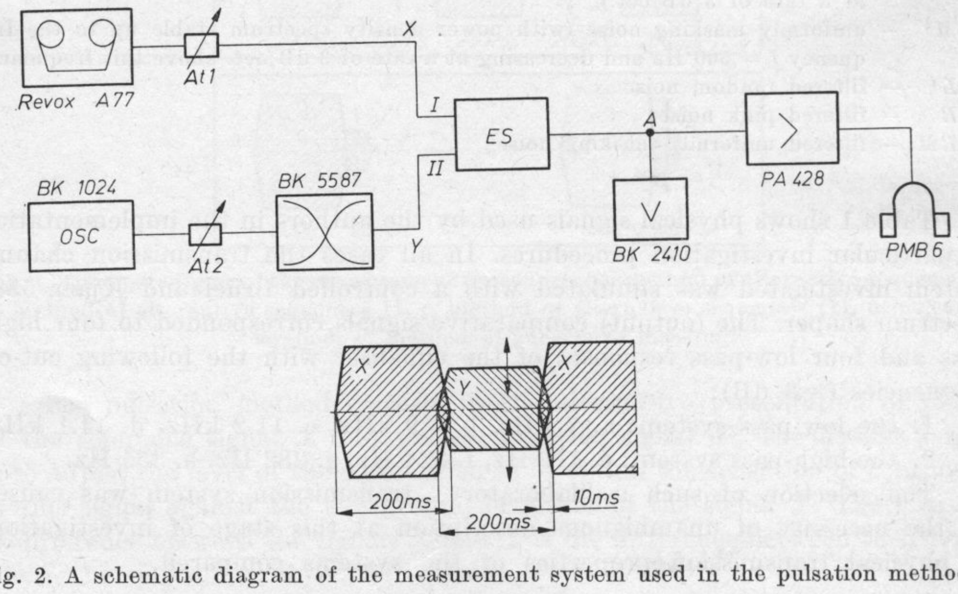

Fig. 2. A schematic diagram of the measurement system used in the pulsation method

were fed to the corresponding inputs of a multi-channel electronic switch (*ES*). At the output of the switch the two signals were presented in the form of alternate pulses. The duration of each pulse was 200 ms (the rise time being 10 ms). When appropriately amplified (by the amplifier Fonica PA 428), the signals were heard by the listener through the orthodynamic headphones PMB-6. In the course of the experiment the listener adjusted with a decade attenuator At2 the level of the comparative signal *Y* so that is was heard as continuous. The corresponding frequency responses of channel II were created, analogously to the preliminary experiments, by means of a set of 1/3 octave filters. 3 listeners, workers of the Sound Engineering Department, took part in the investigations. After 3 trial series each of the listeners made 5 measurements for each frequency response of channel II. The level of the reference signal at point *A* was 290 mV, while its loudness level at the output of the headphones was 60 phons. The level of the comparative signal at which the listener set the pulsation threshold was read from a Brüel and Kjaer 2410 voltmeter. It should be noted that the pilot experiments showed that the use of pulses with shorter duration than the assumed one (higher switch frequency) and loudspeaker monitoring decreases the reliability of evaluation.

4. Method of alternative sequences

The measurement system used in the method of alternative sequences is shown in Fig. 3. White noise from a Brüel and Kjaer 1024 generator was supplied to two inputs (II and III) of the electronic switch through two separate

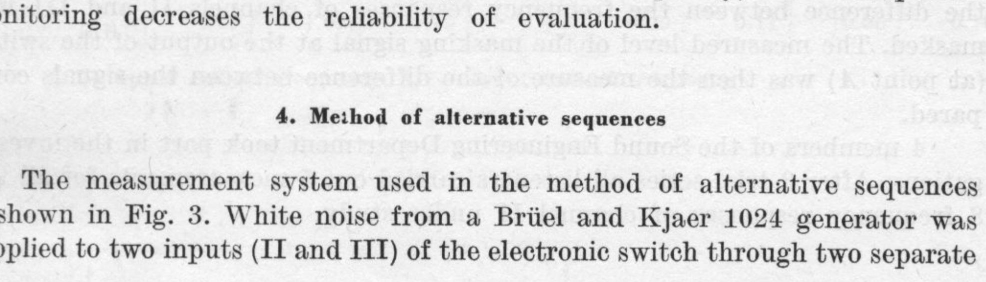

Fig. 3. A schematic diagram of the measurement system used in the method of alternative sequences

channels. One of the channels (III) had a flat frequency response in the whole acoustic band. The other channel (II) included an adjustable spectrum shaper. The pulses of the signals X and Y (with 200 ms duration) thus obtained were presented alternatively at the output of the switch. The interstimulus interval was 200 ms. The next sequence was repeated after 400 ms. The rise and decay times of the signals were 10 ms. A signal of uniformly masking noise (M), reproduced from magnetic tape, was added to both signals. The signal M was supplied to the input of the electronic switch. Summed-up pulses with the same duration were presented to listeners through the PMB-6 headphones. The voltage of the reference signal X (channel III) at the output of the switch was kept at a constant level of 80 m V (point A). Before the signal M was switched on, the loudness level of the pulses X and Y was adjusted subjectively and was about 60 phons.

A system of filters was used to control the frequency response of channel II, as in the preliminary experiments. In the course of the experiment the listener, using the attenuator $At1$, increased the level of the masking signal M until he recognized the timbre of the pulses as the same. This signified that the difference between the frequency responses of channels II and III was masked. The measured level of the masking signal at the output of the switch (at point A) was then the measure of the difference between the signals compared.

4 members of the Sound Engineering Department took part in the investigations. After 3 trial series all listeners carried out 5 measurements for all the 8 frequency responses of channel II under study.

5. Discussion of the results

The results obtained in the investigations by the pulsation method are given in Table 2 and Fig. 4. Particular rows of Table 2 and Fig. 4 show the mean values of the voltage level of the comparative signal at which the listener set the pulsation threshold.

The results were obtained for 3 listeners and 8 frequency responses. Table 2 also gives the values of the corresponding standard deviations (dB). For these results the significance of differences among the mean values was examined by the t-test for uncorrelated data

$$t = \frac{Y_1 - Y_2}{\sqrt{\frac{\sigma_1^2+\sigma_2^2}{n-1}}},$$

where Y_1 and Y_2 are the mean values of the voltage of the comparative signal for the pulsation threshold determined by the listener for two different frequency responses of channel II (a, b, c, d and e, f, g, h), σ_1^2 and σ_2^2 are the values

Table 2. The results for the pulsation method. The mean values of the values of the level of the pulsation threshold and the values of the corresponding standard deviations

Listener		Cut-off frequency of channel II							
		f_g [kHz]				f_d [Hz]			
		7 kHz	8.9 kHz	11.2 kHz	14.1 kHz	179 Hz	224 Hz	282 Hz	355 Hz
		a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
1	*Y* [dB]	−14.52	−13.43	−12.4	−11.4	−10.23	−10.03	−9.9	−9.79
	σ [dB]	0.71	0.63	0.21	0.74	0.21	0.12	0.17	0.13
2	*Y* [dB]	−14.52	−13.19	−12.22	−10.99	−10.12	− 9.95	−9.87	−9.84
	σ [dB]	0.27	0.54	1.0	0.18	0.11	0.11	0.05	0.11
3	*Y* [dB]	−15.29	−13.64	−12.92	−12.04	−10.31	−10.26	−10.2	−10.28
	σ [dB]	0.77	1.02	0.15	0.34	0.09	0.11	0.06	0.16

f_g — upper cut-off frequency, f_d — lower cut-off frequency

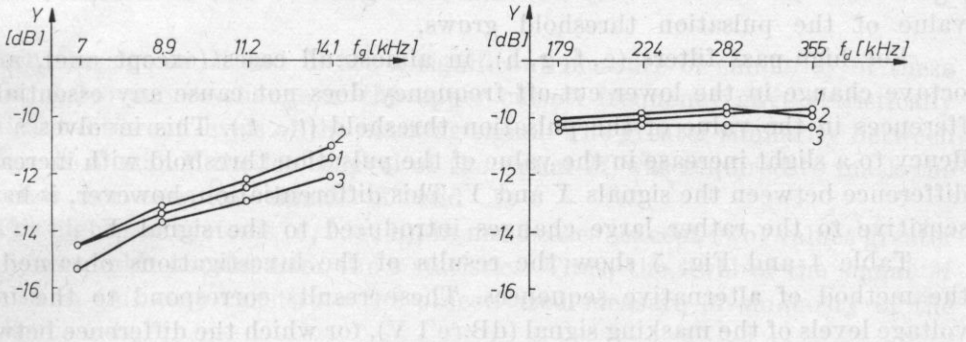

Fig. 4. The mean values of the level of the signal *Y* corresponding to the pulsation threshold. Results for 3 listeners as a function of the cut-off frequencies of the frequency response of channel II

of the corresponding variances and n is the number of results for a measurement point ($n = 5$). Table 3 shows the results of the t — test obtained for neighbouring measurement points. For $2n-2 = 8$ degrees of freedom and the significance level $a = 0.1$ the critical value of the t-test is $t_a = 1.86$.

Table 3. The values of the t statistics in comparison of different frequency responses of channel II in the pulsation method

Comparison of frequency responses of channel II	Listener 1	2	3
$a-b$	8.84	13.86	11.11
$b-c$	12.37	7.84	6.57
$c-d$	9.86	12.33	12.83
$e-f$	1.61	2.12	0.78
$f-g$	1.30	1.34	0.89
$g-h$	1.0	0.45	0.97

For low-pass systems (a, b, c, d) the results — $t \geqslant t_a$ — indicate the significance of differences between the values of pulsation thresholds with a 1/3 octave change in the cut-off frequency. The value of the voltage level of the comparative signal Y at which the pulsation threshold was set can in this case be the objectivized measure of similarity between the signal Y and the reference signal X, since the similarity between the signals X and Y increases as the value of the pulsation threshold grows.

For high-pass filters (e, f, g, h), in almost all cases (except one), a 1/3 octave change in the lower cut-off frequency does not cause any essential differences in the value of the pulsation threshold ($t < t_a$). This involves a tendency to a slight increase in the value of the pulsation threshold with increasing difference between the signals X and Y. This differentiation, however, is hardly sensitive to the rather large changes introduced to the signal Y.

Table 4 and Fig. 5 show the results of the investigations obtained for the method of alternative sequences. These results correspond to the mean voltage levels of the masking signal (dB re 1 V), for which the difference between the signals X and Y was masked. Table 4 and Fig. 5 compare results for 4 listeners and 8 frequency responses of channel II; Table 4 also gives the corresponding values of the standard deviation (dB).

On the basis of the mean values obtained, analogously to the pulsation method, the investigation results were verified statistically. Table 5 shows the values of the t test obtained.

The results obtained ($t \geqslant t_a$) for low-pass systems (a, b, c, d) indicate that the value of the voltage of the signal M required to mask the differences

Table 4. The results for the method of alternative sequences. The mean values of the voltage level of the masking signal and the values of the corresponding standard deviations

Listener		Cut-off frequency of channel II f_g [kHz] 7 kHz a	8.9 kHz b	11.2 kHz c	14.1 kHz d	f_d [Hz] 179 Hz e	224 Hz f	282 Hz g	355 Hz h
1	Y [dB]	−16.8	−18.6	−20.9	−24.7	−23.7	−23.2	−20.6	−18.9
	σ [dB]	0.29	0.58	0.64	0.28	0.60	0.52	0.54	0.71
2	Y [dB]	−15.1	−13.8	−15.3	−19.6	−24.4	−22.6	−18.6	−15.8
	σ [dB]	0.29	0.19	0.29	0.96	1.15	0.27	0.49	0.51
3	Y [dB]	−13.2	−16.7	−21.6	−26.1	−26.9	−22.5	−18.1	−14.3
	σ [dB]	0.08	0.36	0.41	0.57	0.15	0.67	0.65	0.33
4	Y [dB]	−15.4	−19.7	−21.2	−25.5	−18.0	−15.3	−13.8	−13.1
	σ [dB]	0.60	0.47	0.36	0.65	1.10	0.41	0.66	0.42

f_g — upper cut-off frequency, f_d — lower cut-off frequency

between the signals X and Y are a significant measure of similarity of these signals. A 1/3 octave change in the upper cut-off frequency gave statistically significant different levels of the masking signal. The greater similarity between the signals X and Y the lower level of the signal M was required to mask the difference between the signals X and Y.

For high-pass systems (e, f, g, h), in most cases (except two) values greater than t_a were also obtained for the t statistics. Thus, the level of the signal M can also in this case constitute the objectivized measure of similarity of the signals X and Y.

6. Conclusions

1. On the basis of experiments, two measurement procedures characterized by the highest sensitivity of differentiating the evaluated objects were chosen of 11 procedures. For the method of alternative sequences II (variants G, H and J), the best results in the preliminary investigations were obtained

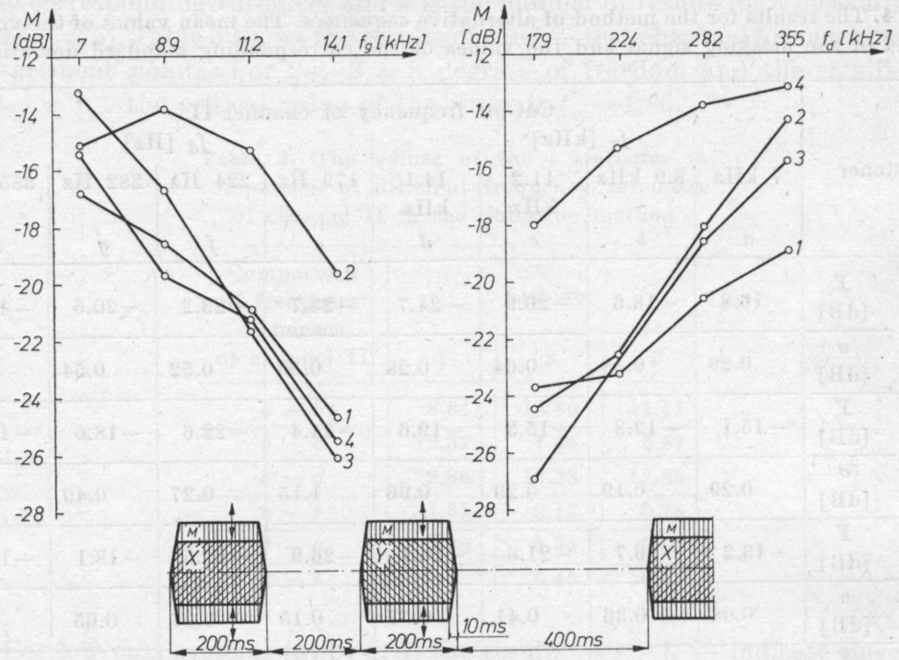

Fig. 5. The results of the method of alternative sequences. The mean values of the level of the signal M as a function of the cut-off frequencies of the frequency response of channel II obtained for 4 listeners

Table 5. The values of the t statistics in comparison of different frequency responses of channel II in the method of alternative sequences

Comparison of frequency responses of channel II	Listener 1	2	3	4
$a - b$	5.69	10.95	22.14	9.74
$b - c$	5.15	9.25	17.28	4.98
$c - d$	8.88	9.62	12.83	12.27
$e - f$	1.33	3.07	9.77	4.52
$f - g$	6.49	11.97	8.54	3.63
$g - h$	3.47	7.72	11.10	1.54

using uniformly masking noise as the masking noise. The use of pink or white noise decreased the sensitivity of differentiating objects.

2. In the case of the pulsation method the characteristics of changes in the pulsation threshold as a function of the cut-off frequency of channel II are very close to one another for all the three listeners. The differences between the values of the pulsation threshold obtained for particular listeners do not exceed 1.1 dB.

3. In the case of the pulsation method a 1/3 octave change in the upper cut-off frequency causes an essential change in the value of the sensation threshold. This method can thus serve to differentiate the frequency responses of the electroacoustic channel over the high frequency range. The determination of the sensitivity of the method requires further investigations.

4. Fig. 6 shows the mean values of the pulsation threshold calculated on the basis of the results for 3 listeners. The response of the pulsation threshold obtained as a function of frequency was approximated by linear regression

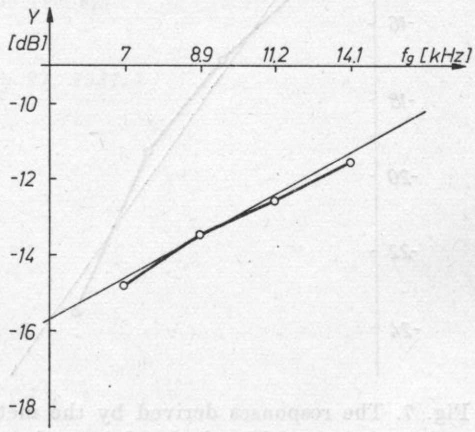

Fig. 6. The response of the pulsation threshold as a function of the cut-off frequency f_g of channel II, calculated from the results for 3 listeners (thick line) and its linear approximation (thin line)

(with the correlation coefficient $r = 0.996$). The directional coefficient of the straight line is 1.081. The approximation shown in Fig. 6 is based on the results obtained for 4 values of f_g of channel II. However, the determination of the frequency response of the pulsation threshold requires further investigation with denser measurement points (with a change in t_g by values less than 1/3 octave).

5. The values of the pulsation threshold obtained with adjustment of the lower cut-off frequency indicate a low differentiating sensitivity. The results obtained are characterized by very high repeatability (low variance), which in turn indicates the sharp character of the occurrence of the pulsation threshold.

6. In the case of alternative sequences III a 1/3 octave change in the cut-off frequency response of channel II causes an essential change in the value of the signal M required to mask the differences between the signals X and Y. The responses obtained for particular listeners (Fig. 5) are however, much more differentiated than those obtained using the pulsation method. It follows from Fig. 5 that with upper limiting of the transmission band the curves derived for listeners 1, 3 and 4 are close. In turn, the curve for listener 2, which is different from them, seems to indicate his sharper differentiation of timbre changes over the high frequency range. Analogously, with lower limiting of the signal, listener 4 shows a better than average differentiation of timbre.

7. Fig. 7 shows the mean responses obtained by the method of alternative sequences. With lower limiting of the band, linear regression (with the correlation coefficient $r = 0.999$) gives the directional coefficient of 2.181. With upper limiting, linear regression (with the correlation coefficient $r = 0.982$) gives

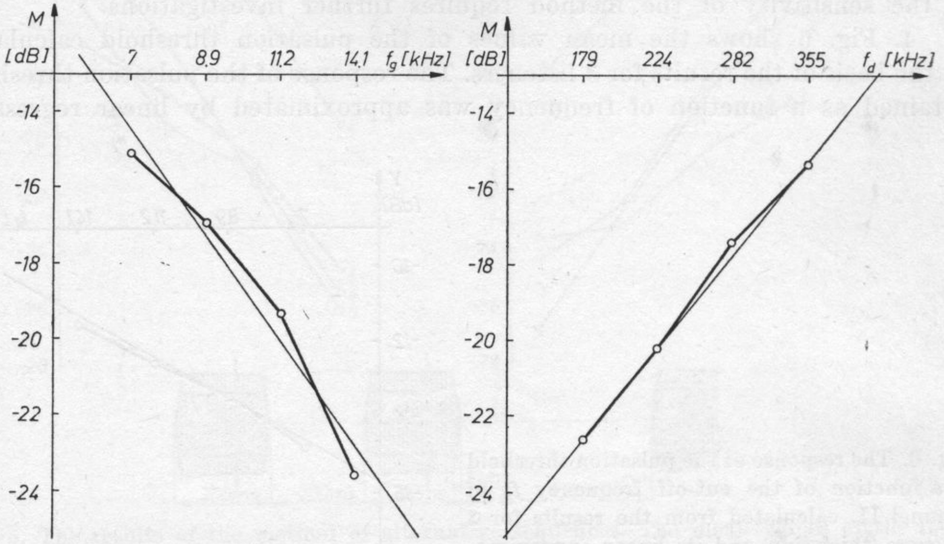

Fig. 7. The responses derived by the method of alternative sequences as a function of f_g and f_d of channel II, calculated from the results for 4 listeners (thick lines) and their linear approximation (thin lines)

the directional coefficient of − 2.802. The determination of the exact behaviour of the frequency responses obtained by the method of alternative sequences requires further investigation

8. The results obtained suggest that the method of alternative sequences assures better sensitivity of auditory differentiation of spectrum changes than the pulsation method does.

9. On the basis of the investigations carried out, it seems that the method of alternative sequences can be used to differentiate the frequency responses of electroacoustic devices.

References

[1] C. Bordene-Sacerdote, C. Modena, *Prove d'ascolto su sistemi di altoparlanti*, Elettronica a Telecomunicazioni, **6**, 1-20 (1971).

[2] B. W. Kulesza, *Method of auditory evaluation of radio sets in industrial conditions* (in Polish), Proc. XXII Open Seminar on Acoustics, Świeradów 1975.

[3] T. Łętowski, *Auditory evaluation of electroacoustic devices* (in Polish), Zesz. Nauk. PRiTV, 27 (1976).

[4] W. A. Munson, J. W. Karlin, *Isopreference method for evaluating speech transmission circuits,* J. Acoust. Soc. Am., **34**, 762-774 (1962).

[5] L. H. Nakatani, K. D. Dukes, *A sensitive test of speech communication quality,* J. Acoust. Soc. Am., **53**, 1083-1092 (1973).

[6] L. C. W. Pols, L. J. T. van der Kamp, R. Plomp, *Perceptual and physical space of vowel sounds,* J. Acoust. Soc. Am., **46**, 458-467 (1969).

[7] D. L. Richards, *Design and analysis of subjective acoustical experiments which involve a quantal response,* Acoustica, **2**, 83 (1952).

[8] E. M. Rothauser, G. E. Urbanek, W. P. Pachl, *Isopreference method for speech evaluation,* J. Acoust. Soc. Am., **44**, 408-418 (1968).

[9] S. G. Stevens, *Cross modality validation of subjective scales for loudness vibration and electric shock,* J. Exp. Psychol., **57**, 201-209 (1959).

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THE METHOD OF THE DEFORMATION OPERATOR IN QUANTUM ACOUSTICS — A FORMULATION OF PERTURBATION CALCULUS

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This paper gives a formulation of perturbation calculus which is useful for the description of coherent states related to the propagation of ultrasonic waves in crystals. This formulation is based on the results of the theory of coherent states, particularly on the properties of the deformation operator. The method of the construction of the initial state, which is used in perturbation calculus, is verified through comparison with the results of the method of the quasi-equilibrium density matrix based on the use of information theory in statistical physics. The method of perturbation calculus which is presented in this paper describes the time dependence of the mean value of any physical quantity for a crystal which undergoes dynamic deformation. This method makes it possible to grasp the dependence of phenomena observed on the phase and amplitude of the initially excited acoustic wave.

1. Introduction

This investigation used the properties of the deformation operator which is known from the theory of coherent states [1, 3, 6, 7, 9] to formulate perturbation calculus which describes effects related to the propagation of travelling ultrasonic waves in crystals. The method of the construction of the initial state, to be used in perturbation calculus, was verified through comparison with the results of the method of the quasi-equilibrium density matrix [2].

It was found that both methods lead to similar results, with the results being the same in the case of harmonic crystals. The difference which occurs in the general case is related to the fact that the method of the deformation operator does not account for the irreversible processes which occur in the dynamic deformation of crystals. An essential advantage of the method of the deformation operator is that it leads directly to a formulation of perturbation calculus.

The considerations given in this paper concern an infinite continuous medium; they can, however, be naturally extended to the case of a continuous medium.

2. Method of the deformation operator

The method of the deformation operator consists in the description of medium deformation by means of a unitary operator,

$$D(\{a_{q,p}\}) = \exp\Big[\sum_{q,p} (a_{q,p}\hat{a}^{+}_{q,p} - \bar{a}_{q,p}\hat{a}_{q,p})\Big], \tag{1}$$

where q and p are wave vectors and phonon vibration branches [4, 5], $\hat{a}_{q,p}$ and $\hat{a}^{+}_{q,p}$ are the operators of phonon annihilation and creation [4, 5], $a_{q,p}$ are complex numbers, and the dash over the variable denotes its complex conjugate.

As a result of crystal deformation, the wave function ψ which describes the state of the crystal before deformation passes into the function $\hat{D}(\{a_{q,p}\})\psi$; thus, the deformation of the crystal described by the density matrix $\hat{\varrho}$ corresponds to the transformation of the density matrix

$$\hat{\varrho} \to \hat{D}(\{a_{q,p}\})\hat{\varrho}\hat{D}^{-1}(\{a_{q,p}\}). \tag{2}$$

Paper [3] gave a deformation operator which corresponds to static deformation; the considerations in [3] lead to the determination of the value of the coefficients $a_{q,p}$,

$$a_{q,p} = -\sum_{b,l,j} u\binom{l}{b}_j i\sqrt{\frac{m_b\omega(q,p)}{2NV\hbar}}\,\bar{e}\binom{q}{b}_{j,p} \exp(-iql), \tag{3}$$

where N is the number of elementary cells in unit volume, V is the volume of the crystal, l is a vector which defines the elementary cell [4, 5], i is imaginary unity (i.e. $i = \sqrt{-1}$), m_b is the mass of the atom which occupies the place in the elementary cell defined by the vector b [4, 5], $u\binom{l}{b}_j$ is the jth component of the displacement of the corresponding atom, related to the deformation of the crystal, $e\binom{q}{b}_{j,p}$ is a complex polarisation vector of the vibration mode corresponding to the wave vector q and the vibration branch p [4, 5], $\omega(q, p)$ is the vibration frequency in the particular mode, and $\hbar$ is the Planck constant divided by 2π.

From the point of view of acoustics, the deformation operator with parameters defined by formula (3) has nevertheless a serious disadvantage. The initial stage which it serves to derive is useless in describing travelling waves.

This can be shown in the case of a harmonic crystal which is in a state of termodynamic equilibrium before deformation. Let $\hat{\varrho}$ be a density matrix which describes the crystal in a state of termodynamic equilibrium. The dependence of the deformation field on time can be found in the case when at $t = 0$ the crystal undergoes deformation which is represented by the deformation operator with parameters determined by formula (3).

The use of formula (2), the representation of the deformation field operator by the operators of phonon creation and annihilation [4, 5],

$$\hat{R}\binom{\boldsymbol{l}}{\boldsymbol{b}}_j = \sum_{\boldsymbol{q},p} \bar{e}\binom{\boldsymbol{q}}{\boldsymbol{b}}_{j,p} i\sqrt{\frac{\hbar}{2NVm_{\boldsymbol{b}}\,\omega(\boldsymbol{q},p)}}\,(\hat{a}_{-\boldsymbol{q},p} - \hat{a}^+_{\boldsymbol{q},p})\exp(-i\boldsymbol{q}\boldsymbol{i}), \tag{4}$$

where the notation is as in (3), the invariance of the trace with respect to the cyclic representation of the operators and formulae which follow from the commutation rules [6],

$$\begin{aligned} \hat{D}^{-1}(\{\alpha_{\boldsymbol{q},p}\})\,\hat{a}_{\boldsymbol{k},g}\,\hat{D}(\{\alpha_{\boldsymbol{q},p}\}) &= \hat{a}_{\boldsymbol{k},g} + \alpha_{\boldsymbol{k},g};\\ \hat{D}^{-1}(\{\alpha_{\boldsymbol{q},p}\})\,\hat{a}^+_{\boldsymbol{k},g}\,\hat{D}(\{\alpha_{\boldsymbol{q},p}\}) &= \hat{a}_{\boldsymbol{k},g} + \bar{\alpha}_{\boldsymbol{k},g}; \end{aligned} \tag{5}$$

give

$$\begin{aligned} \left\langle\left\langle \hat{R}\binom{\boldsymbol{l}}{\boldsymbol{b}}_j(t)\right\rangle\right\rangle &= \mathrm{Tr}\left\{\hat{D}(\{\alpha_{\boldsymbol{q},p}\})\,\hat{\varrho}\,\hat{D}^{-1}(\{\alpha_{\boldsymbol{q},p}\})\hat{R}\binom{\boldsymbol{l}}{\boldsymbol{b}}_j(t)\right\}\\ &= \mathrm{Tr}\left\{\hat{\varrho}\,\hat{D}^{-1}(\{\alpha_{\boldsymbol{q},p}\})\hat{R}\binom{\boldsymbol{l}}{\boldsymbol{b}}_j(t)\hat{D}(\{\alpha_{\boldsymbol{q},p}\})\right\}\\ &= \sum_{\boldsymbol{q},p}\bar{e}\binom{\boldsymbol{q}}{\boldsymbol{b}}_{j,p} i\sqrt{\frac{\hbar}{2NVm_{\boldsymbol{b}}\,\omega(\boldsymbol{q},p)}}\{\alpha_{-\boldsymbol{q},p}\exp[-i\omega(-\boldsymbol{q},p)t] -\\ &\qquad - \bar{\alpha}_{\boldsymbol{q},p}\exp[i\omega(\boldsymbol{q},p)t]\exp(-i\boldsymbol{q}\boldsymbol{l}), \end{aligned} \tag{6}$$

where $\langle\langle\hat{A}\rangle\rangle$ is the mean value of the operator $\hat{A}$ and $\mathrm{Tr}\{\hat{A}\}$ is the trace of this operator.

When Q denotes such a set of wave vectors that out of each pair $\{\boldsymbol{q}, -\boldsymbol{q}\}$ strictly one vector belongs to Q, the mean (6) can be written in the form

$$\begin{aligned} \left\langle\left\langle \hat{R}\binom{\boldsymbol{l}}{\boldsymbol{b}}_j(t)\right\rangle\right\rangle &= \sum_{p,\boldsymbol{q}\in Q} i\sqrt{\frac{\hbar}{2NVm_{\boldsymbol{b}}\,\omega(\boldsymbol{q},p)}}\left\{\bar{e}\binom{\boldsymbol{q}}{\boldsymbol{b}}_{j,p}\alpha_{-\boldsymbol{q},p}\exp[-i\omega(-\boldsymbol{q},p)t]\times\right.\\ &\times\exp(-i\boldsymbol{q}\boldsymbol{l}) - \bar{e}\binom{\boldsymbol{q}}{\boldsymbol{b}}_{j,p}\bar{\alpha}_{\boldsymbol{q},p}\exp[i\omega(\boldsymbol{q},p)t]\exp(-i\boldsymbol{q}\boldsymbol{l}) +\\ &+\bar{e}\binom{\boldsymbol{q}}{\boldsymbol{b}_{j,p}}\alpha_{\boldsymbol{q},p}\exp[-i\omega(\boldsymbol{q},p)t]\exp(i\boldsymbol{q}\boldsymbol{l}) - \bar{e}\binom{-\boldsymbol{q}}{\boldsymbol{b}}_{j,p}\bar{\alpha}_{-\boldsymbol{q},p}\exp[i\omega(-\boldsymbol{q},p)t]\times\\ &\left.\times\exp(i\boldsymbol{q}\boldsymbol{l})\right\}. \end{aligned} \tag{7}$$

Taking into account the identities [4, 5]

$$\omega(-\boldsymbol{q}, p) = \omega(\boldsymbol{q}, p); \qquad \bar{e}\begin{pmatrix} -\boldsymbol{q} \\ \boldsymbol{b} \end{pmatrix}_{j,p} = e\begin{pmatrix} \boldsymbol{q} \\ \boldsymbol{b} \end{pmatrix}_{j,p} \tag{8}$$

and the following relation which results from (3)

$$\bar{a}_{\boldsymbol{q},p} = -a_{-\boldsymbol{q},p}, \tag{9}$$

gives

$$\left\langle\left\langle \hat{R}\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j (t) \right\rangle\right\rangle = \sum_{p,\boldsymbol{q}\in Q} -\sqrt{\frac{\hbar}{NVm_{\boldsymbol{b}}\omega(\boldsymbol{q},p)}}\, \mathrm{Im}\left\{e\begin{pmatrix} \boldsymbol{q} \\ \boldsymbol{b} \end{pmatrix}_{j,p} a_{\boldsymbol{q},p}\exp(i\boldsymbol{q}\boldsymbol{l})\right\}\cos\omega(\boldsymbol{q},p)t, \tag{10}$$

where Im is the imaginary part of the complex number.

In expression (10) no more than one standing wave corresponds to each vibration mode and to each pair $\{\boldsymbol{q}, -\boldsymbol{q}\}$. It thus follows that the initial state which leads to the formation of a single travelling wave in the crystal cannot be derived with deformation represented by the deformation operator with the parameters $a_{\boldsymbol{q},p}$ defined by formula (3).

An attempt can now be made to modify the present procedure so that it may also be used in the case of tne travelling wave. The analogy to quantum optics where arbitrary complex numbers can become parametres which occur in the deformation operator may be used for this purpose [7, 8].

Let $R'\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j$ and $P'\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j$ denote predetermined values of displacement and momentum of atoms, let $\hat{D}(\{a_{\boldsymbol{q},p}\})$ be a deformation operator with arbitrary complex coefficients (particularly those which do not necessarily satisfy relation (9)) and let $\hat{\varrho}$ denote some density matrix which describes the crystal. The mean values of position and momentum of the atoms in the state $\hat{\varrho}$ will be represented by $r\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j$ and $p\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j$.

The parameters of the deformation operator can be determined so that the operation

$$\hat{\varrho} \to \hat{D}(\{a_{\boldsymbol{q},p}\})\hat{\varrho}\,\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})$$

leads to a change in the mean values of the position of the atoms from $r\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j$ to $r\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j + R'\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j$ and a change in the mean momentum values from $p\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j$ to $p\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j + P'\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j$. The use of formulae (5) and (8) and the representation of the momentum operators by the operators of phonon creation and annihilation [4, 5]

$$\hat{P}\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j = \sum_{\boldsymbol{q},p} \sqrt{\frac{m_b \hbar\omega(\boldsymbol{q}, p)}{2NV}} \left\{ e\begin{pmatrix} \boldsymbol{q} \\ \boldsymbol{b} \end{pmatrix}_{j,p} \hat{a}_{\boldsymbol{q},p} \exp(i\boldsymbol{q}\boldsymbol{l}) + \bar{e}\begin{pmatrix} \boldsymbol{q} \\ \boldsymbol{b} \end{pmatrix}_{j,p} \hat{a}^+_{\boldsymbol{q},p} \exp(-i\boldsymbol{q}\boldsymbol{l}) \right\}, \tag{11}$$

give

$$\left\langle\left\langle \hat{R}\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j \right\rangle\right\rangle = \mathrm{Tr}\left\{ \hat{D}(\{a_{\boldsymbol{q},p}\})\hat{\varrho}\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})\hat{R}\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j \right\}$$

$$= \mathrm{Tr}\{\hat{\varrho}\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})\hat{R}\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j \hat{D}(\{a_{\boldsymbol{q},p}\})\}$$

$$= r\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j + \sum_{\boldsymbol{q},p} \bar{e}\begin{pmatrix} \boldsymbol{q} \\ \boldsymbol{b} \end{pmatrix}_{j,p} i\sqrt{\frac{\hbar}{2NVm_{\boldsymbol{b}}\omega(\boldsymbol{q}, p)}} (a_{-\boldsymbol{q},p} - \bar{a}_{\boldsymbol{q},p}) \exp(-i\boldsymbol{q}\boldsymbol{l});$$

$$\left\langle\left\langle \hat{P}\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j \right\rangle\right\rangle = \mathrm{Tr}\left\{ \hat{D}(\{a_{\boldsymbol{q},p}\})\hat{\varrho}\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})\hat{P}\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j \right\} = \mathrm{Tr}\{\hat{\varrho}\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\}) \times$$

$$\times \hat{P}\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j \hat{D}(\{a_{\boldsymbol{q},p}\})\}$$

$$= p\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j + \sum_{\boldsymbol{q},p} \sqrt{\frac{m_b \hbar\omega(\boldsymbol{q}, p)}{2NV}} e\begin{pmatrix} \boldsymbol{q} \\ \boldsymbol{b} \end{pmatrix}_{j,p} (a_{\boldsymbol{q},p} + \bar{a}_{-\boldsymbol{q},p}) \exp(i\boldsymbol{q}\boldsymbol{l}). \tag{12}$$

From the system of equations

$$\sum_{\boldsymbol{q},p} \bar{e}\begin{pmatrix} \boldsymbol{q} \\ \boldsymbol{b} \end{pmatrix}_{j,p} i\sqrt{\frac{\hbar}{2NVm_{\boldsymbol{b}}\omega(\boldsymbol{q}, p)}} (a_{\boldsymbol{q},p} - \bar{a}_{\boldsymbol{q},p}) \exp(-i\boldsymbol{q}\boldsymbol{l}) = R'\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j;$$

$$\sum_{\boldsymbol{q},p} e\begin{pmatrix} \boldsymbol{q} \\ \boldsymbol{b} \end{pmatrix}_{j,p} \sqrt{\frac{\hbar}{2NVm_{\boldsymbol{b}}\omega(\boldsymbol{q}, p)}} (a_{\boldsymbol{q},p} + \bar{a}_{-\boldsymbol{q},p}) \exp(i\boldsymbol{q}\boldsymbol{l}) = P'\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j; \tag{13}$$

the following formula can be obtained

$$a_{\boldsymbol{q},p} = \sum_{\boldsymbol{l},\boldsymbol{b},j} \left\{ \sqrt{\frac{1}{2NVm_{\boldsymbol{b}}\hbar\omega(\boldsymbol{q}, p)}} P'\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j - i\sqrt{\frac{m_b\omega(\boldsymbol{q}, p)}{2NV\hbar}} R'\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j \right\} \bar{e}\begin{pmatrix} \boldsymbol{q} \\ \boldsymbol{b} \end{pmatrix}_{j,p} \times$$

$$\times \exp(-i\boldsymbol{q}\boldsymbol{l}). \tag{14}$$

It can be seen that in a specific case of static deformation when $P'\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j = 0$ formula (14) can be reduced to (3). In a general case $P'\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j \neq 0$ and the parameters $a_{\boldsymbol{q},p}$ defined by formula (4) can take any complex values. It can readily be shown that the deformation operator with its parameters defined by (14) can describe perturbations which lead to the formation of travelling waves in the crystal. The assumption of a harmonic crystal and the repetition of the train of thought which previously gave formula (10) now give

$$\left\langle\left\langle \hat{R}\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j (t) \right\rangle\right\rangle = -\sum_{\boldsymbol{q},p} \sqrt{\frac{2\hbar}{NVm_{\boldsymbol{b}}\omega(\boldsymbol{q},p)}}\ \mathrm{Im}\left\{a_{\boldsymbol{q},p} e\begin{pmatrix} \boldsymbol{q} \\ \boldsymbol{b} \end{pmatrix}_{j,p} \exp\{-i[\omega(\boldsymbol{q},p)t - \boldsymbol{q}\boldsymbol{l}]\}\right\}. \tag{15}$$

In particular, the choice of $a_{\boldsymbol{q},p} = \delta_{\boldsymbol{q},\boldsymbol{d}}\delta_{f,s}$ ($\delta_{\boldsymbol{q},\boldsymbol{d}}$ and $\delta_{p,s}$ denote particular Kronecker deltas) leads to an evolution of the deformation field which corresponds to the propagation of a single S-type travelling wave with the wave vector $\boldsymbol{d}$. It can thus be seen that in effect the present procedure of the "imposition" on the crystal of arbitrary fields of the displacements $R'\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j$ and velocities $P'\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j / m_{\boldsymbol{b}}$ of the atoms permits the construction of initial states which are useful for describing travelling waves.

This approach is extremely convenient, since it will be seen that it leads in a natural way to the formulation of perturbation calculus.

Let the crystal in a state described by the density matrix undergo at a time $t = 0$ dynamic deformation represented by the deformation operator $\hat{D}(\{a_{\boldsymbol{q},p}\})$, where the parameters $a_{\boldsymbol{q},p}$ are defined by predetermined fields of displacement and momentum of atoms according to formula (14). After deformation the crystal is therefore described by the density matrix $\hat{D}(\{a_{\boldsymbol{q},p}\})\hat{\varrho}\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})$ and thus the mean value of the arbitrary operator $\hat{A}$ at a time t, $t > 0$, is

$$\langle\langle \hat{A}(t) \rangle\rangle = \frac{\mathrm{Tr}\{\hat{D}(\{a_{\boldsymbol{q},p}\})\hat{\varrho} D^{-1}(\{a_{\boldsymbol{q},p}\})\hat{A}(t)\}}{\mathrm{Tr}\{\hat{D}(\{a_{\boldsymbol{q},p}\})\hat{\varrho}\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})} = \frac{\mathrm{Tr}\left\{\hat{D}(\{a_{\boldsymbol{q},p}\})\hat{\varrho}\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})\exp\left(\frac{i}{\hbar}Ht\right) A \exp\left(-\frac{i}{\hbar}Ht\right)\right\}}{\mathrm{Tr}\{\hat{D}\{(a_{\boldsymbol{q},p}\})\hat{\varrho}\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})\}}. \tag{16}$$

Cyclic transpositions under the trace sign and the use of the unitariness of the deformation operator give

$$\langle\langle A(t) \rangle\rangle = \frac{\mathrm{Tr}\left\{\hat{D}(\{a_{\boldsymbol{q},p}\})\hat{\varrho}\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})\exp\left(\frac{i}{\hbar}Ht\right) A \exp\left(-\frac{i}{\hbar}Ht\right)\right\}}{\mathrm{Tr}\{\hat{D}(\{a_{\boldsymbol{q},p}\})\hat{\varrho}\hat{D}^{-1}(a\{_{\boldsymbol{q},p}\})\}}$$

$$= \mathrm{Tr}\left\{\hat{\varrho}\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})\exp\left(\frac{i}{\hbar}\hat{H}t\right)\hat{D}(\{a_{\boldsymbol{q},p}\})\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})\hat{A}\hat{D}(\{a_{\boldsymbol{q},p}\})\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\}) \times \times\exp\left(-\frac{i}{\hbar}Ht\right)\hat{D}(\{a_{\boldsymbol{q},p}\})\right\}$$

$$= \mathrm{Tr}\left\{\hat{\varrho}\exp\left[\frac{i}{\hbar}\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})\hat{H}\hat{D}(\{a_{\boldsymbol{q},p}\})t\right]\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})\hat{A}\hat{D}(\{a_{\boldsymbol{q},p}\})\times \right. \tag{17}$$
$$\left.\times\exp\left[-\frac{i}{\hbar}\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})\hat{H}\hat{D}(\{a_{\boldsymbol{q},p}\})t\right]\right\}.$$

It can be seen that formally expression (17) can be interpreted as the mean value of the operator $\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})\hat{A}\hat{D}(\{a_{\boldsymbol{q},p}\})$ at a time $t, t > 0$, when the time dependence of the operator $\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})\hat{A}\hat{D}(\{a_{\boldsymbol{q},p}\})$ is defined by Heisenberg representation corresponding to the "hamiltonian" $\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})\hat{H}\hat{D}(\{a_{\boldsymbol{q},p}\})$ and the state of the crystal is described by the density matrix $\hat{\varrho}$.

All hamiltonians which occur in quantum acoustics are, in terms of the operators of phonon creation and annihilation, polynomials of finite order. In this case, it follows from formulae (5) that the "new" hamiltonian $\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})\hat{H}\hat{D}(\{a_{\boldsymbol{q},p}\})$ consists of the "old" hamiltonian $\hat{H}$ and the "remainder" $\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})\hat{H}\hat{D}(\{a_{\boldsymbol{q},p}\})-\hat{H}$. The remainder is the sum of terms each of which, in terms of the numbers $a_{\boldsymbol{q},p}$ and $\bar{a}_{\boldsymbol{q},p}$, is a polynomial of at least the first order.

In the case when

$$\hat{H}' = \hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})\hat{H}\hat{D}(\{a_{\boldsymbol{q},p}\})-\hat{H} \tag{18}$$

is much smaller than $\hat{H}$, it is possible, treating $\hat{H}'$ as perturbation and $\hat{H}$ as unperturbed hamiltonian, to represent the mean value of the operator $\hat{A}$ for $t > 0$ by the response of the system to the sudden introduction of the perturbation $\hat{H}'$ at a time $t = 0$ [2],

$$\langle\!\langle \hat{A}(t)\rangle\!\rangle = \mathrm{Tr}\left\{\hat{\varrho}\exp\left(\frac{i}{\hbar}\hat{H}t\right)\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})\hat{A}\hat{D}(\{a_{\boldsymbol{q},p}\})\exp\left(-\frac{i}{\hbar}\hat{H}t\right)\right\}+$$
$$+\sum_{n=1}^{\infty}\left(\frac{1}{i\hbar}\right)^n\int_0^t\int_0^{t_1}\dots\int_0^{t_{n-1}}dt_1\dots dt_n\,\mathrm{Tr}\{(\hat{D}^{-1}(\{a_{\boldsymbol{q},p}\})\hat{A}\hat{D}(\{a_{\boldsymbol{q},p}\}))t\times$$
$$\times[\hat{H}'_{t_1}[\dots[\hat{H}'_{t_n},\hat{\varrho}]\dots]\}. \tag{19}$$

(The time dependence of the operators which occur in the subintegral function is defined by Heisenberg representation corresponding to the hamiltonian $\hat{H}$.)

3. Method of the quasi-equilibrium density matrix

The method of the quasi-equilibrium density matrix [2] can serve to determine the statistical quantum state representing a crystal with predetermined mean values of the displacement and velocity fields and with predetermined mean energy. In terms of information theory it can be stated that the task is to eliminate "excess" information (related to the description of the microscopic state of the system) from the density matrix describing the crystal and at the same time to retain that information which corresponds to the given mean values of the predetermined physical quantities.

The solution results from the taking into account of the fact that in practice the crystal is never fully isolated from the environment which plays the role of the thermostat. The interaction between the crystal and the thermostat is random and leads to part of the information contained in the density matrix being "forgotten" [2]. (In classical statistical physics this process involves a "fuzziness" of the trajectories describing the evolution of the system in the phase space, which leads to a smoothing of the distribution function).

The information which corresponds to the predetermined mean values of displacement and momentum of atoms and the predetermined mean energy of the crystal is imposed by the generator exciting ultrasonic waves and the temperature of the thermostat, and therefore it is not "forgotten". It can thus be seen that the density matrix of interest can be determined by the variational method, seeking such a statistical state to which there corresponds the maximum value of information entropy [2]

$$S = -\mathrm{Tr}\{\hat{\varrho}\ln\hat{\varrho}\}, \tag{20}$$

under the condition of a definite mean value of the total energy of the crystal

$$\langle\!\langle \hat{H} \rangle\!\rangle = E, \tag{21}$$

the conditions corresponding to the predetermined mean values of the displacement of the atoms from the equilibrium positions

$$\left\langle\!\left\langle \hat{R}\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j \right\rangle\!\right\rangle = R\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j, \tag{22}$$

the conditions corresponding to the predetermined mean values of the momentum of the atoms

$$\left\langle\!\left\langle \hat{P}\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j \right\rangle\!\right\rangle = P\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j \tag{23}$$

and the condition of normalisation of the density matrix

$$\langle\!\langle \hat{\varrho} \rangle\!\rangle = 1. \tag{24}$$

The solution of the present variational problem is the density matrix [2]

$$\hat{\varrho} = \exp\left[-\Phi - \beta\hat{H} - \sum_{\boldsymbol{l},\boldsymbol{b},j} f\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j \hat{R}\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j - \sum_{\boldsymbol{l},\boldsymbol{b},j} h\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j \hat{P}\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j\right], \tag{25}$$

where $f\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j$ are Lagrange multipliers corresponding to the conditions of the predetermined mean values of the displacement of atoms, $h\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j$ are Lagrange multipliers corresponding to the conditions of the predetermined mean values of the momentum of atoms, β is a Lagrange multiplier corresponding to the condition of the predetermined mean energy of the crystal and with interpretation $1/kT$ (k being a Boltzmann constant and T the absolute temperature of the crystal), and $\Phi - 1$ is a Lagrange multiplier corresponding to the condition of normalisation of the density matrix.

It can readily be shown that differentiation of $-\Phi$ with respect to β, $h\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j$ and $f\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j$ gives respectively the mean value of the total energy of the crystal, the mean values of the momentum of atoms and the mean values of the displacement of atoms from the equilibrium positions. It can be seen that in a specific case when the predetermined mean values of the momentum of atoms and the mean values of the displacement of atoms from the equilibrium positions are zero, the Lagrange factors $h\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j$ and $f\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j$ are also zero and the density matrix (25) describes the state of thermodynamic equilibrium (corresponding to the canonical distribution).

4. Comparison of the results of the two methods

The results obtained by the method of the deformation operator and by the method of the quasi-equilibrium density matrix can first be compared for the case of a harmonic crystal. The method of the deformation operator can serve to find a density matrix which describes a dynamically deformed harmonic crystal which before deformation was in the state of thermodynamic equilibrium described by the canonical distribution.

Consideration of formulae (2) and (5) and the identity, which results from definition (1),

$$\hat{D}(\{a_{\boldsymbol{q},p}\}) = \hat{D}^{-1}(\{-a_{\boldsymbol{q},p}\}), \tag{26}$$

gives

$$\begin{aligned} \hat{\varrho} &= \hat{D}(\{a_{\boldsymbol{q},p}\}) \exp\Big[-\Phi - \beta \sum_{\boldsymbol{k},g} \hbar\omega(\boldsymbol{k}, g)\, \hat{a}^{+}_{\boldsymbol{k},g} \hat{a}_{\boldsymbol{k},g}\Big] \hat{D}^{-1}(\{a_{\boldsymbol{q},p}\}) \\ &= \hat{D}^{-1}(\{-a_{\boldsymbol{q},p}\}) \exp\Big[-\Phi - \beta \sum_{\boldsymbol{k},g} \hbar\omega(\boldsymbol{k}, g)\, \hat{a}^{+}_{\boldsymbol{k},g} \hat{a}_{\boldsymbol{k},g}\Big] \hat{D}(\{-a_{\boldsymbol{q},p}\}) \\ &= \frac{\exp\big[-\beta \sum_{\boldsymbol{k},g} \hbar\omega(\boldsymbol{k}, g)\, \hat{a}^{+}_{\boldsymbol{k},g} \hat{a}_{\boldsymbol{k},g} + \beta \sum_{\boldsymbol{k},g} a_{\boldsymbol{k},g} \hat{a}^{+}_{\boldsymbol{k},g} + \beta \sum_{\boldsymbol{k},g} \bar{a}_{\boldsymbol{k},g} \hat{a}_{\boldsymbol{k},g}\big]}{\mathrm{Tr}\big\{\exp\big[-\beta \sum_{\boldsymbol{k},g} \hbar\omega(\boldsymbol{k}, g)\, \hat{a}^{+}_{\boldsymbol{k},g} \hat{a}_{\boldsymbol{k},g} + \beta \sum_{\boldsymbol{k},g} a_{\boldsymbol{k},g} \hat{a}^{+}_{\boldsymbol{k},g} + \beta \sum_{\boldsymbol{k},g} \bar{a}_{\boldsymbol{k},g} \hat{a}_{\boldsymbol{k},g}\big]\big\}}, \end{aligned} \tag{27}$$

(the values of the parameters $a_{\boldsymbol{q},p}$ in (27) are defined by formulae (14) and the other notation remains as above).

Using formula (4) and (11) the quasi-equilibrium density matrix which describes a dynamically deformed harmonic crystal can be expressed by the operators of phonon creation and annihilation

$$\hat{\varrho} = \exp\left[-\Phi' - \beta' \sum_{\boldsymbol{k},g} \hbar\omega(\boldsymbol{k},g)\,\hat{a}^+_{\boldsymbol{k},g}\hat{a}_{\boldsymbol{k},g} - \sum_{\boldsymbol{k},g} f\begin{pmatrix}\boldsymbol{l}\\ \boldsymbol{b}\end{pmatrix}_j \hat{R}\begin{pmatrix}\boldsymbol{l}\\ \boldsymbol{b}\end{pmatrix}_j - \sum_{\boldsymbol{k},g} h\begin{pmatrix}\boldsymbol{l}\\ \boldsymbol{b}\end{pmatrix}_j \hat{P}\begin{pmatrix}\boldsymbol{l}\\ \boldsymbol{b}\end{pmatrix}_j\right.$$

$$= \frac{\exp\left[-\beta' \sum_{\boldsymbol{k},g} \hbar\omega(\boldsymbol{k},g)\,\hat{a}^+_{\boldsymbol{k},g}\hat{a}_{\boldsymbol{k},g} - \sum_{\boldsymbol{k},g} \bar{\gamma}_{\boldsymbol{k},g}\hat{a}_{\boldsymbol{k},g} - \sum_{\boldsymbol{k},g}\gamma_{\boldsymbol{k},g}\hat{a}^+_{\boldsymbol{k},g}\right]}{\mathrm{Tr}\left\{\exp\left[-\beta' \sum_{\boldsymbol{k},g} \hbar\omega(\boldsymbol{k},g)\,\hat{a}^+_{\boldsymbol{k},g}\hat{a}_{\boldsymbol{k},g} - \sum_{\boldsymbol{k},g} \bar{\gamma}_{\boldsymbol{k},g}\hat{a}_{\boldsymbol{k},g} - \sum_{\boldsymbol{k},g}\gamma_{k,g}\hat{a}^+_{k,g}\right]\right\}}, \tag{28}$$

where

$$\gamma_{\boldsymbol{k},g} = \sum_{\boldsymbol{b},\boldsymbol{l},j}\left(-f\begin{pmatrix}\boldsymbol{l}\\ \boldsymbol{b}\end{pmatrix}_j i\sqrt{\frac{\hbar}{2NVm_{\boldsymbol{b}}\omega(\boldsymbol{k},g)}} + h\begin{pmatrix}\boldsymbol{l}\\ \boldsymbol{b}\end{pmatrix}_j\sqrt{\frac{m_{\boldsymbol{b}}\hbar\omega(\boldsymbol{k},g)}{2NV}}\;\bar{e}\begin{pmatrix}\boldsymbol{k}\\ \boldsymbol{b}\end{pmatrix}_{j,g}\times\right.$$
$$\times\exp(-i\boldsymbol{kl}). \tag{29}$$

It can be seen that the density matrices (27) and (28) have the same form and differ only in terms of parameters which occur in them. The imposition of the condition of equal mean values of the total energy of the crystal, of the position and momentum of the atoms will permit equation of particular parameters and, as a result, determination of the values of the Lagrange factors which occur in (28).

Let us calculate the mean value of the total energy of the crystal in the state described by the density matrix (27)

$$\langle\!\langle \hat{H} \rangle\!\rangle = \mathrm{Tr}\left\{\left[\sum_{\boldsymbol{k},g}\hbar\omega(\boldsymbol{k},g)\,\hat{a}^+_{\boldsymbol{k},g}\hat{a}_{\boldsymbol{k},g}\right]\hat{D}(\{\alpha_{\boldsymbol{q},p}\})\exp\left[-\Phi - \beta\sum_{\boldsymbol{k},g}\hbar\omega(\boldsymbol{k},g)\times\right.\right.$$
$$\left.\left.\times\hat{a}^+_{\boldsymbol{k},g}\hat{a}_{\boldsymbol{k},g}\right]\hat{D}^{-1}(\{\alpha_{\boldsymbol{q},p}\})\right\}$$

$$= \mathrm{Tr}\left\{\hat{D}^{-1}(\{\alpha_{\boldsymbol{q},p}\})\left[\sum_{\boldsymbol{k},g}\hbar\omega(\boldsymbol{k},g)\,\hat{a}^+_{\boldsymbol{k},g}\hat{a}_{\boldsymbol{k},g}\right]\hat{D}(\{\alpha_{\boldsymbol{q},p}\})\exp\left[-\Phi - \beta\times\right.\right.$$

$$\left.\left.\times\sum_{\boldsymbol{k},g}\hbar\omega(\boldsymbol{k},g)\,\hat{a}^+_{\boldsymbol{k},g}\hat{a}_{\boldsymbol{k},g}\right]\right\} = \sum_{\boldsymbol{k},g}\frac{\hbar\omega(\boldsymbol{k},g)}{\exp[\beta\hbar\omega(\boldsymbol{k},g)]-1} + \sum_{\boldsymbol{k},g}\hbar\omega(\boldsymbol{k},g)\,\bar{\alpha}_{\boldsymbol{k},g}\alpha_{\boldsymbol{k},g}. \tag{30}$$

Using formulae (14) and taking into account the relations serving to "diagonalize" the hamiltonian of a harmonic crystal [4, 5], the second term of expression (30) can be expressed by the changes in the mean values of the displacement and momentum of atoms caused by dynamic deformation

$$\sum_{\boldsymbol{k},g}\hbar\omega(\boldsymbol{k},g)\,\bar{\alpha}_{k,g}\alpha_{k,g} = \sum_{\boldsymbol{k},g}\frac{P^{2'}\begin{pmatrix}\boldsymbol{l}\\ \boldsymbol{b}\end{pmatrix}_j}{2m_{\boldsymbol{b}}} + \sum_{\boldsymbol{l},\boldsymbol{b},j}\frac{\partial^2 W}{\partial R\begin{pmatrix}\boldsymbol{l}\\ \boldsymbol{b}\end{pmatrix}_j \partial R\begin{pmatrix}\boldsymbol{l}'\\ \boldsymbol{b}'\end{pmatrix}_{j'}} R'\begin{pmatrix}\boldsymbol{l}\\ \boldsymbol{b}\end{pmatrix}_j R'\begin{pmatrix}\boldsymbol{l}'\\ \boldsymbol{b}'\end{pmatrix}_{j'}, \tag{31}$$

where W is the potential energy of a crystal described in adiabatic approximation [4, 5], $R'\begin{pmatrix}\boldsymbol{l}\\ \boldsymbol{b}\end{pmatrix}_j$ and $R'\begin{pmatrix}\boldsymbol{l}'\\ \boldsymbol{b}'\end{pmatrix}_j$ are atom displacements from the equilibrium positions (understood "classically", i.e. in the sense corresponding to the description of a crystal "before quantization"), and the partial derivatives are assumed for zero displacements.

It can be seen that expression (31) can be interpreted as deformation energy and that the value of the expression $\beta = 1/kT$ in formula (27) corresponds to such temperature of the crystal at which the mean energy in the state of thermodynamic equilibrium is equal to the difference between the predetermined total energy and the deformation energy. This determines $\beta = \beta'$ as a function of the mean energy of the crystal, the mean positions and momenta of the atoms. From the equation

$$\beta\hbar\omega(\boldsymbol{k}, g)\, a_{\boldsymbol{k},g} = -\gamma_{\boldsymbol{k},g} \tag{32}$$

it is possible to determine

$$\begin{aligned} h\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j &= -\sum_{\boldsymbol{k},g} \beta \sqrt{\frac{\hbar\omega(\boldsymbol{k}, g)}{2NVm_{\boldsymbol{b}}}}\, (\bar{a}_{\boldsymbol{k},g} + a_{-\boldsymbol{k},g})\, \bar{e}\begin{pmatrix} \boldsymbol{k} \\ \boldsymbol{b} \end{pmatrix}_{j,g} \exp(-i\boldsymbol{k}\boldsymbol{l}); \\ f\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j &= \sum_{\boldsymbol{k},g} i\beta \sqrt{\frac{m_{\boldsymbol{b}}\hbar\omega^3(\boldsymbol{k}, g)}{2NV}}\, (\bar{a}_{\boldsymbol{k},g} - a_{-\boldsymbol{k},g})\, \bar{e}\begin{pmatrix} \boldsymbol{k} \\ \boldsymbol{b} \end{pmatrix}_{j,g} \exp(-i\boldsymbol{k}\boldsymbol{l}). \end{aligned} \tag{33}$$

Consideration of formula (14) leads to an explicit form of the Lagrange multipliers which occur in (28),

$$\begin{aligned} h\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j &= -\frac{\beta}{m_{\boldsymbol{b}}} P'\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j, \\ f\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j &= -\beta \sum_{\boldsymbol{b},\boldsymbol{l},j} \frac{\partial^2 W}{\partial R\begin{pmatrix} \boldsymbol{l} \\ \boldsymbol{b} \end{pmatrix}_j \partial R\begin{pmatrix} \boldsymbol{l}' \\ \boldsymbol{b}' \end{pmatrix}_{j'}} R'\begin{pmatrix} \boldsymbol{l}' \\ \boldsymbol{b}' \end{pmatrix}_{j'}. \end{aligned} \tag{34}$$

(with notation as in (31) and previous formulae).

Thus, it can be seen that in the case of a harmonic crystal the two methods lead to the same results.

In a general case when the hamiltonian of the crystal is not a square function of the operators of phonon creation and annihilation, the results of the two methods are slightly different. E.g. for the hamiltonian of the form [4, 5]

$$\begin{aligned} \hat{H} &= \sum_{\boldsymbol{k},g} \hbar\omega(\boldsymbol{k}, g)\, \hat{a}^{+}_{\boldsymbol{k},g} \hat{a}_{\boldsymbol{k},g} + \\ &+ \sum_{\substack{q_1,q_2,q_3,\\ p_1,p_2,p_3,}} V\begin{pmatrix} q_1 q_2 q_3 \\ p_1 p_2 p_3 \end{pmatrix} (\hat{a}^{+}_{q_1,p_1} - \hat{a}_{-q_1,p_1})(\hat{a}^{+}_{q_2,p_2} - \hat{a}_{-q_2,p_2})(\hat{a}^{+}_{q_3,p_3} - \hat{a}_{-q_3,p_3}) \end{aligned} \tag{35}$$

the density matrix of the deformed crystal obtained by the method of the deformation operator will have the form

$$\hat{\varrho} = \hat{D}(\{\alpha_{\boldsymbol{q},p}\})\exp[-\Phi-\beta\hat{H}]\hat{D}^{-1}(\{\alpha_{\boldsymbol{q},p}\}) =$$
$$=\exp\Big\{-\Phi-\beta\sum_{\boldsymbol{k},g}\hbar\omega(\boldsymbol{k},g)\,\hat{a}^{+}_{\boldsymbol{k},g}\hat{a}_{\boldsymbol{k},g}+\beta\sum_{\boldsymbol{k},g}\hbar\omega(\boldsymbol{k},g)\,\bar{\alpha}_{\boldsymbol{k},g}\hat{a}_{\boldsymbol{k},g}+\beta\sum_{\boldsymbol{k},g}\hbar\omega(\boldsymbol{k},g)\,\alpha_{\boldsymbol{k},g}\hat{a}^{+}_{\boldsymbol{k},g}-$$
$$-\beta\sum_{\boldsymbol{k},g}\hbar\omega(\boldsymbol{k},g)\,\bar{\alpha}_{\boldsymbol{k},g}\alpha_{\boldsymbol{k},g}-\beta\sum_{\substack{\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3\\p_1,p_2,p_3}}V\begin{pmatrix}\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3\\p_1,p_2p_3\end{pmatrix}(\hat{a}^{+}_{\boldsymbol{q}_1,p_1}-\bar{\alpha}_{\boldsymbol{q}_1,p_1}\hat{a}_{-\boldsymbol{q}_1,p_1}+$$
$$+\alpha_{-\boldsymbol{q}_1p_1})(\hat{a}^{+}_{\boldsymbol{q}_2,p_2}-\bar{\alpha}_{\boldsymbol{q}_2,p_2}-\hat{a}_{-\boldsymbol{q}_2,p_2}+\alpha_{-\boldsymbol{q}_2,p_2})(\hat{a}^{+}_{\boldsymbol{q}_3,p_3}-\bar{\alpha}_{\boldsymbol{q}_3p_3}-\hat{a}_{-\boldsymbol{q}_3,p_3}+\alpha_{-\boldsymbol{q}_3,p_3})\Big\} \tag{36}$$

(the assumption being that before deformation the crystal was in the state of thermodynamic equilibrium, described by the canonical distribution).

The method of the quasi-equilibrium density matrix leads to the result

$$\hat{\varrho} = \exp\Big\{-\Phi'-\beta'\sum_{\boldsymbol{k},g}\hbar\omega(\boldsymbol{k},g)\,\hat{a}^{+}_{\boldsymbol{k},g}\hat{a}_{\boldsymbol{k},g}-\sum_{\boldsymbol{k},g}\bar{\gamma}_{\boldsymbol{k},g}\hat{a}_{\boldsymbol{k},g}-\sum_{\boldsymbol{k},g}\gamma_{\boldsymbol{k},g}\hat{a}^{+}_{\boldsymbol{k},g}-$$
$$-\beta'\sum_{\substack{\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3\\p_1,p_2,p_3}}V\begin{pmatrix}\boldsymbol{q}_1\boldsymbol{q}_2\boldsymbol{q}_3\\p_1p_2p_3\end{pmatrix}(\hat{a}^{+}_{\boldsymbol{q}_1,p_1}-\hat{a}_{-\boldsymbol{q}_1,p_1})(\hat{a}^{+}_{\boldsymbol{q}_2,p_2}-\hat{a}_{-\boldsymbol{q}_2,p_2})(\hat{a}^{+}_{\boldsymbol{q}_2,p_3}-\hat{a}_{-\boldsymbol{q}_3,p_3})\Big\} \tag{37}$$

($\boldsymbol{k}, g$ are defined as in (29)).

It can be seen that the density matrices (36) and (37) are different even when they lead to the same mean values of the total energy of the crystal, the positions and momenta of the atoms.

The description of dynamic deformation by means of the deformation operator was based on the assumption that the deformation of the crystal involves the following transformation of the density matrix which describes the crystal

$$\hat{\varrho} \to \hat{D}(\{\alpha_{\boldsymbol{q},p}\})\,\hat{\varrho}\,\hat{D}^{-1}(\{\alpha_{\boldsymbol{q},p}\}),$$

where $\hat{D}(\{\alpha_{\boldsymbol{q},p}\})$ is a unitary operator defined from formula (1).

This assumption made it possible to find a relation between the values of the changes in the mean values of the positions and momenta of the atoms and the values of the parameters $\alpha_{\boldsymbol{q},p}$ which occur in the deformation operator. The physical sense of the above assumption is particularly conspicuous in the case of static deformation i.e. such that leads to changes in the mean values of the positions of the atoms, without simultaneously changing, however, the mean values of the momenta of the atoms.

It follows from the considerations in paper [3] that the description of static deformation by the deformation operator involves the assumption that deformation of each wave function describing the crystal can be reduced to the subtraction of the corresponding displacement vectors from the arguments of the wave function. It can be said, though not very precisely, that with deformation of this type the information is related to the form of the wave functions describing the crystal remains. The mathematical counterpart of this is the fact that no deformation represented by the deformation operator changes the value of information entropy (this can readily be shown when $\hat{\varrho}' = \hat{D}(\{a_{q,p}\}) \hat{\varrho} \hat{D}^{-1}(\{a_{q,p}\})$ is inserted into (20) and the operators which occur under the trace sign are cyclically rearranged).

The case is different with the method of the quasi-equilibrium density matrix. This method accounts indirectly for the processes which are related to the interaction between the crystal and the environment and lead to the destruction of information contained in the density matrix. It can thus be expected that the results obtained by the method of the quasi-equilibrium density matrix are closer to reality than those of the method of the deformation operator An essential advantage of the method of the deformation operator, however, is that it leads directly to the formulation of perturbation calculus.

References

[1] R. J. Glauber, *Coherent and incoherent states of the radiation field*, Phys. rev., **131**, 6, 2766 (1963).

[2] N. D. Zubarev, *Neravnovesnaya staticheskaya termodynamika*, Nauka, Moscow 1971.

[3] K. Szumilin, *The theory of externally deformed crystals at low temperatures*, Bull. Ac. Pol. Sc., sc. tec., **20**, 12, 509 (1972).

[4] M. Born, K. Huang, *Dynamical theory of crystal lattices*, Clarendon Press, Oxford 1954.

[5] J. M. Ziman, *Electrons and phonons*, Clarendon Press, Oxford 1960.

[6] J. Perina, *Coherence of light*, Nostrand, London 1972.

[7] J. R. Klauder, E. C. G. Sudarshan, *Fundamentals of quantum optics*, Benjamin, New York 1968.

[8] W. H. Louisell, *Quantum statistical properties of radiation*, J. Wiley, New York 1973.

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C H R O N I C L E

THE INTERNATIONAL CONFERENCE NOISE CONTROL 82

The International Conference NOISE CONTROL 82 took place in Krakow on 20-22 September, 1982. It was organised by the Commitee on Acoustics of the Polish Academy of Sciences, the Polish Acoustical Society and the Institute of Mechanics and Vibroacoustics of the Academy of Mining and Metallurgy, in cooperation with the Central Institute of Occupational Safety, the Institute of Building Technology and the Institute of Fundamental Technological Research.

NOISE CONTROL 82 was the sixth scientific conference devoted to noise and abatement in our country. The preceding conferences were organised in 1964, 1970, 1973, 1976 and 1979, mainly headed by Prof. Stefan Czarnecki, the vice-chairman of the Organizational Commitee of the present conference. The conference was attended by 152 participants, 28 of whom were representatives of Bulgaria, Czechoslovakia, Denmark, Holland, the GDR and Hungary. The sessions of the conference were divided into four problem groups: *A.* Methods of reducing noise and vibration of industrial machines and appliances, B. Noise in buildings, C. Communication noises, D. Physical foundations of the emission and propagation of sound and methods of analysis and measurement. During the conference 64 lectures were delivered, out of which 8 were plenary lectures, 35 were delivered in the problem groups and 23 in poster sessions. The following lectures were delivered during the plenary meetings:

Z. Engel, *Noise and vibration control in Poland.*
Z. Engel, *Professor Stefan Czarnecki – an eminent Polish vibroacoustician.*
P. Brüel, *Sound intensity measurement.*
P. Brüel, *Room acoustic speech transmission index.*
O. J. Pedersen, *Measurement and description of environmental noise.*
W. Peutz, *Accuracy in noise control measurements and calculations.*
F. Dul, R. Gutowski, R. Maroński, J. Pietrucha, *Relationships between transient aerodynamics and classical acoustics in the aspect of noise radiation by vibrating bodies.*
J. Kirejczyk, *Method for diminishing cavitation noise.*

The following lectures were delivered in the group devoted to the problems of abating noise and vibration of industrial machines and appliances:

D. Augustyńska, *Methods for diminishing infrasound noise.*
S. Bednarz, J. Piotrowski, *Application of shot-oil eliminators in vibrations.*
J. Brewiński, A. Widota, *Possibilities of diminishing machining tool sound levels.*
Z. Engel, H. Panuszka, M. Menżyński, *Investigations of the acoustic field of reduction systems of gas stations.*
Z. Głowacki, *On a certain method for optimizing vibroinsulation systems.*
A. Gołaś, *Analysis of the possibilities of minimizing the vibration of a roller table strip.*
A. Gołaś, J. Kowal, *Synthesis of an optimal controlled insulator.*

J. Kowalak, C. Cempel, *Influence of machine condition on noise level.*
J. Kowal, M. Szepski, *Dynamic synthesis of a passive system of controlled vibroinsulation.*
S. Kulczycki, *On constructional problems of metal-cutting circular saws in the noise criterion aspect.*
B. Niewczas, W. Stojanowski, A. Troszok, *Aerodynamic noise of an air hammer.*
K. Prync-Skotniczy, *Method for calculating the acoustic frequency response of a centrifugal fan.*
J. Sajdak, *Acoustic investigation of a screen with a polyurethane sieve.*
M. Stankov, *Sumovye kharakteristiki i metody znizhenya suma i energeticheskoy promyslennosti.*
K. Tomaszewski, J. Felis, *Criteria and indexes for vibroacoustic evaluation of machines and appliances.*
D. Waszkiewicz, *Methods for diminishing the vibroacoustic activity of machines.*
J. Zydroń, *Some possibilities of diminishing noise in Diesel engines.*

In the group concerned with noise in buildings, the following lectures were delivered:

K. Ciesielska, *Criteria of evaluation and requirements concerning the acoustic insulation of apartment doors.*
A. Iżewska, *Subjective method for evaluating the acoustics of apartment buildings in terms of installation noises.*
Z. Mietliński, *Economic aspects of shaping the acoustic conditions.*
W. Odrzywołek, *Acoustic problems related to the MWWM mechanical ventilation system.*
C. Puzyna, *Methods for evaluating the acoustic quantities of large-volume halls.*
O. Simkova, *Dynamical properties of shell-like enclosures.*
M. Stawicka-Wałkowska, *Analysis of the acoustic properties of "mini" housing estate interiors in the aspect of development in urban planning.*
B. Szudrowicz, *Acoustic insulating power of massive double partitions.*
D. Trynkowska, *Designing ear muffs on the basis of empirical dependencies.*
E. Zalewska-Paciorek, *Digital simulation of selected parameters of the acoustic field.*

The following lectures were devoted to communication noises:

J. Adamczyk, Z. Strzyżakowski, Z. Stojek, *Investigations of the vibroacoustic effects in Warsaw underground crosstown railway line.*
S. Czarnecki, W. Nowakowski, *Investigations of the conditions of vibroacoustic energy propagation on Poniatowski Bridge in Warsaw in real and model conditions.*
R. Danecki, *Evalvation of the results of street-noise research in Częstochowa.*
J. Ejsmont, S. Taryma, *Tire/road, noise road and laboratory measurements relationship.*
R. Kucharski, *Relationship between L_{eg} multi-hour values and short-period values of road noise levels.*
J. Miazga, K. Janicka, *Noise of road vehicles used in Poland in the light of certification investigations.*
M. Wojciechowska, M. Kraszewski, *Initial attempts to evaluate tramway transport in relation to noise emission.*

The following lectures were devoted to the problems of physical foundations of the emission and propagation of sound and methods of analysis and measurement:

J. Adamczyk, M. Łabno, *Envelope analysis of vibroacoustic signals.*
K. Arczewski, R. Gutowski, K. Kulimicz, J. Pietrucha, *Application of sensitivity analysis for modifying the acoustic field of vibrating plate elements.*
J. Bednarski, *Uses of resonance density functions in investigations of the vibrations of continuous systems.*
W. Cholewa, *PAS4/CAMAC — a programmed analyser of signals.*
W. Dąbrowski, W. Marcinkowski, M. Ogorzałek, *Sensitivity analysis of mechanical systems by the coupled-system method.*

A. Gołaś, W. Wszołek, *Investigations of the relationship between the mean velocity of a vibrating surface and the level of emitted noise.*

A. Izworski, R. Tadeusiewicz, *Computer method of extracting noise signal formants.*

J. Kaźmierczak, *Use of an acoustic signal in determining the wear of electric-arc furnace roofs.*

A. Kołodziejski, E. Kozaczka, *Investigations of the vibration and acoustic pressure of a mechanical vibrator.*

E. Kozaczka, *Homomorphous processing of acoustic signals propagated in a bounded medium.*

T. Kwiek-Walasiak, *Acoustic power determination by the intensity method.*

G. Meltzer, *New results on the dynamical behaviour of the human body in sitting position.*

R. Panuszka, T. Uhl, *Analysis of accuracy pressure near-field method for noise source identification.*

W. Rdzanek, *Acoustic resistance of a circular plate for hyper-resonance frequencies.*

J. Rosiński, T. Uhl, *Selected problems of shock noise.*

J. Stenička, *Determination of structure-born noise transmission for machines in dwelling building constructions.*

S. Szyszko, *Methods of constructing spectra patterns of the operational state in the diagnostics of a gear pump.*

R. Tadeusiewicz, *Computer methods for noise signal analysis.*

T. Uhl, H. Łopacz, *Identification of the dynamic properties of mechanical systems by the pulse method in terms of diminishing their vibroactivity.*

R. Zakrzewski, *Post for manual testing of manual tools.*

In evaluating the conference, one can say that, in spite of the state of war, NOISE CONTROL 82 allowed to sustain the scientific activity in the domain of environmental noise control. Nevertheless, the present year did not bring forth the annual open seminar on acoustics. The conference demonstrated a great vigour on the part of the acoustic community in further integration of such organisations as the Commitee on Acoustics, the Polish Acoustical Society, research institutes, design offices and production plants. The numerous sessions and broad discussions allowed for a significant exchange of information and enlarged the knowledge of phenomena related to noise and vibration and countermeasures. It can be readily ascertained that there was a healthy interplay between the results of fundamental and theoretic research and the results of applied work. The discussions that took place after and between lectures showed the necessity for continuing fundamental research, for it provides the basis for applied research and, finally, practical employment. The need for interdisciplinary research in environmental noise control was much stressed. An exposition of Brüel – Kjaer equipment was organised during the conference. From a chronicler's viewpoint one should note two facts. Firstly, Prof. Ignacy Malecki, an eminent Polish acoustician and director of the Institute of Technological Research of the Polish Academy of Sciences, received the title of doctor honoris causa of the Academy of Mining and Metallurgy during the latter's Senate meeting, which took place on 23 September, 1982; secondly, the conference was preceded by the opening of a modern vibroacoustic complex with a large number of laboratories, workshops and lecture rooms.

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